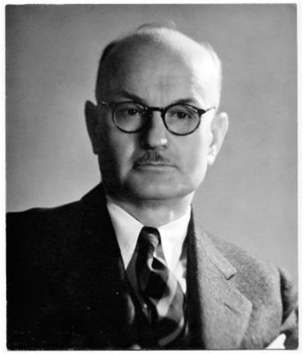
Topological approach to modeling spatial cognition

Yuri Dabaghian

Rice University

How does brain represent space?

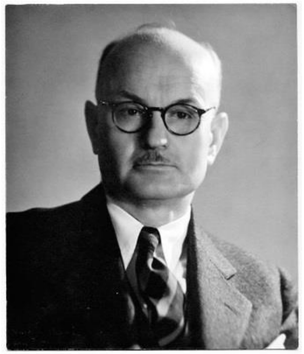
Cognitive map concept



E. Tolman, 1947

Cognitive map – an internalized representation of the environment, that enables spatial navigation and spatial planning

Cognitive map concept



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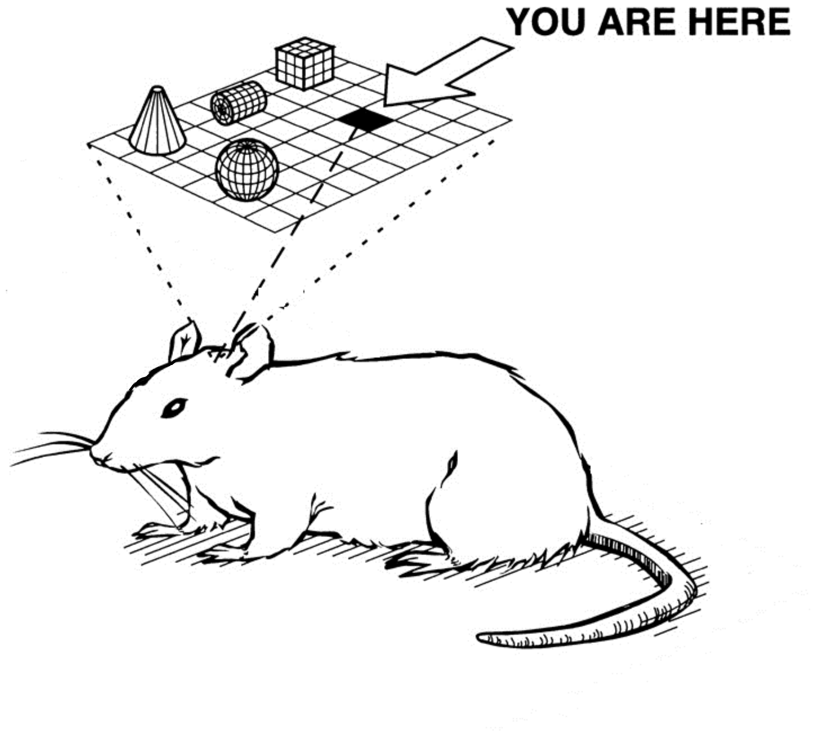
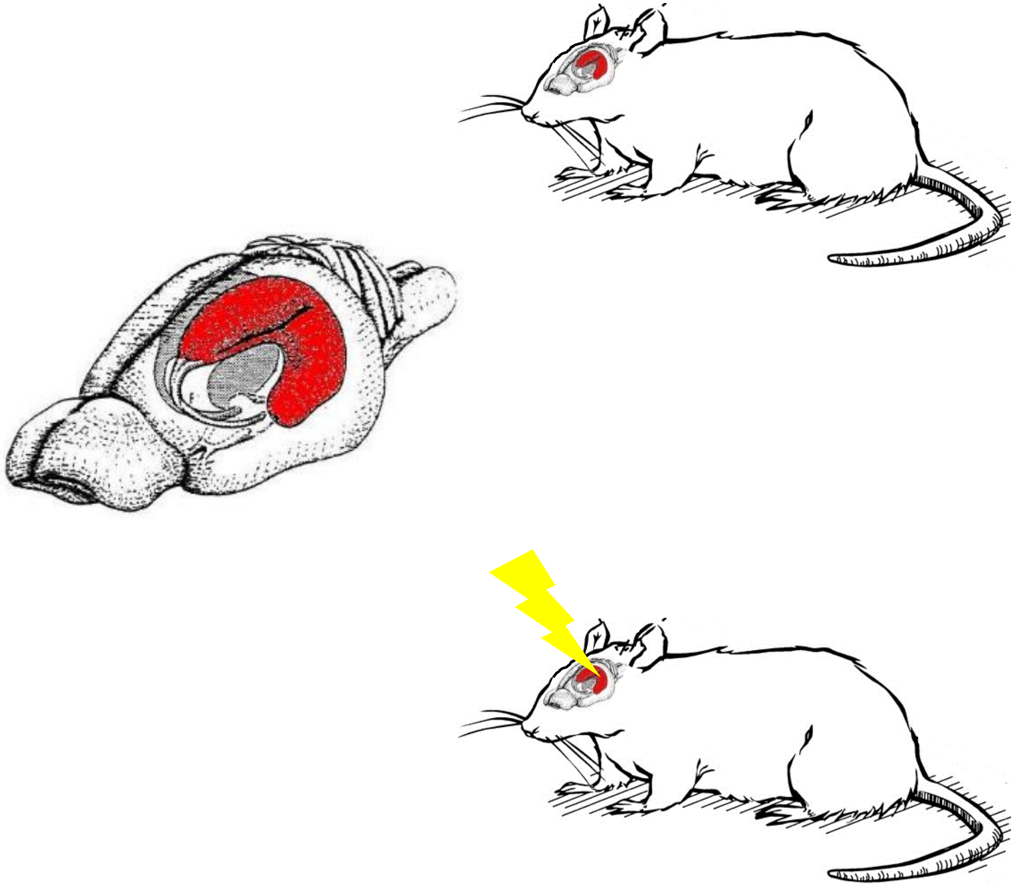
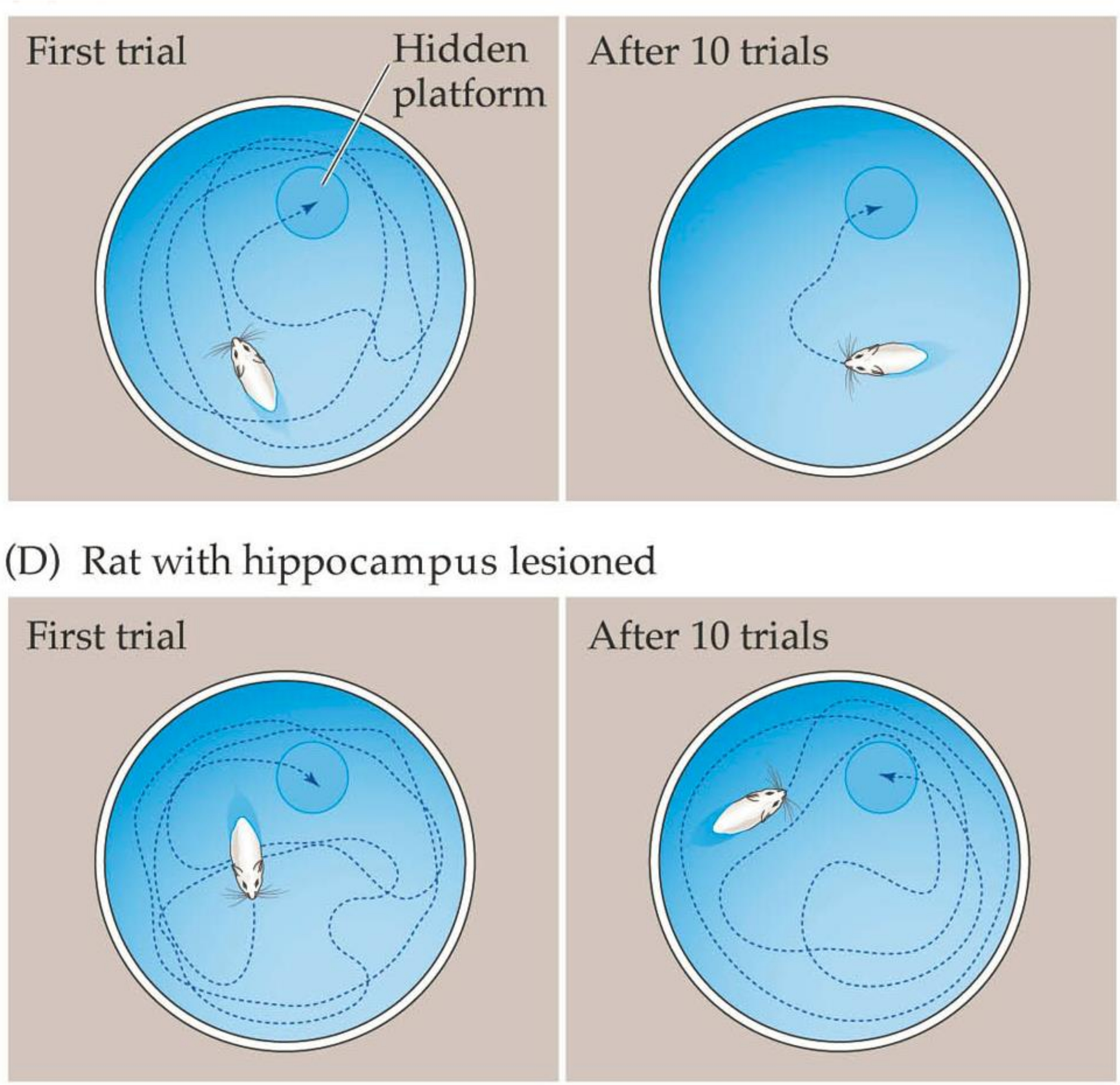
Cognitive map – an internalized representation of the environment, that enables spatial navigation and spatial planning



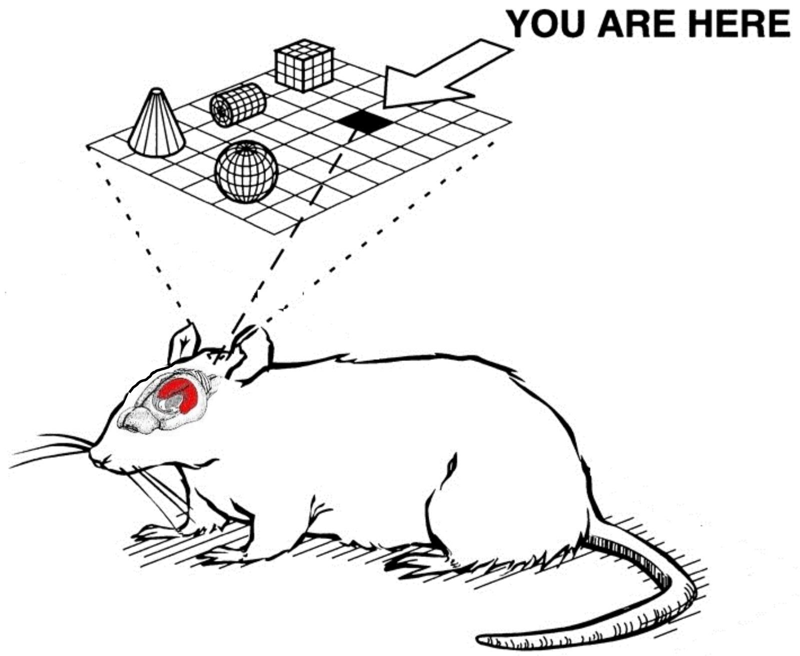
Where is it located?

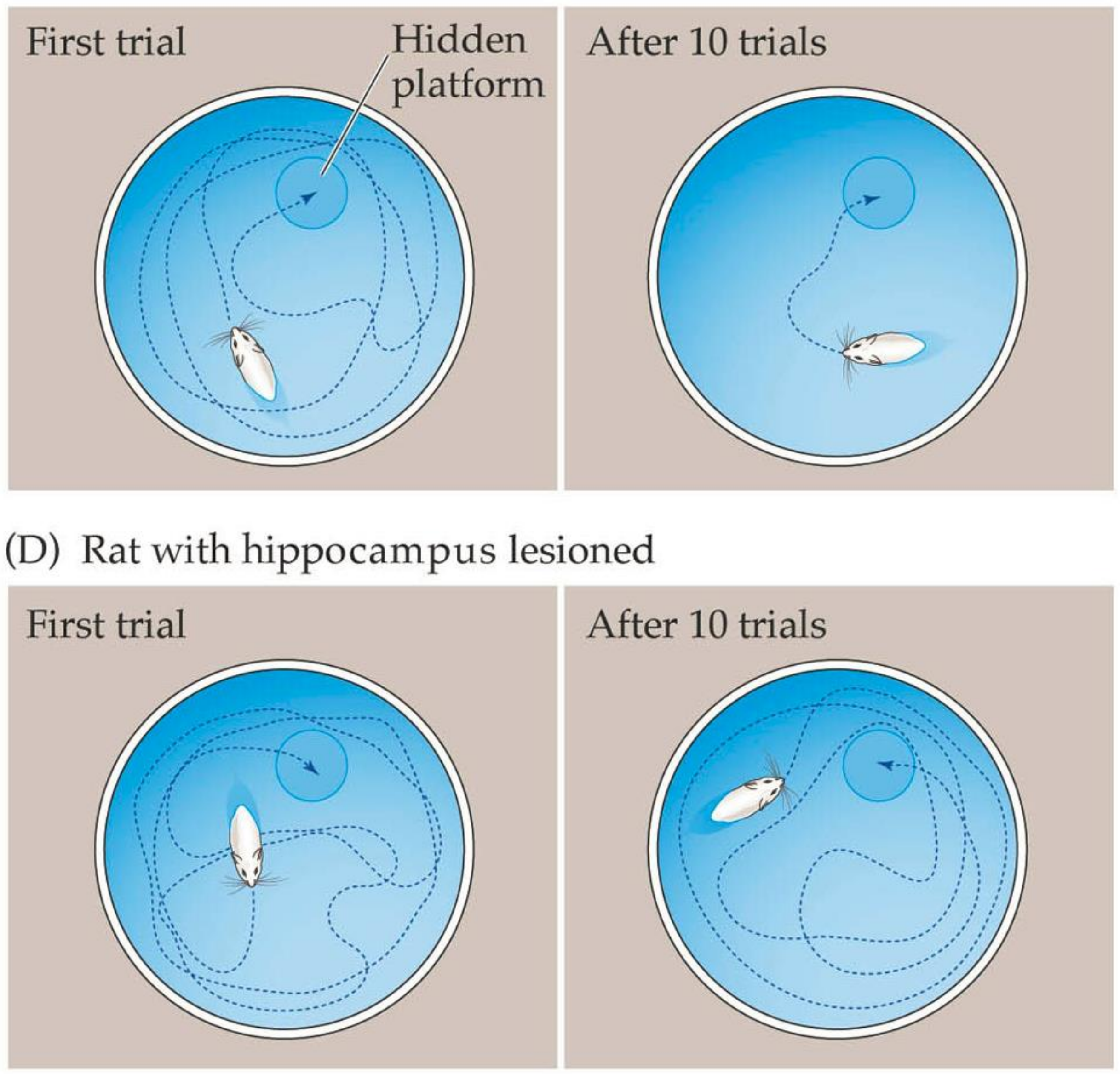
Hippocampus and space coding

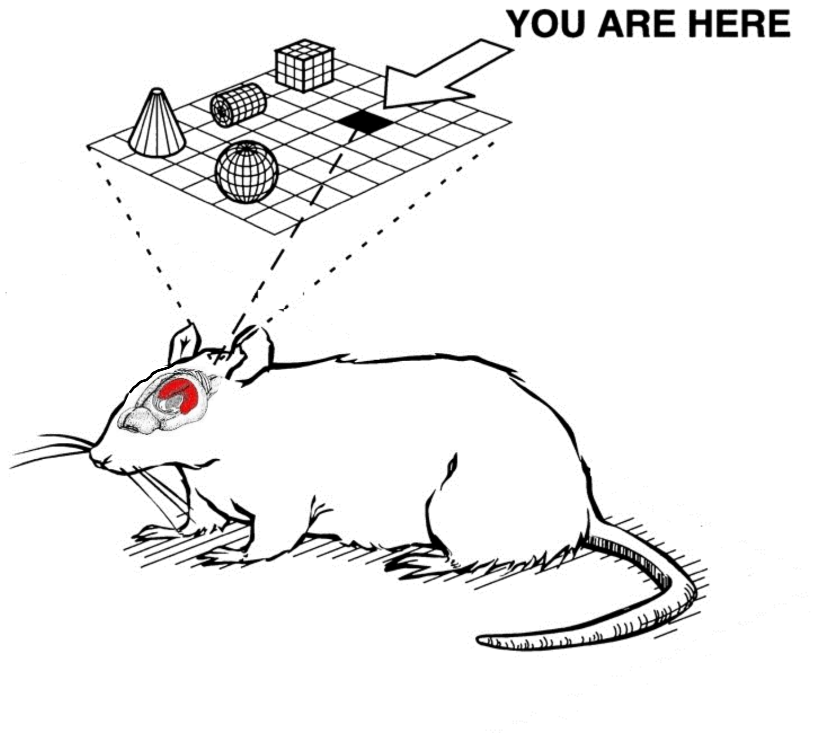




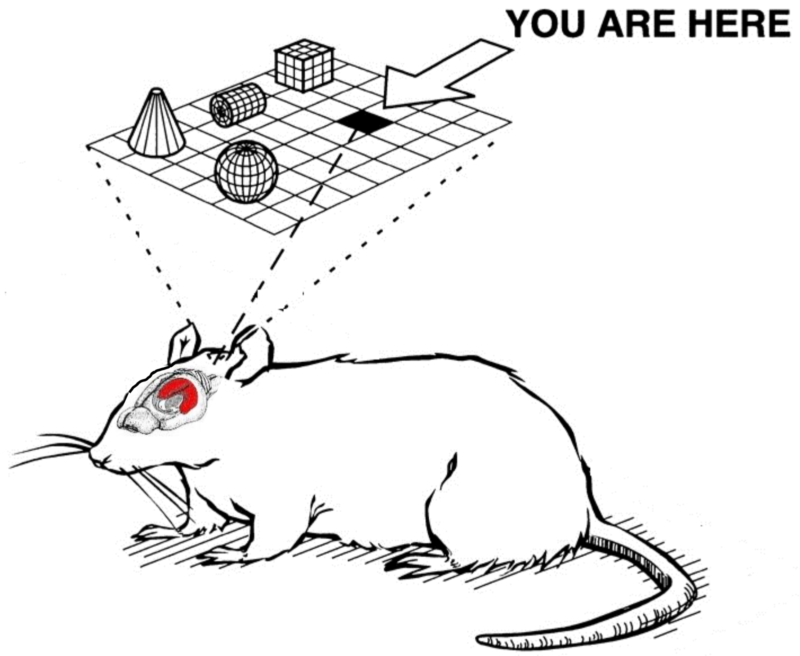
Hippocampus and space coding



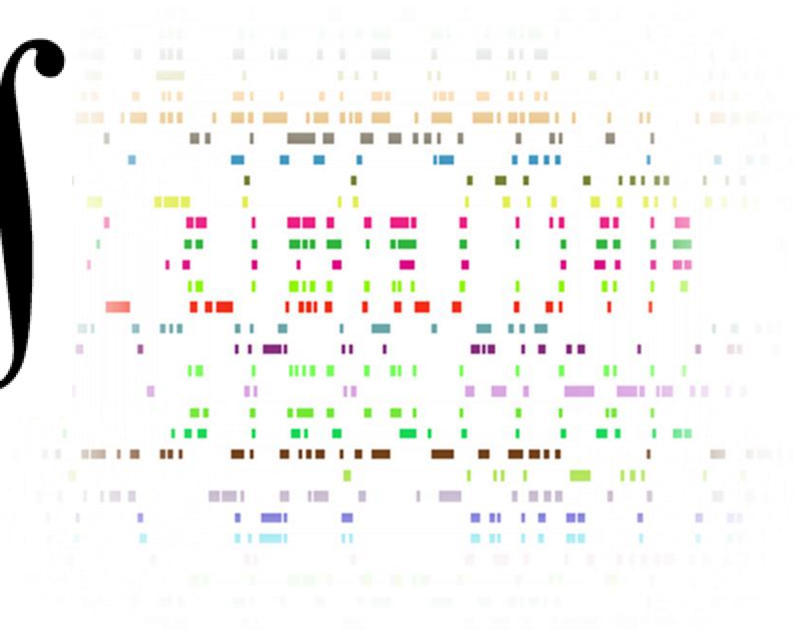




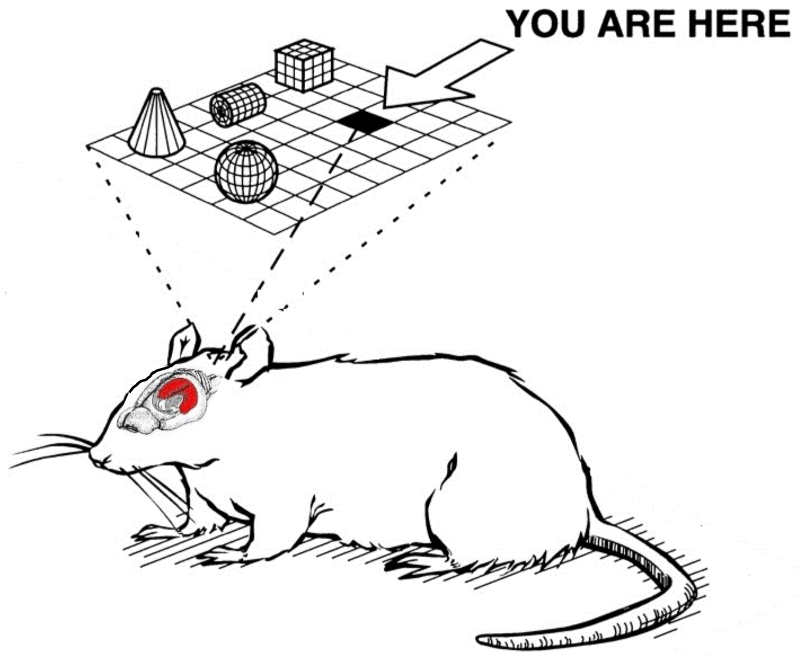
How a cognitive map is produced by the hippocampal network?



$$= \iiint$$



Cognitive representation of space emerges from spiking activity



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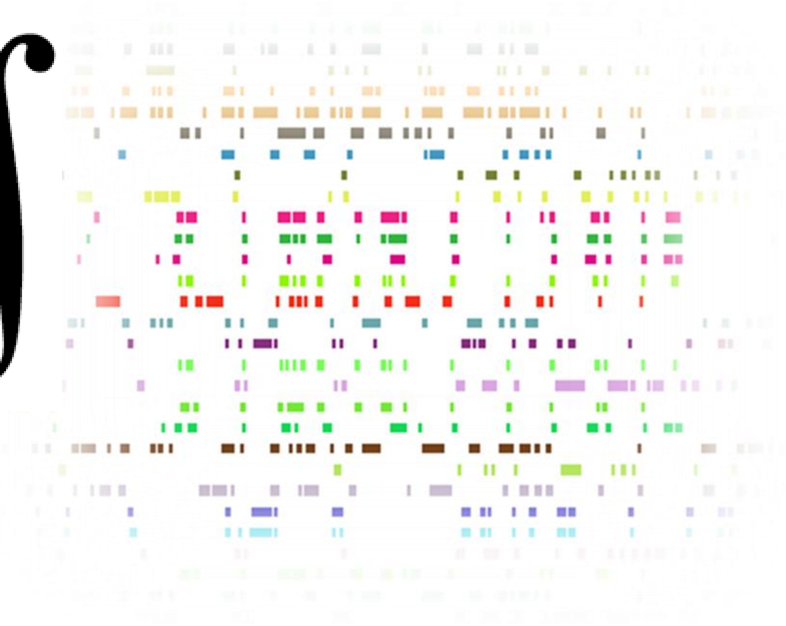
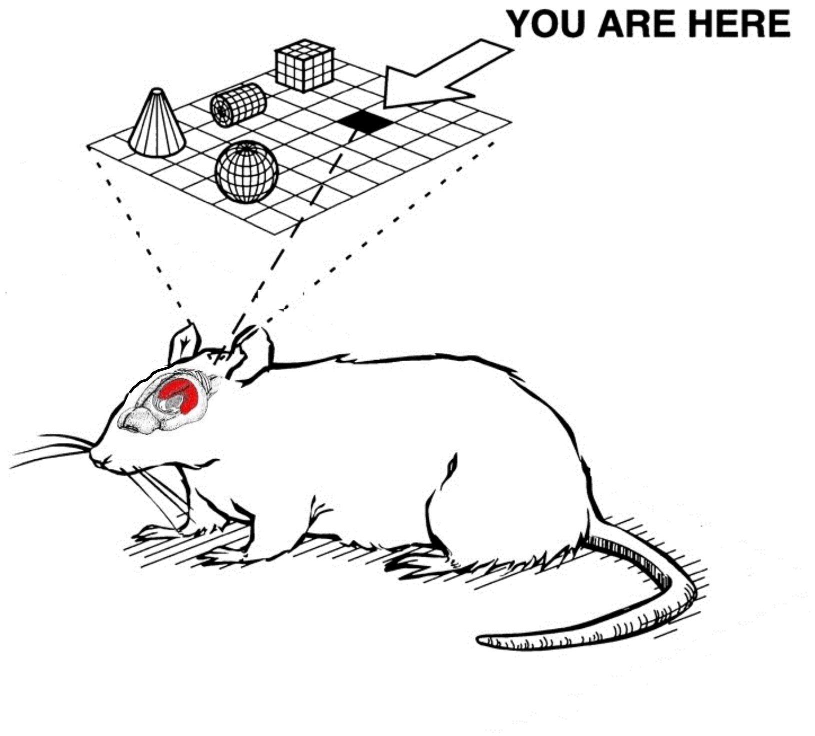


What is the mechanism?

Properties of the cognitive map

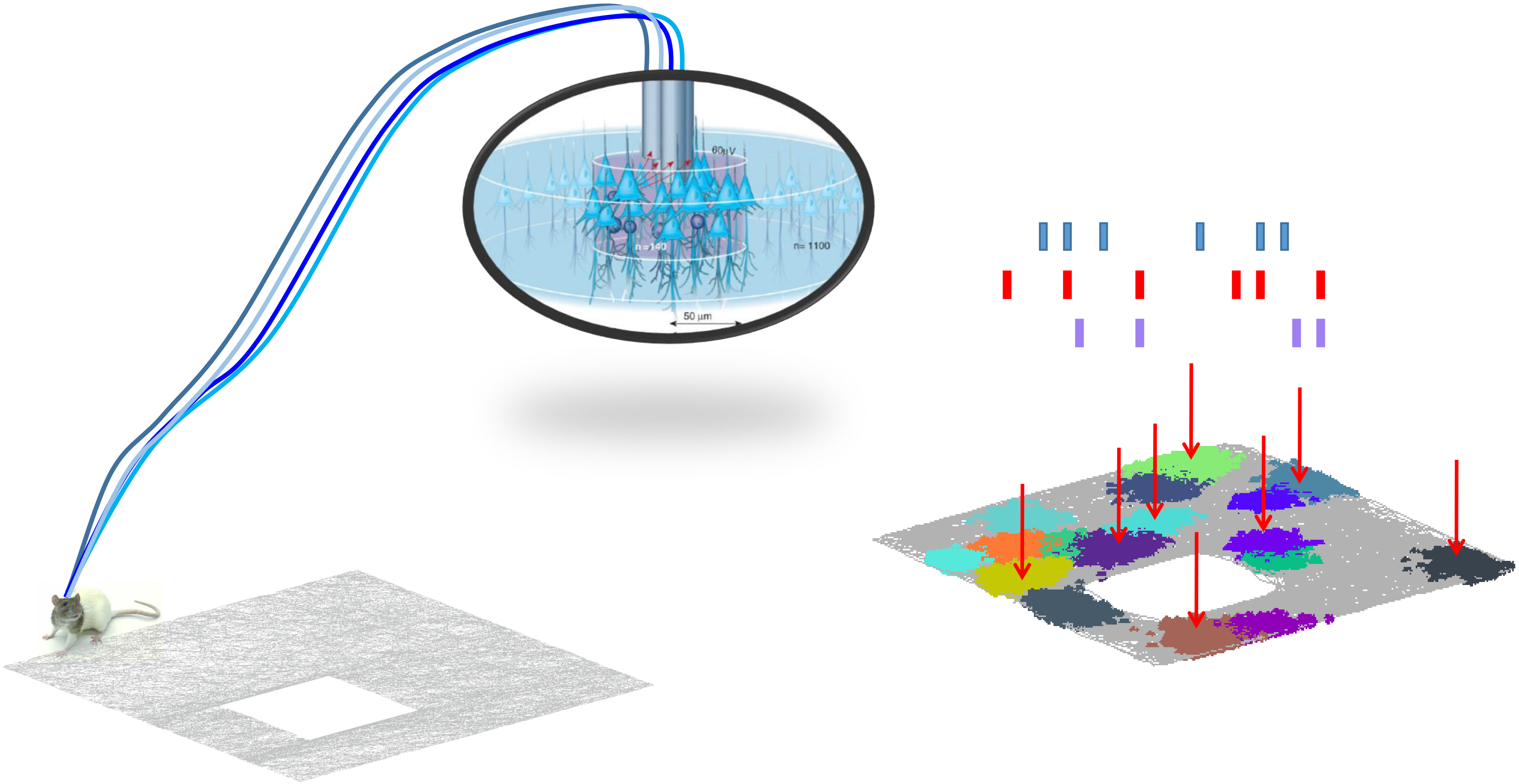


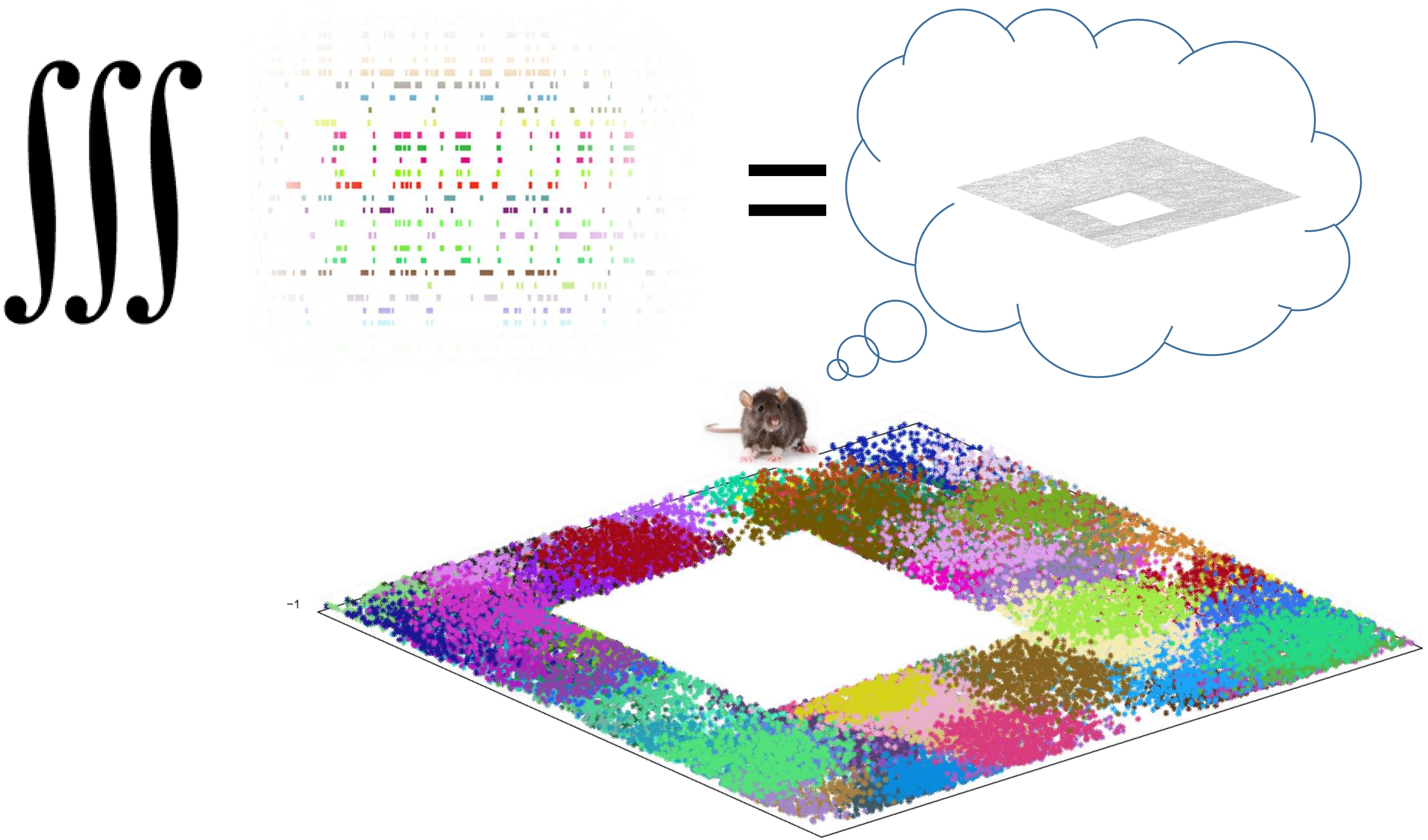
1. What information is represented in the cognitive map?

Distances? Other metrics? Directions? Locations? Spatial order?

2. How this information is read out and processed by the downstream networks?

In vivo recordings of neuronal activity

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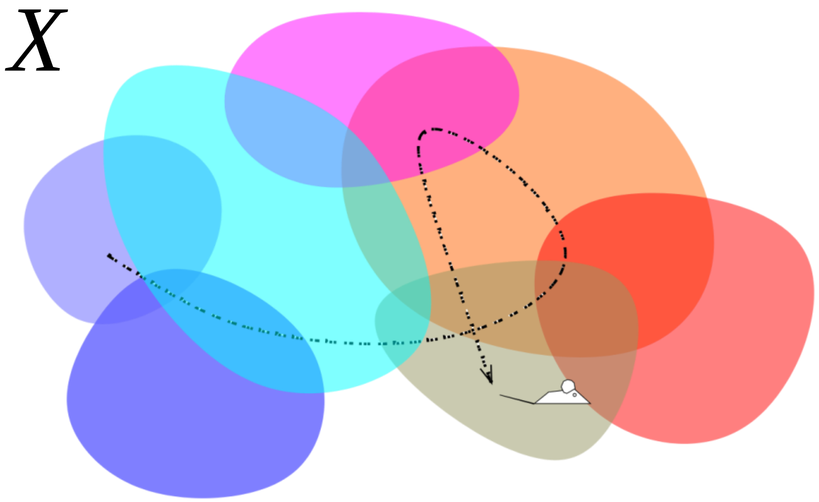
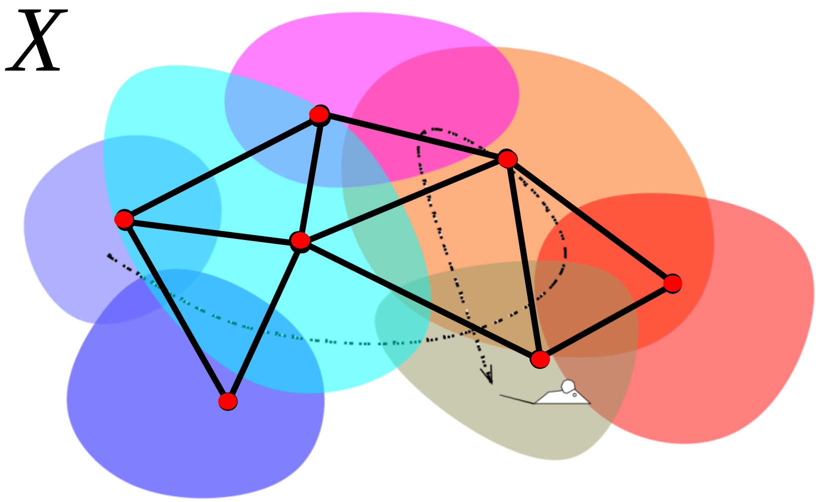
How to proceed?

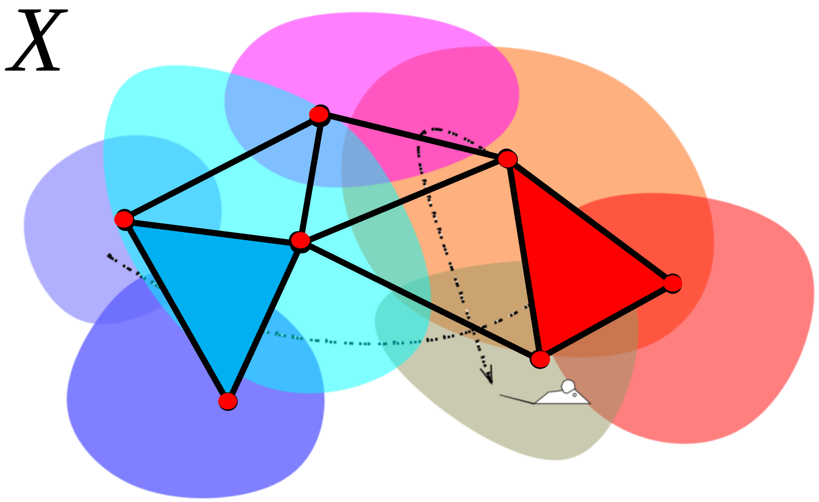
Alexandrov-Čech theorem



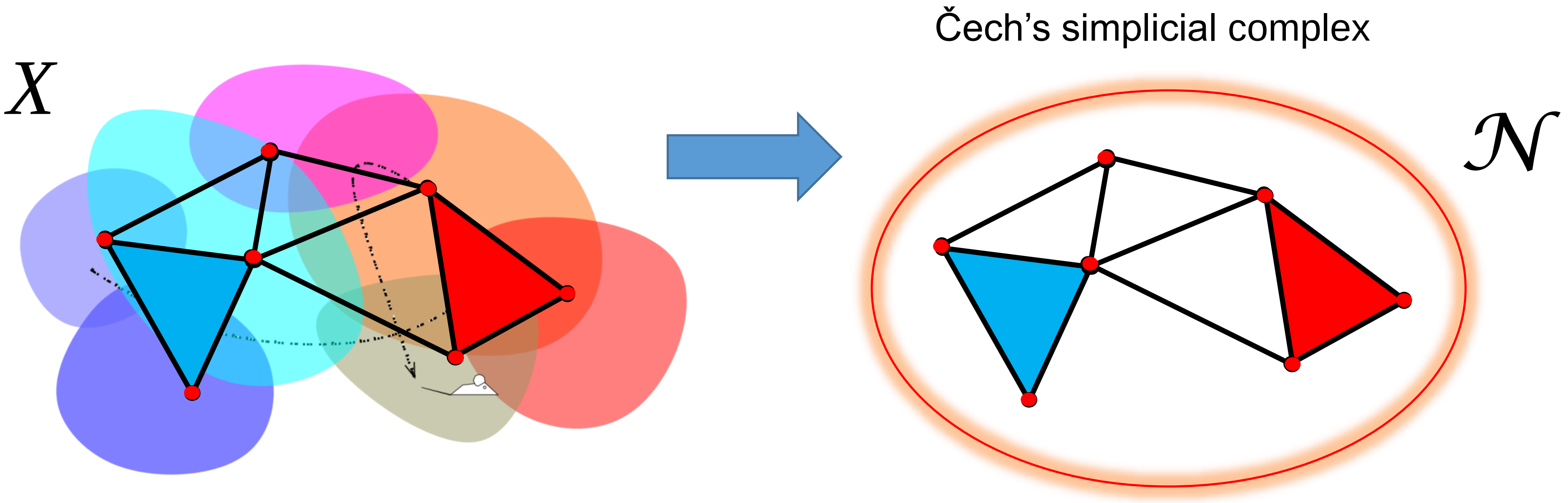
Alexandrov-Čech theorem



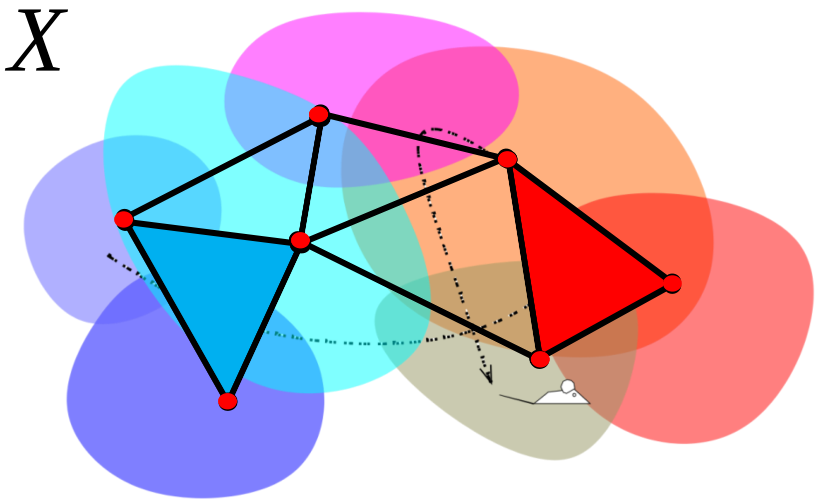
Alexandrov-Čech theorem



Alexandrov-Čech theorem: $H_*(X) = H_*(\mathcal{N})$



Alexandrov-Čech theorem suggests how spiking information could be integrated



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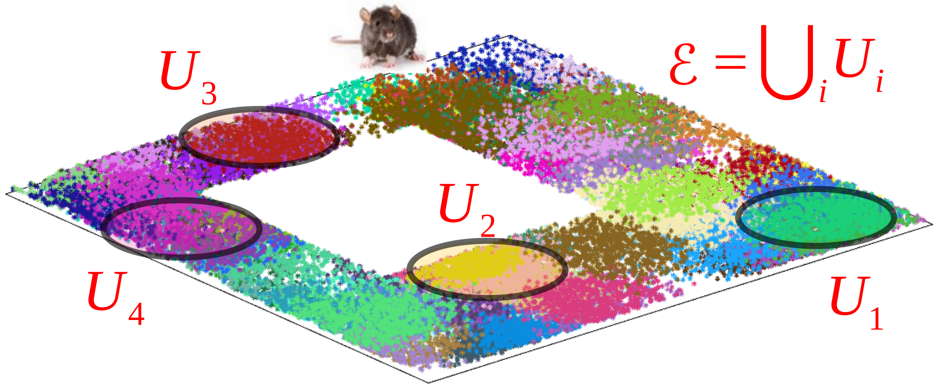
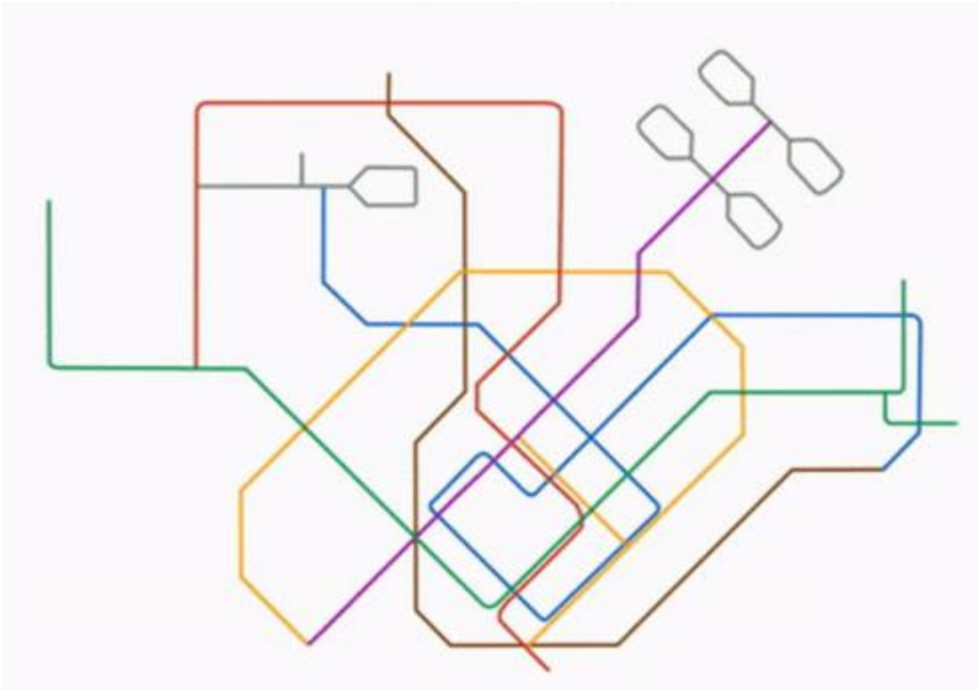
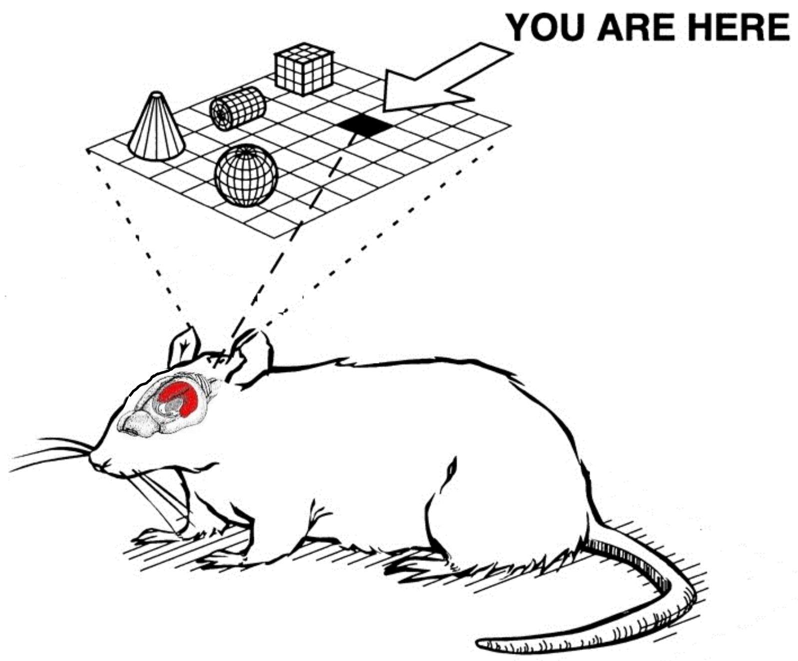


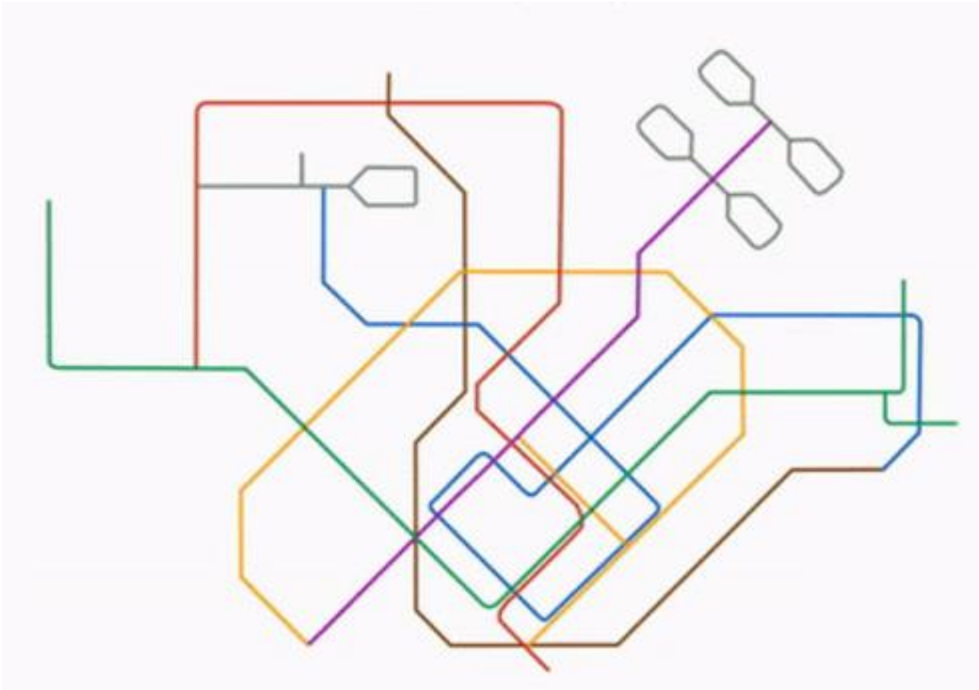
Does it also suggest how hippocampus works?

Topological map



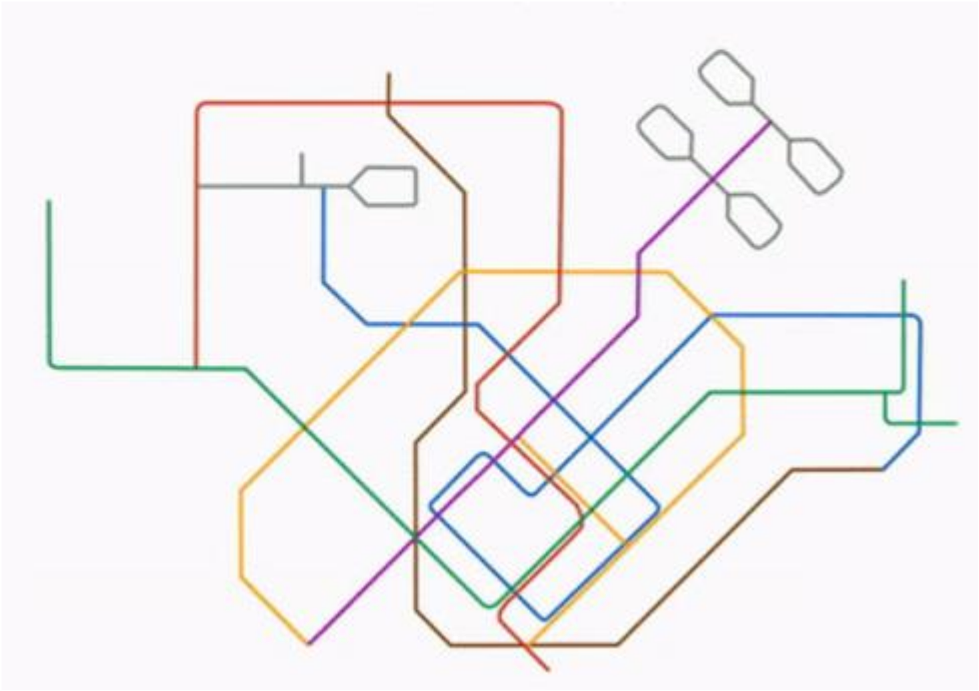


Topological map

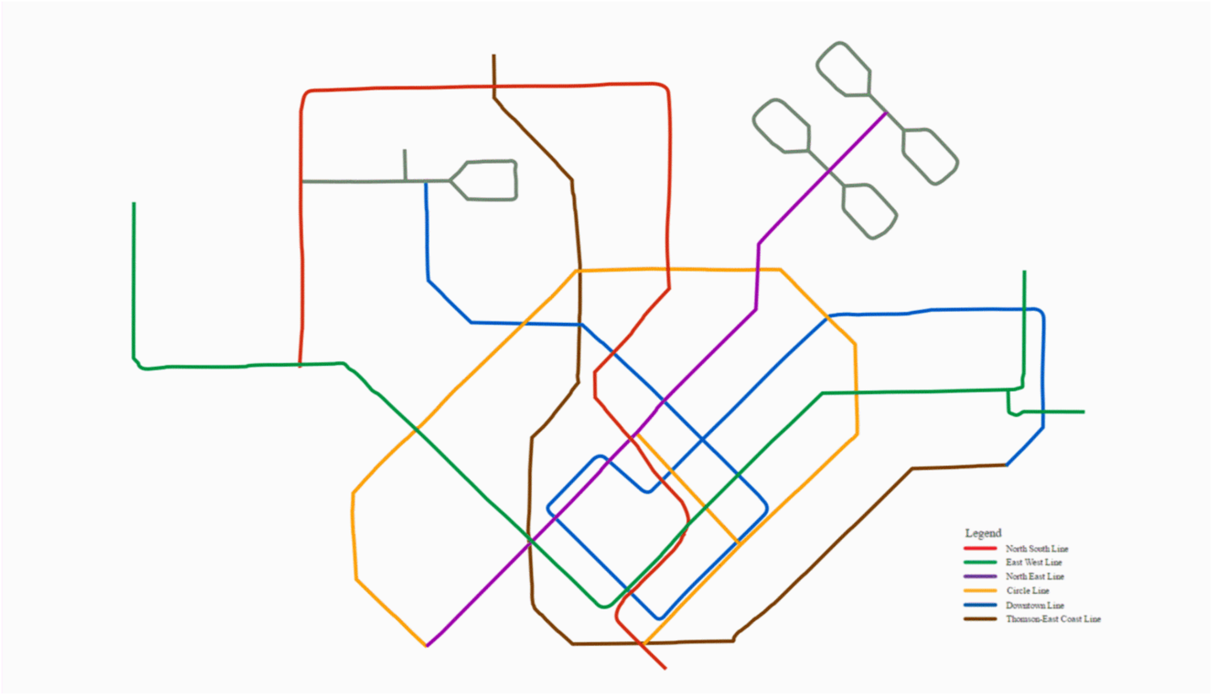


Geometric map

Topological map

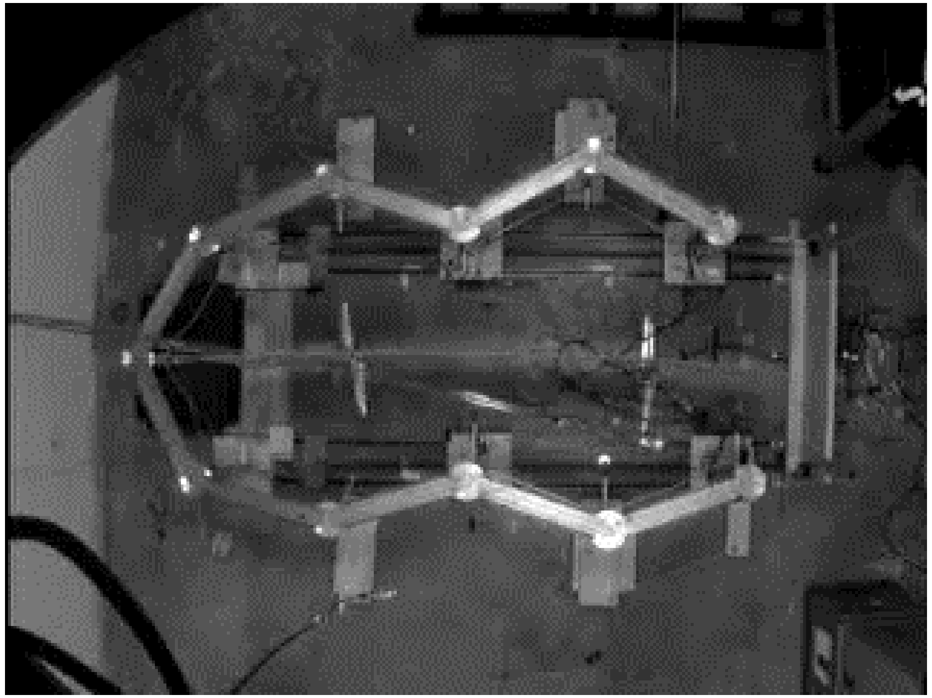


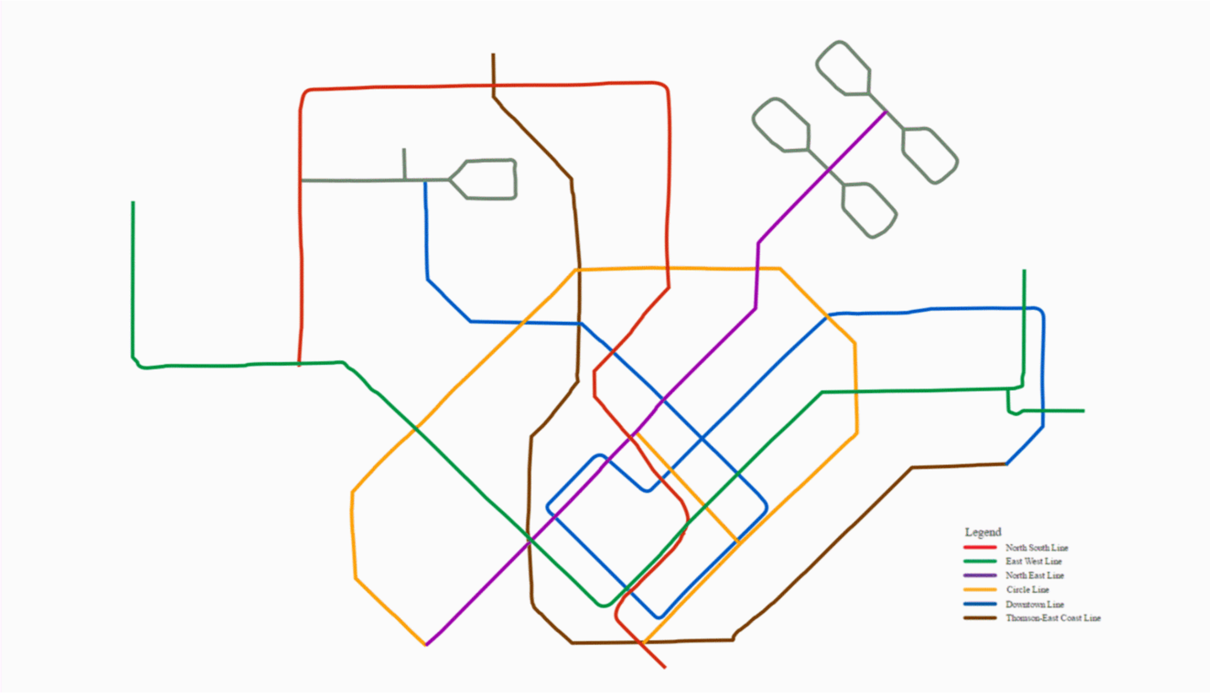
Geometric invariance



Topological map

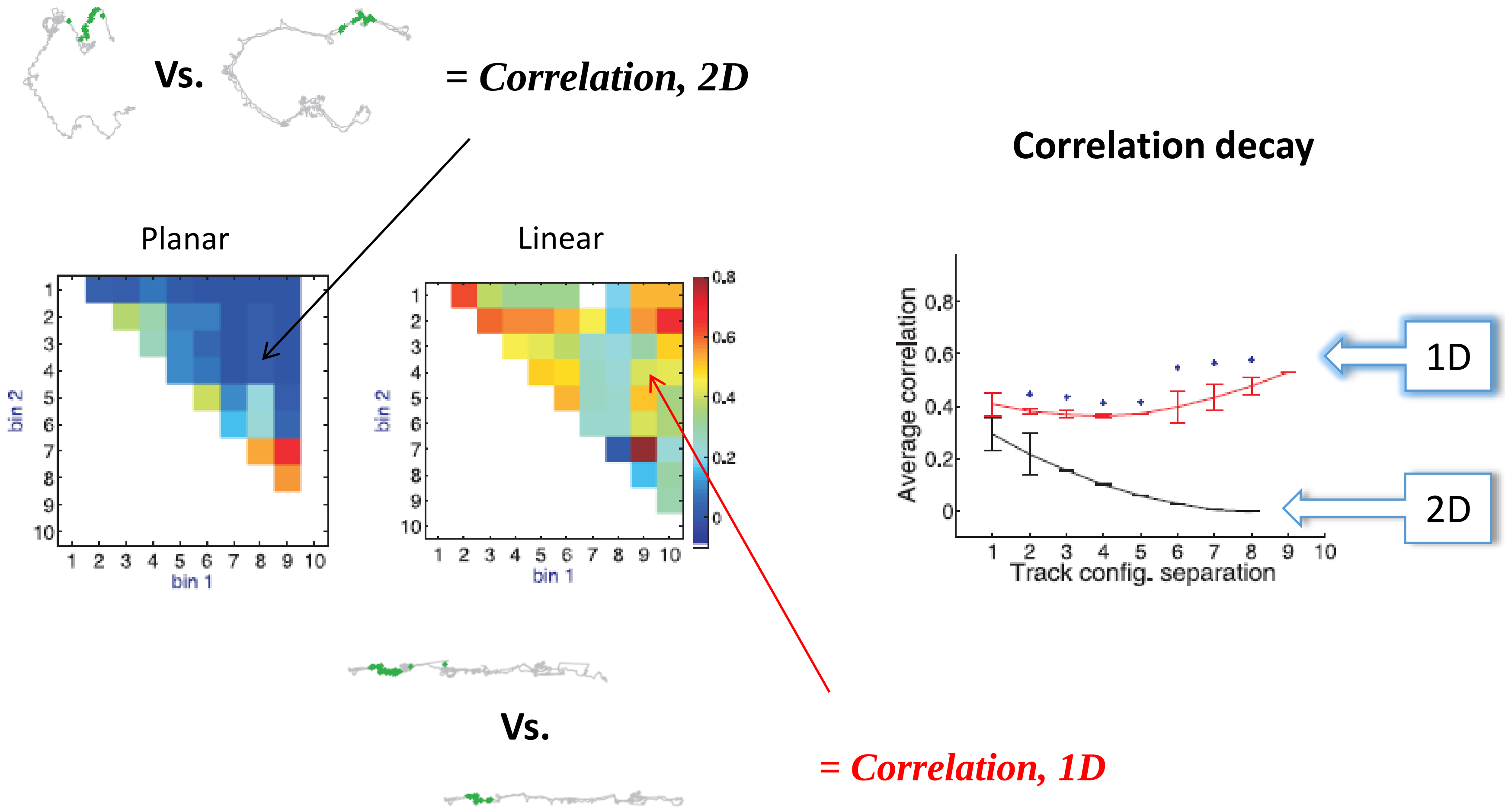
Geometric invariance

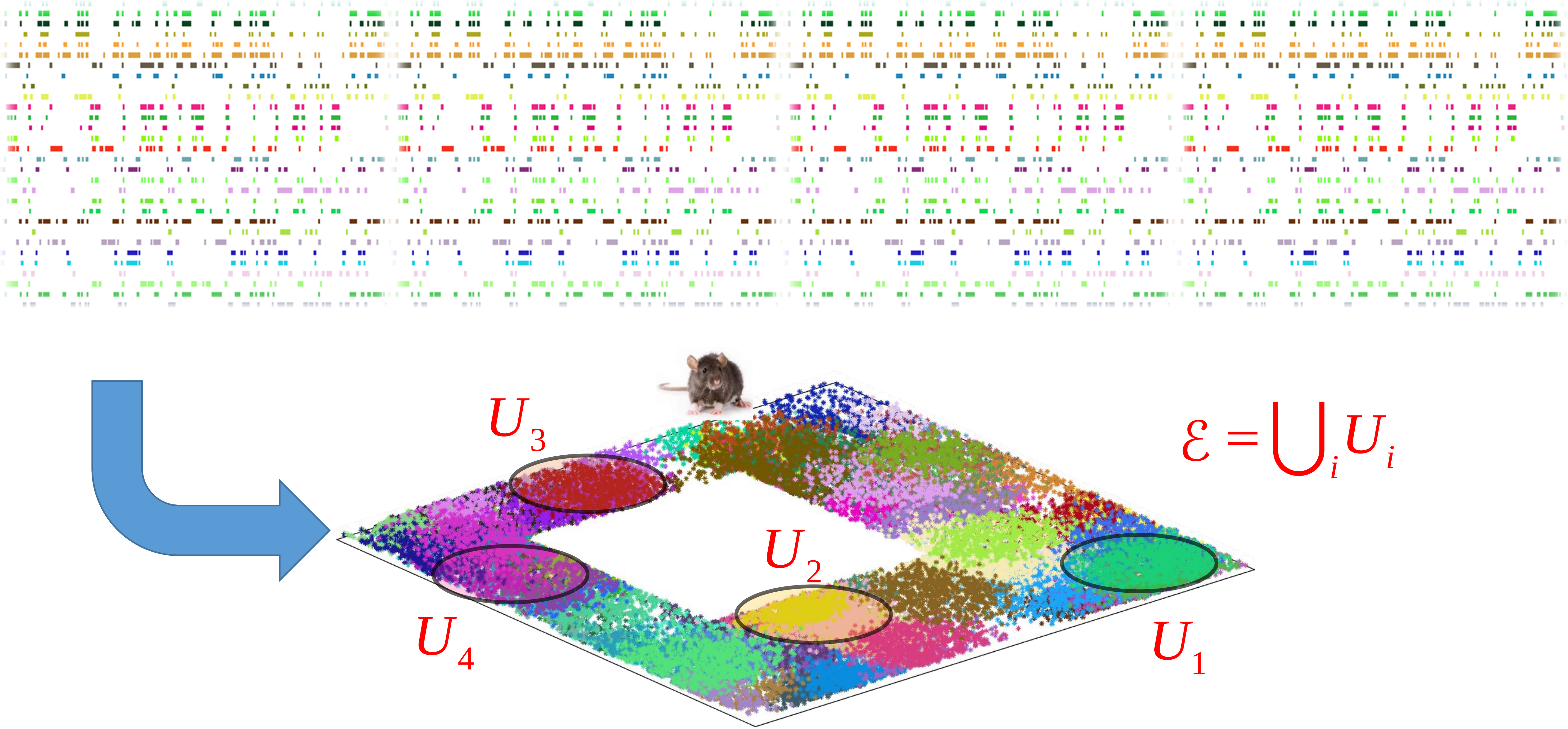




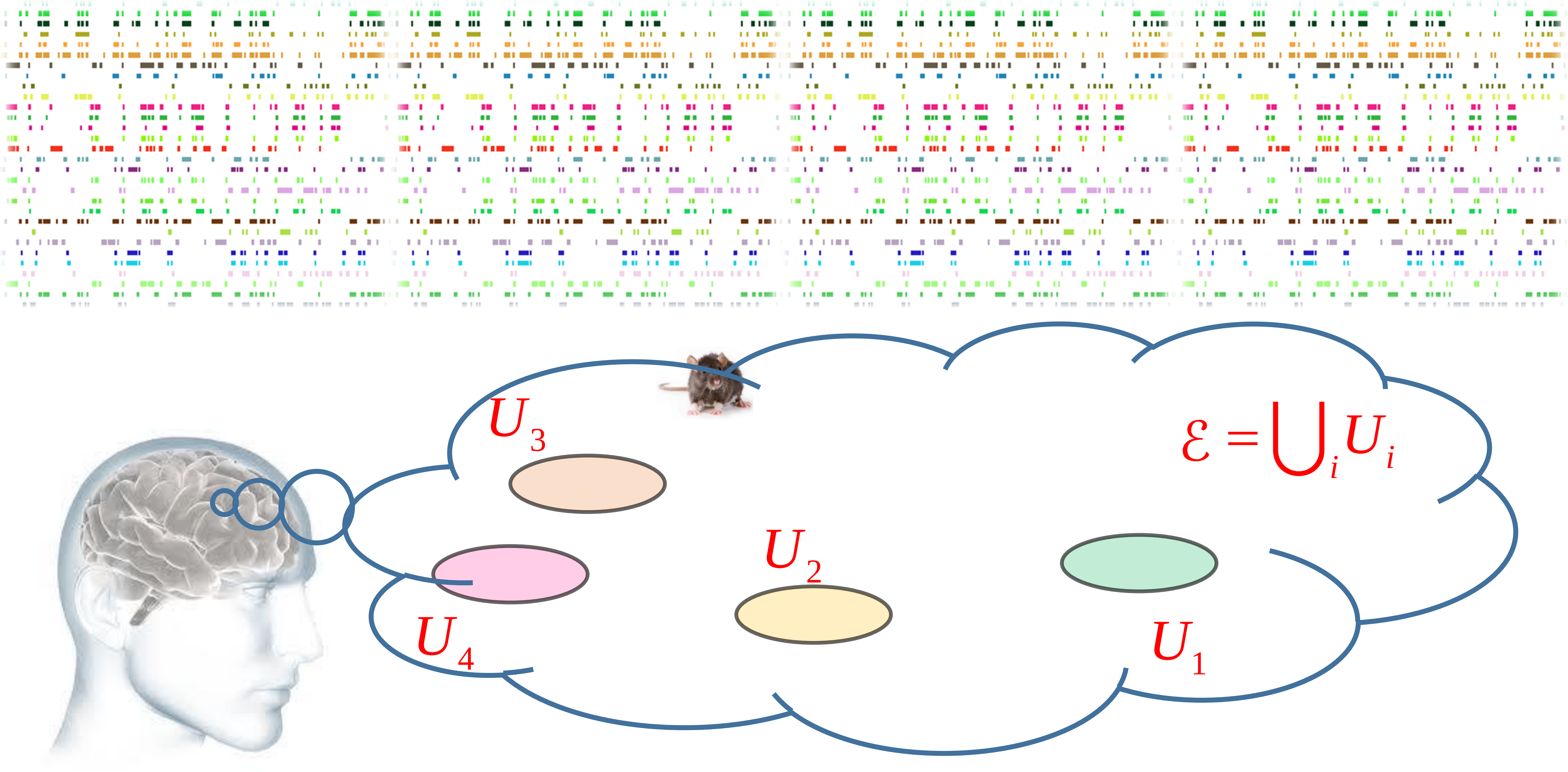
Dabaghian et al, eLife 2014

2D versus 1D spatial frame





U_3
$\mathcal{E} = \bigcup_i U_i$
U_2
U_4
U_1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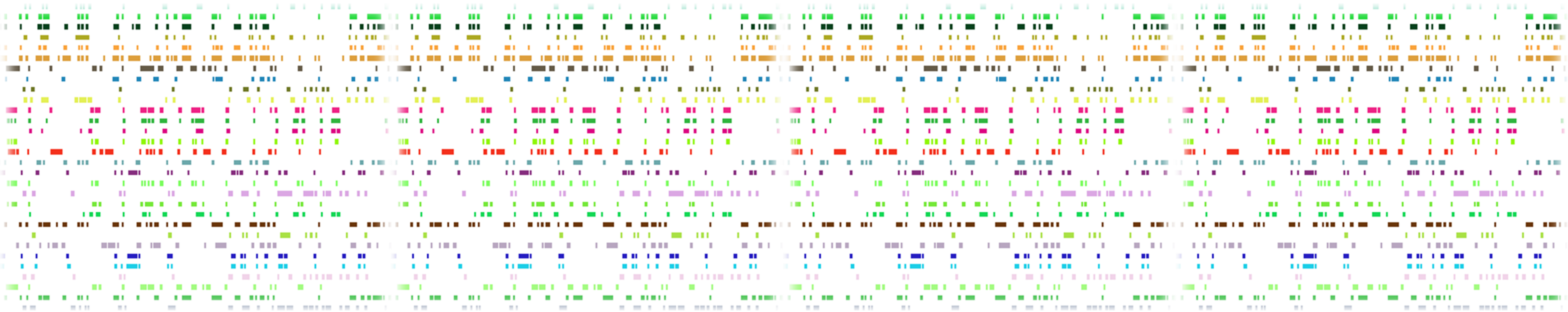
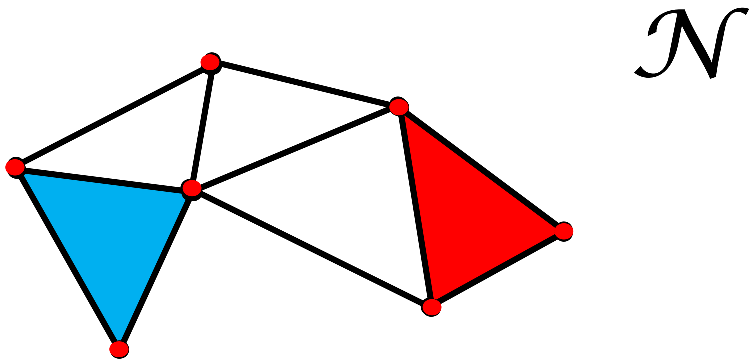
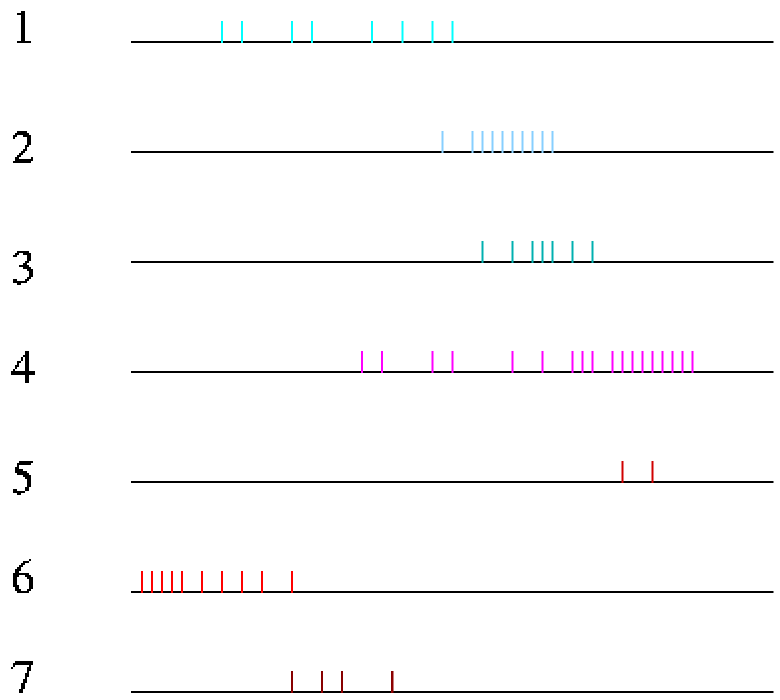


Čech's simplicial complex from spikes



Spikes → Space reconstruction



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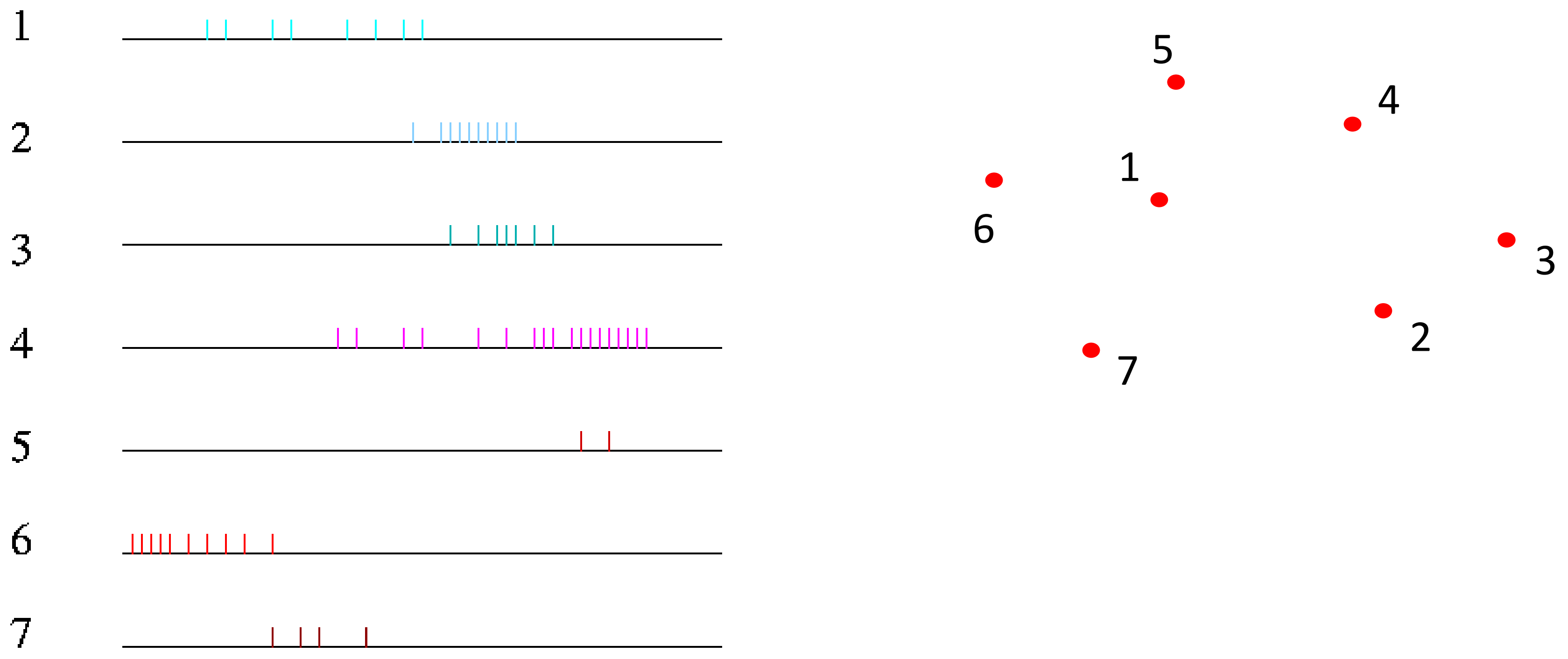
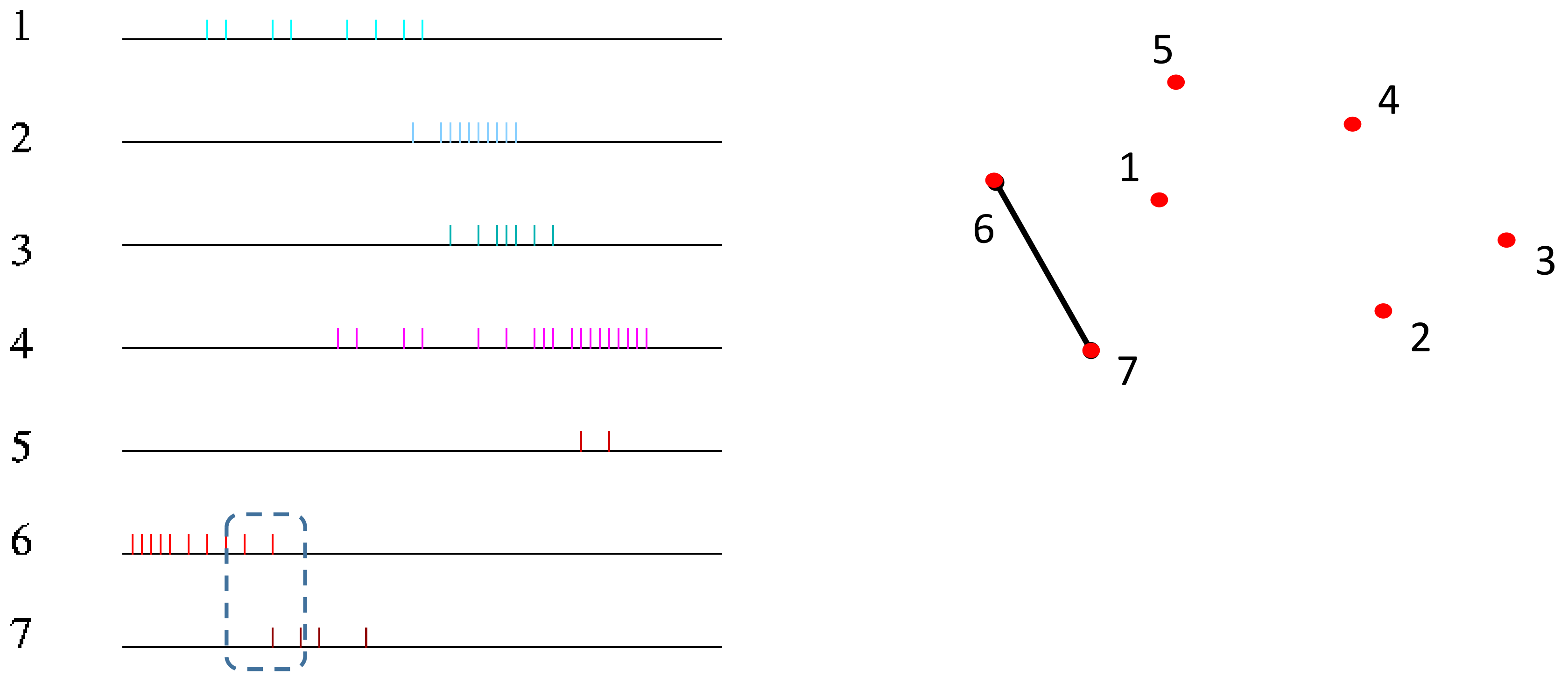


Spikes → Space reconstruction



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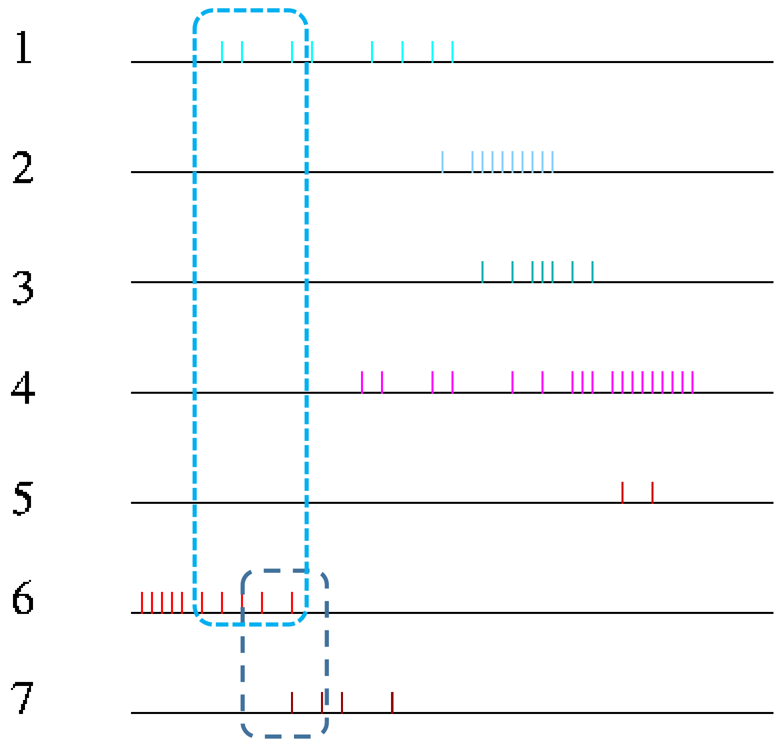

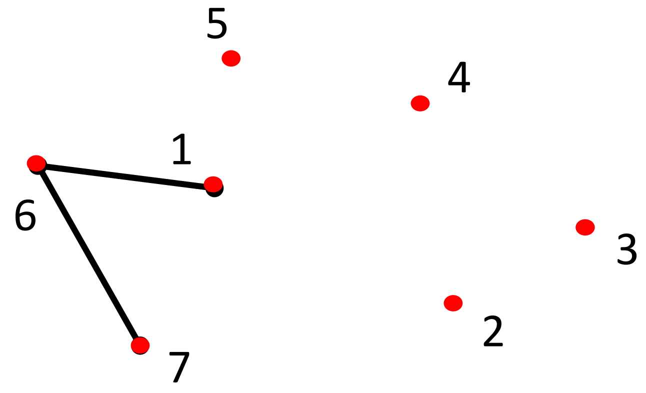

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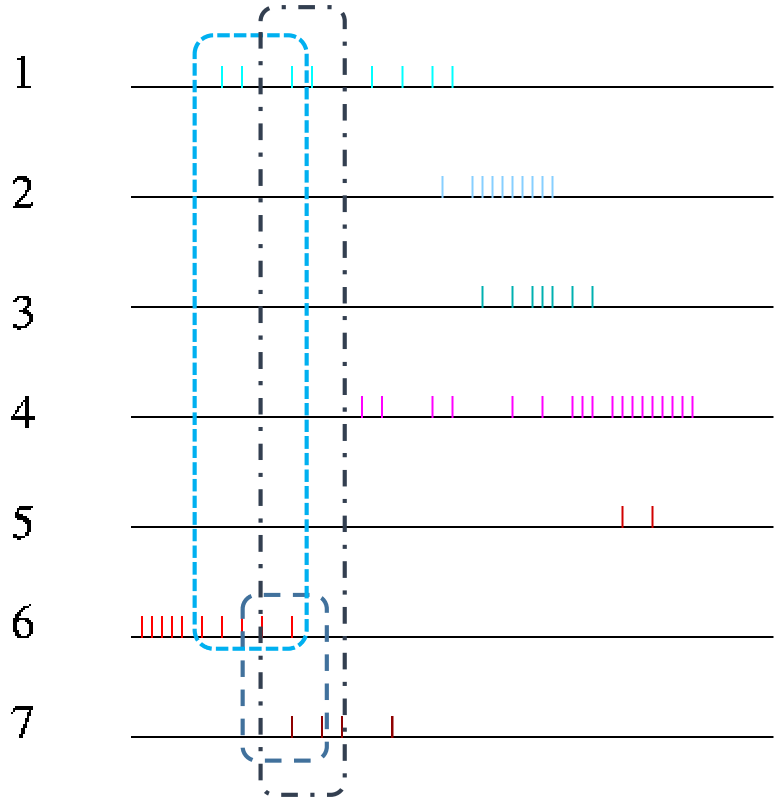

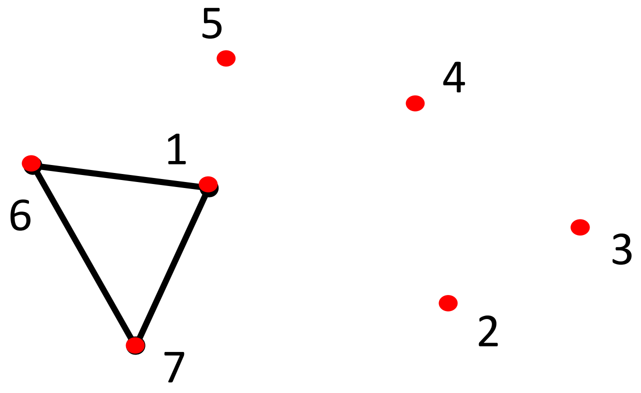

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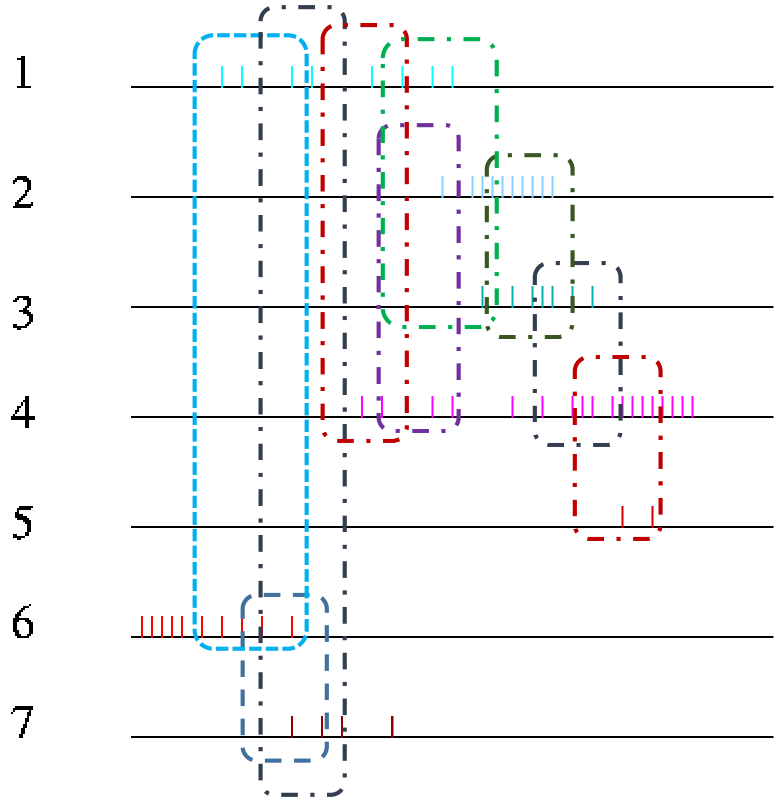
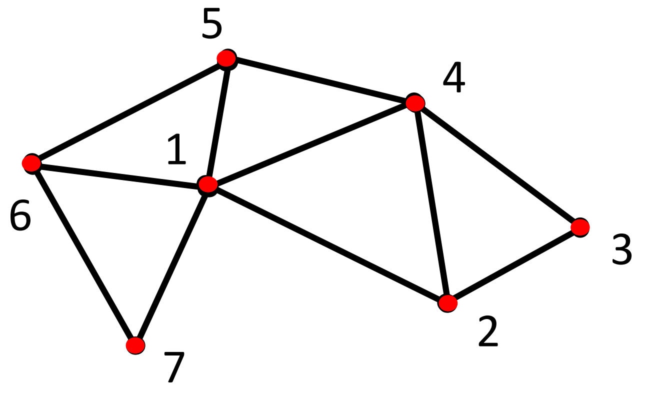
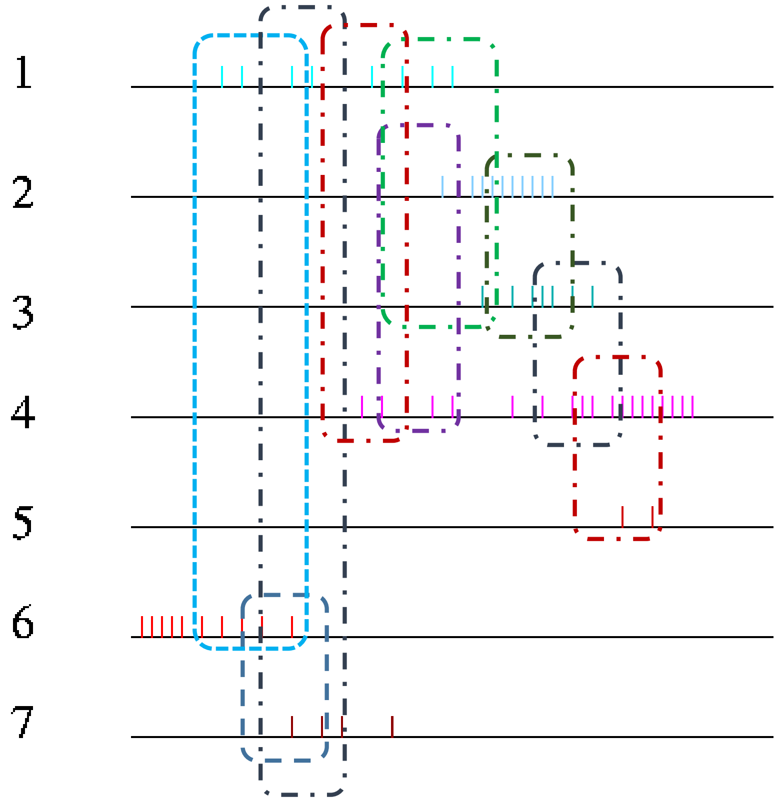
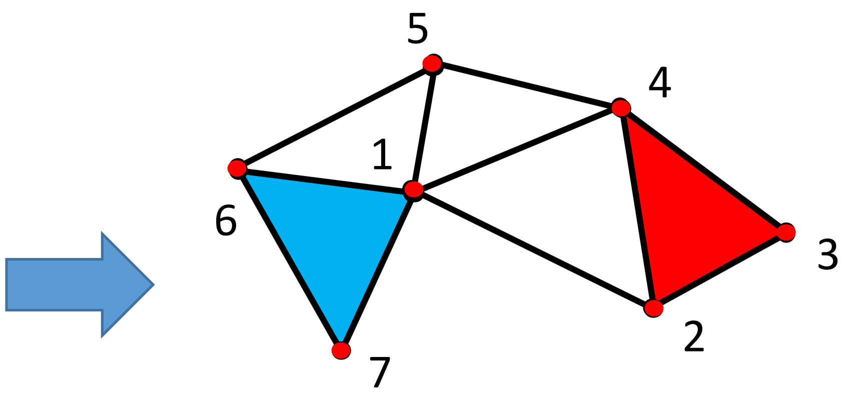




Spikes → Space reconstruction

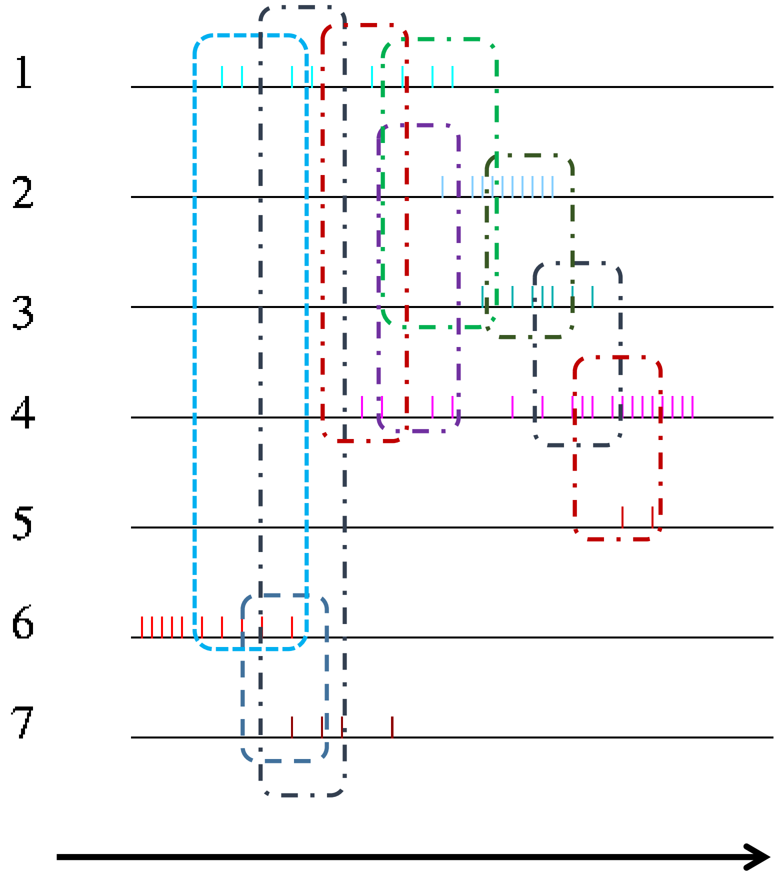


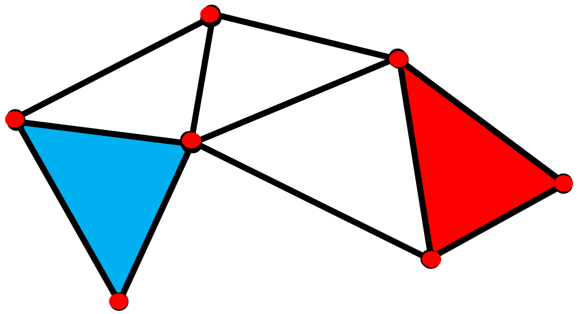


Temporal nerve simplicial complex

Topological information unfolds over time

Accumulation of
topological information

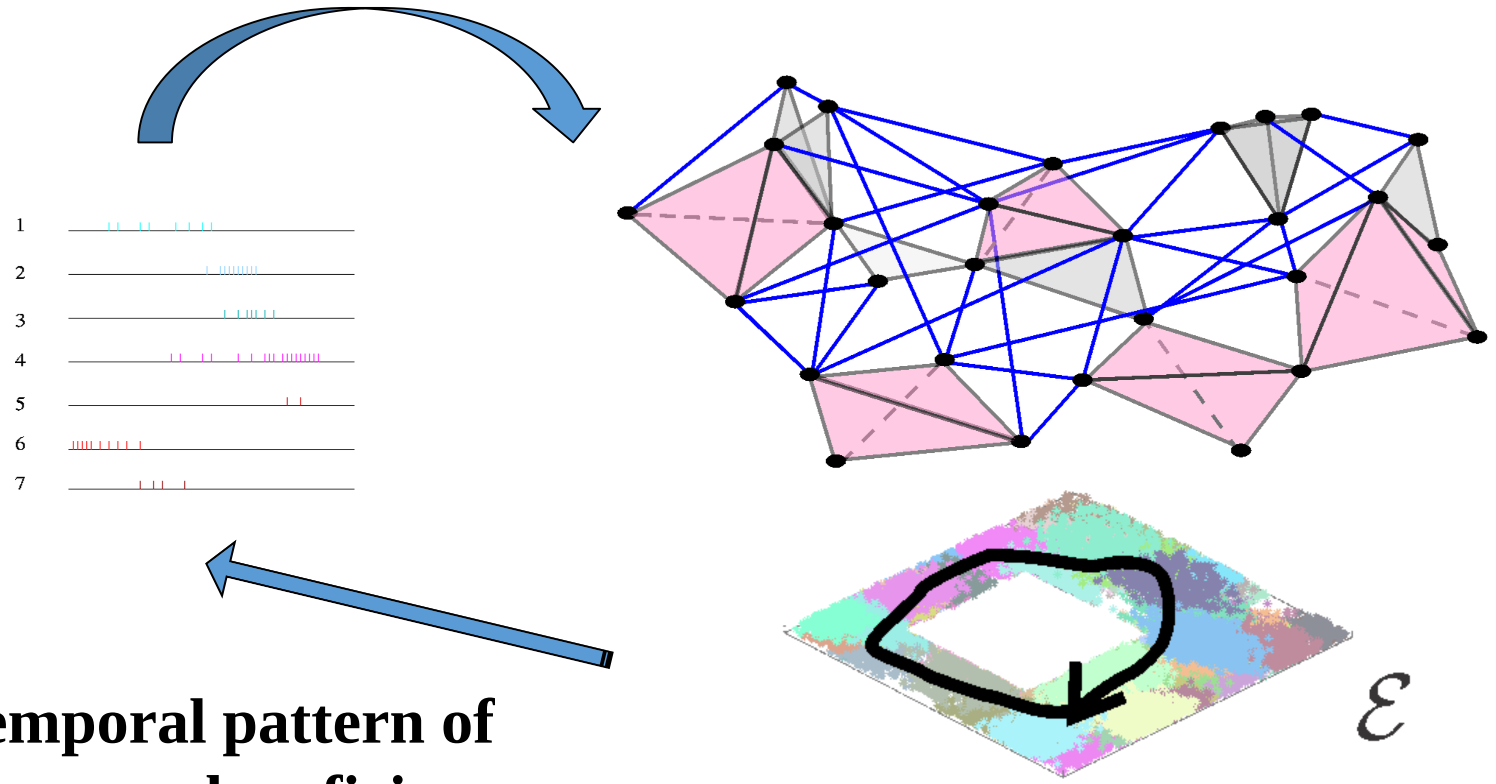




time

Accumulation of spikes

Temporal nerve complex

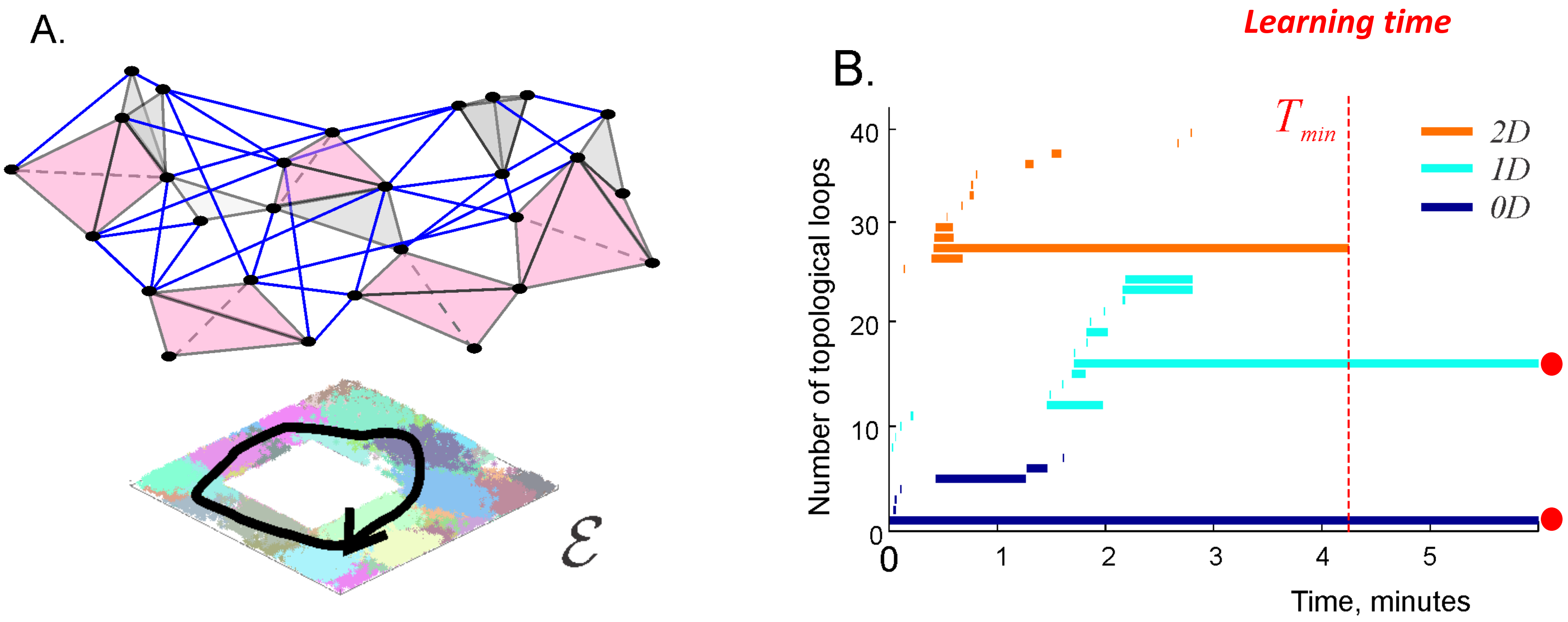


Temporal pattern of neuronal co-firing

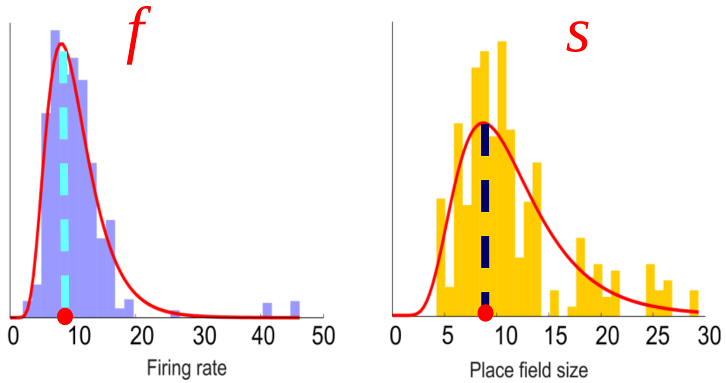
Rat's environment

Topology of $\mathcal{T}$ represents topology of $\mathcal{E}$

Timelines of the topological loops



Persistent loops, persistent Betti numbers: $b_0 = 1$, $b_1 = 1$, $b_{n>1} = 0$



Individual cell's firing rates: $f_1, f_2, \dots, f_N$

Individual place field sizes: $s_1, s_2, \dots, s_N$

$$P_f\left(f, \sigma_F \right)$$

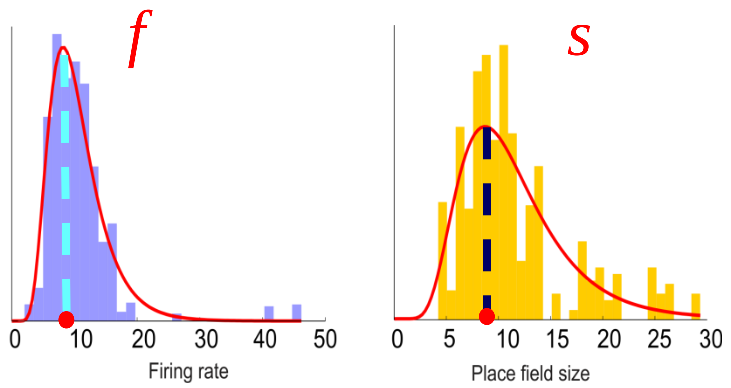
$$P_s\left(s, \sigma_s \right)$$

2N parameters

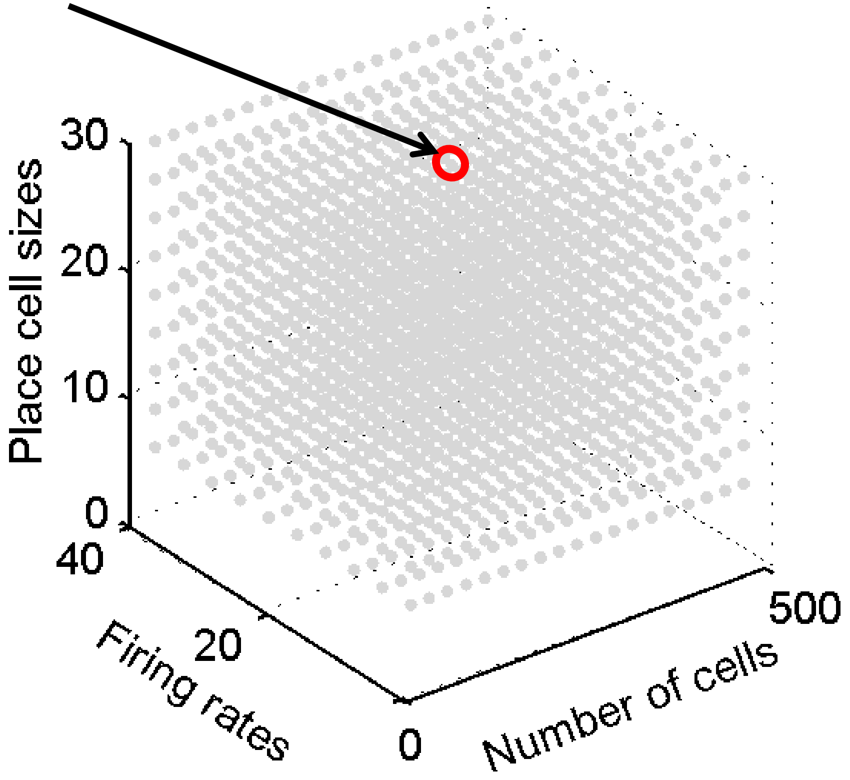
1. mean firing rate, f
2. mean place field size, s
3. number of cells, N

3 parameters per ensemble

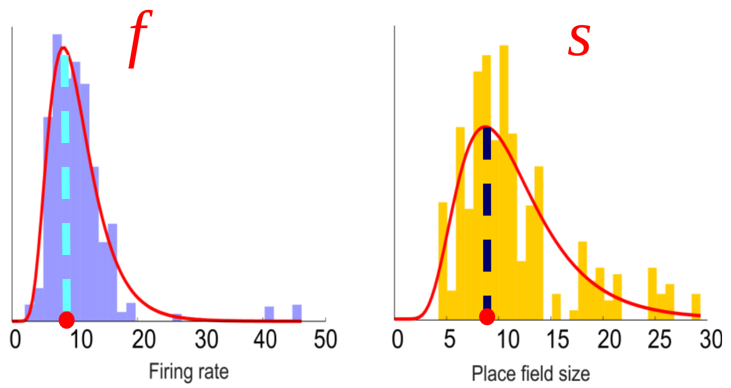
Testing numerically simulated place cell ensembles



Each point (s, f, N) represents a place cell ensemble



Which place cell ensembles produce reliable maps?



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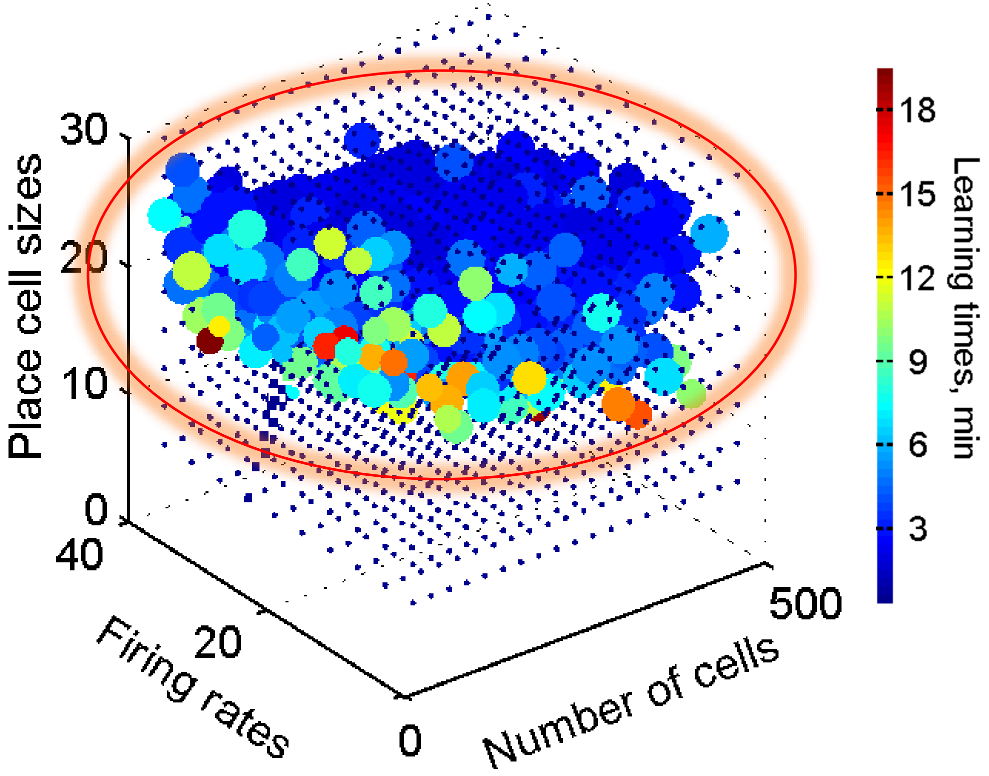
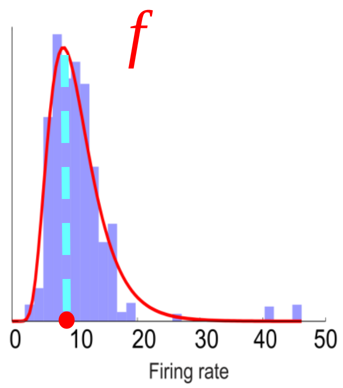
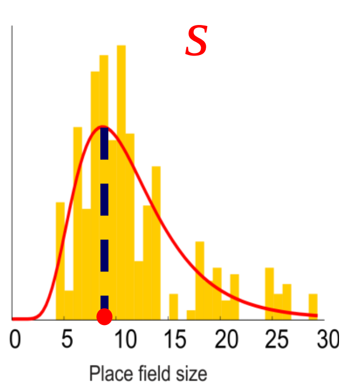


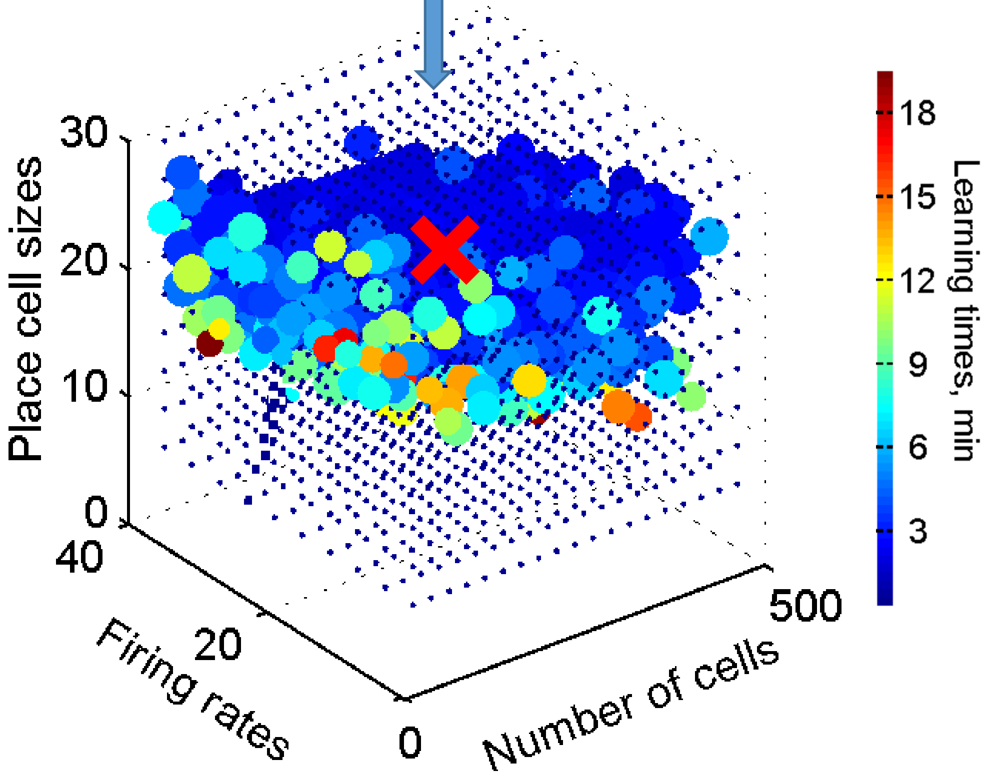
Different neural ensembles acquire information with different efficiencies, depending on firing rate, place field sizes, and size of cell population: the most competent ensembles form the **Learning Region**

Parameters recorded in healthy animals fall into the learning region





$$(s, f, N)$$



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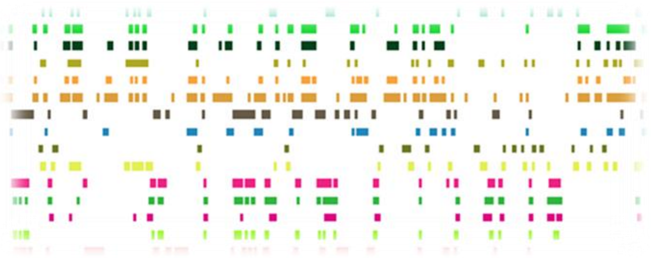
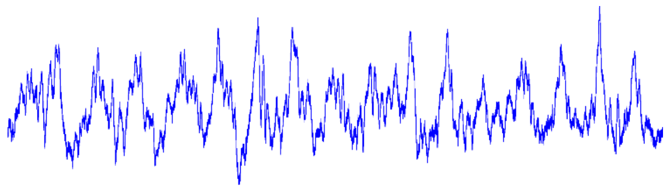
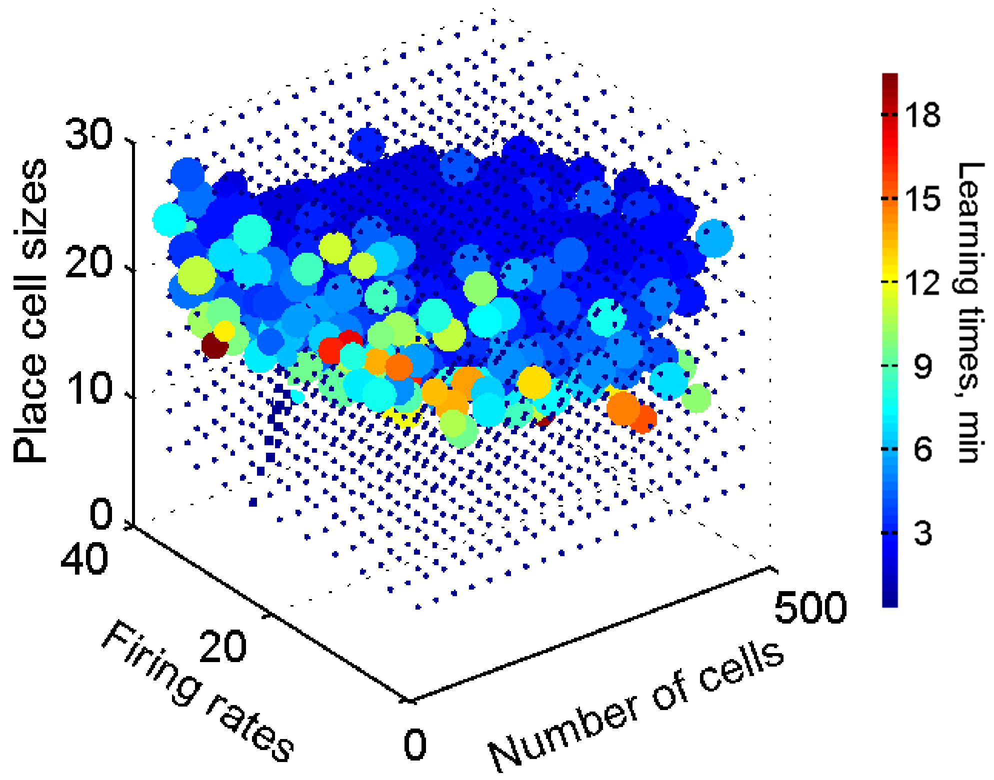
Spike trains

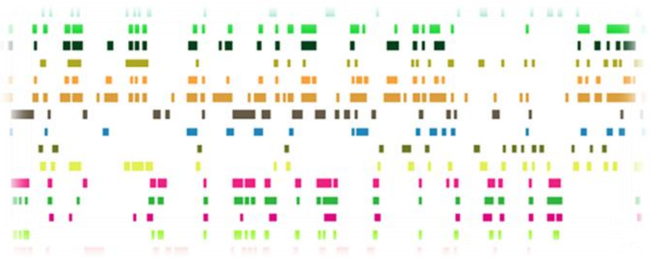


Brain waves



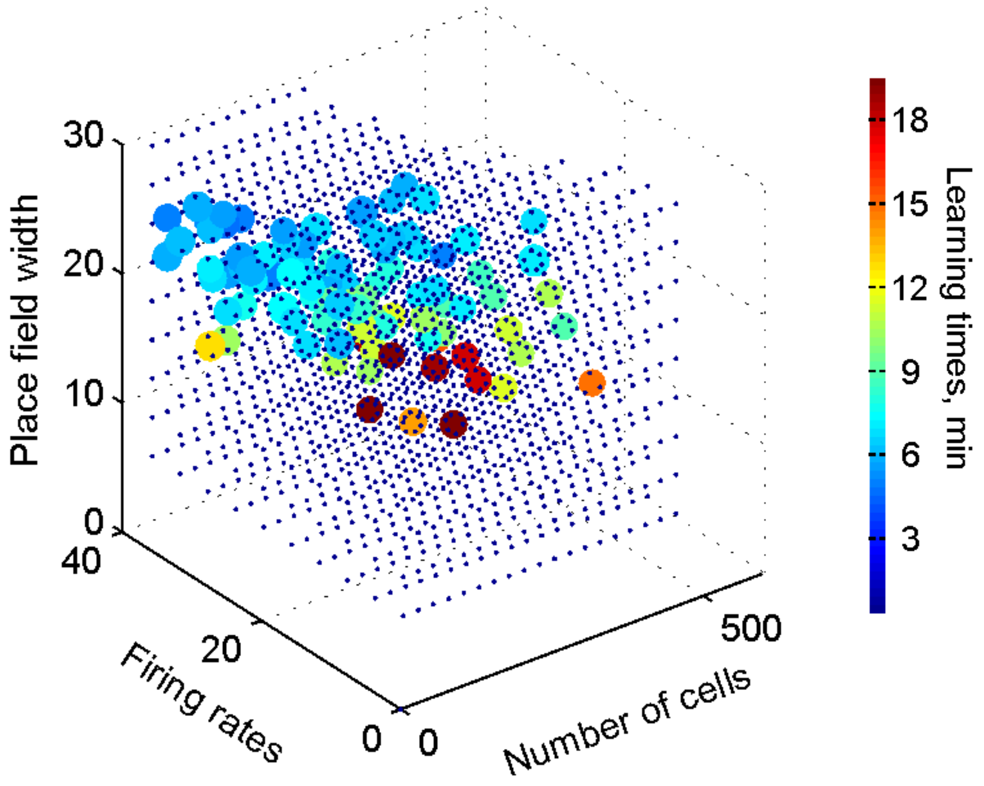


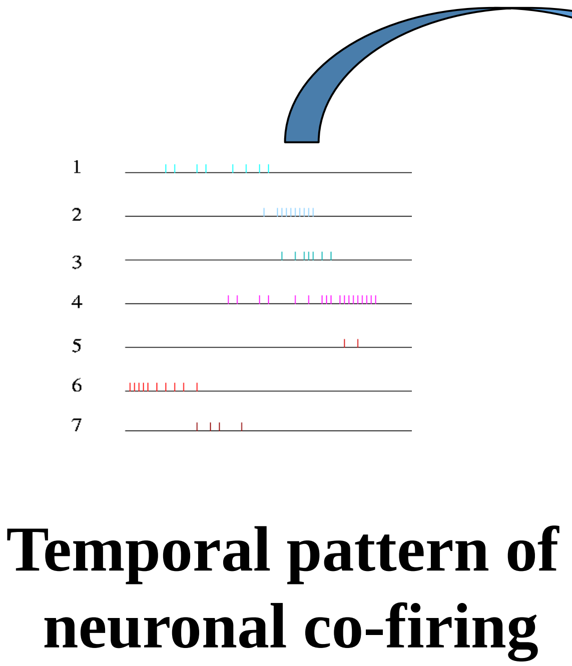
Spike trains

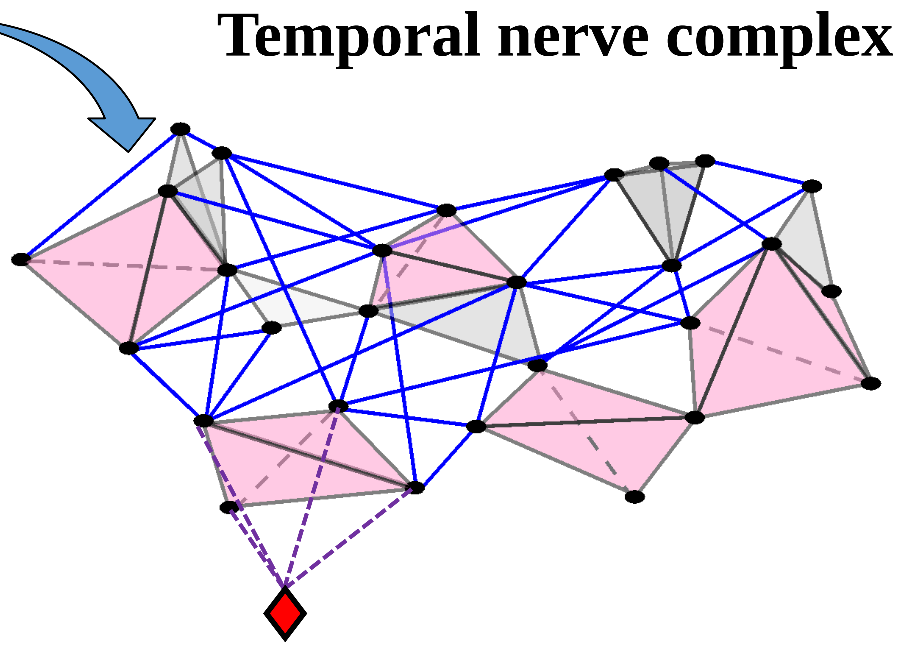


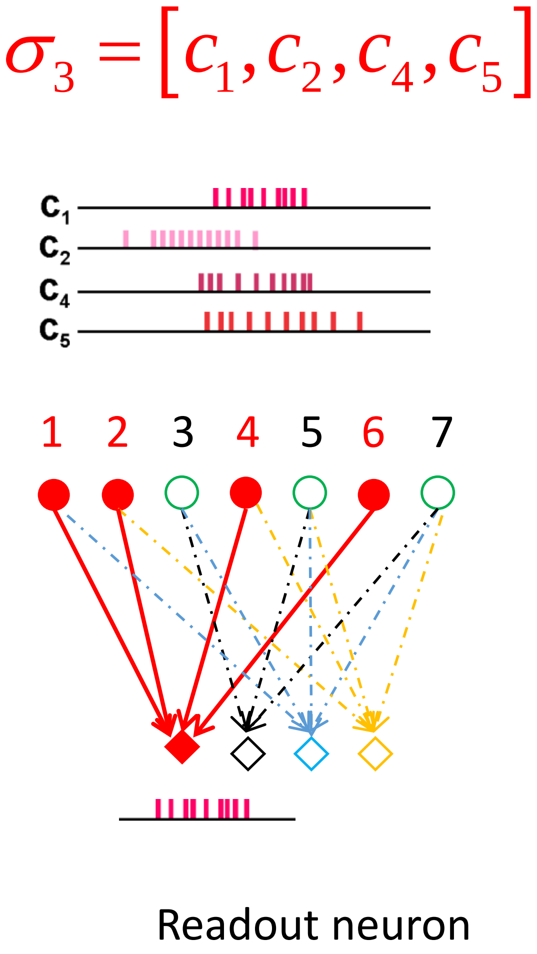
Brain waves

Spike modulation by the brain waves is essential for successful space coding

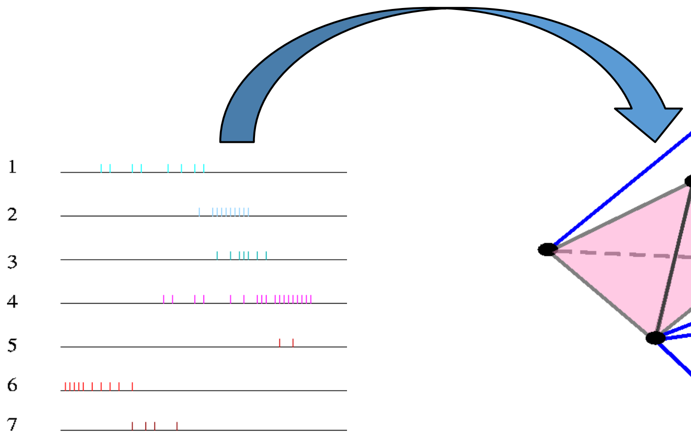






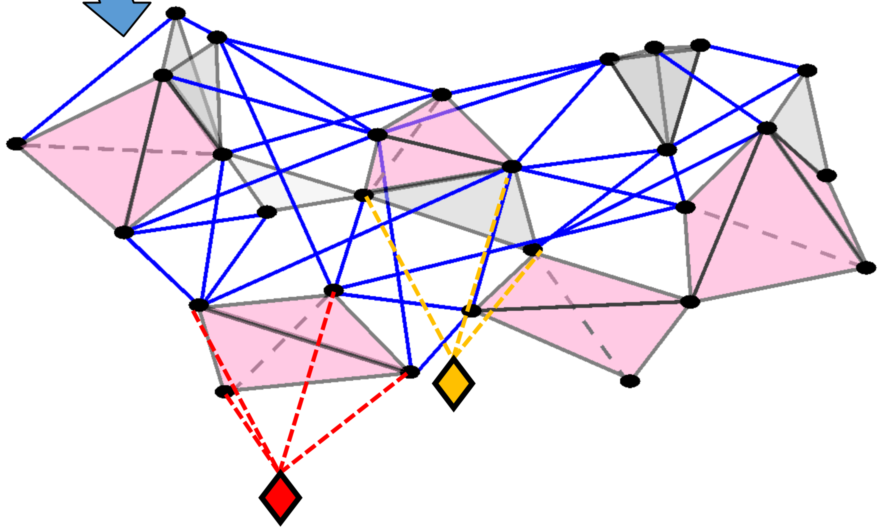


How can this structure be
implemented in the brain?

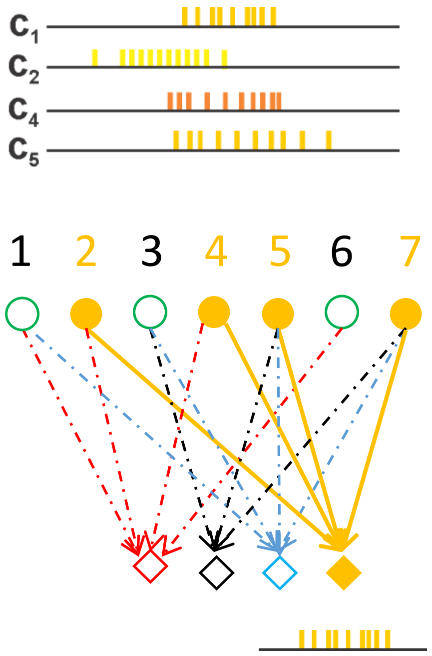


Temporal pattern of neuronal co-firing

Temporal nerve complex

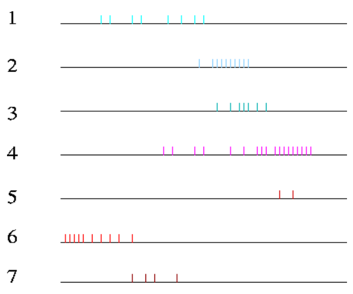


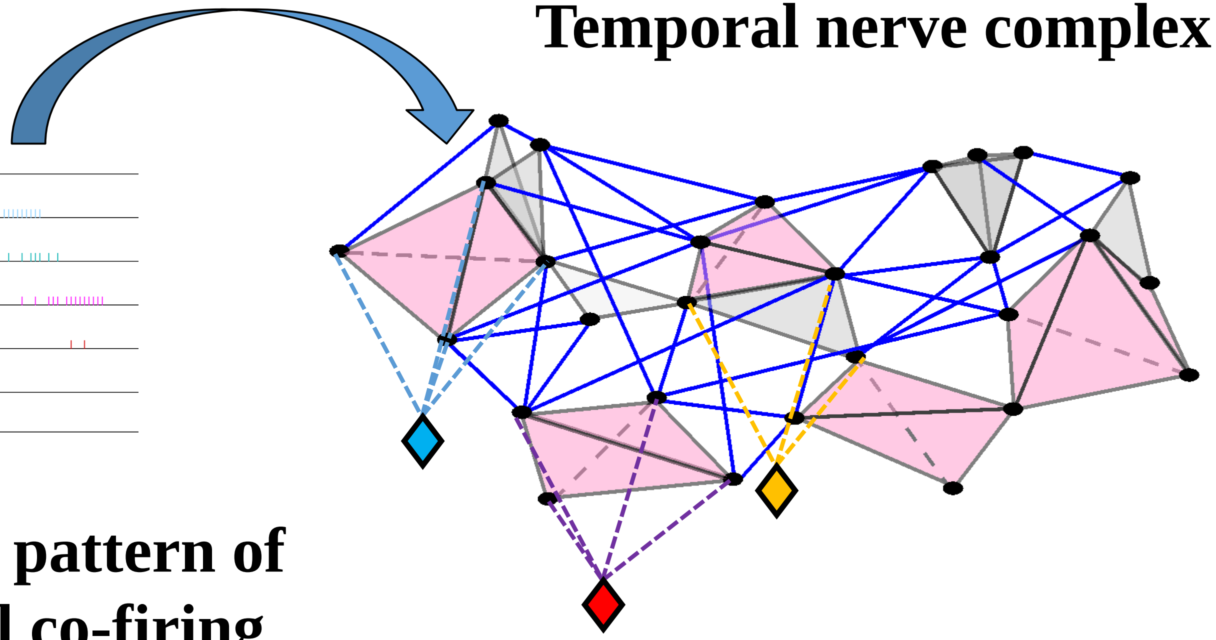
$$\sigma_2 = [c_2, c_4, c_5, c_7]$$



Readout neuron

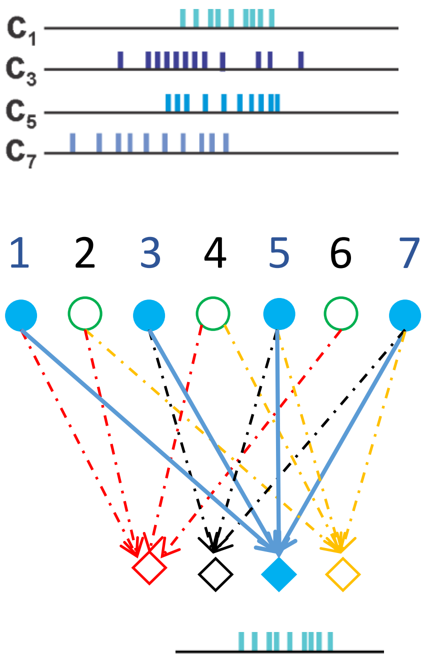
Temporal nerve complex





Temporal pattern of neuronal co-firing

$$\sigma_1 = [c_1, c_3, c_5, c_7]$$

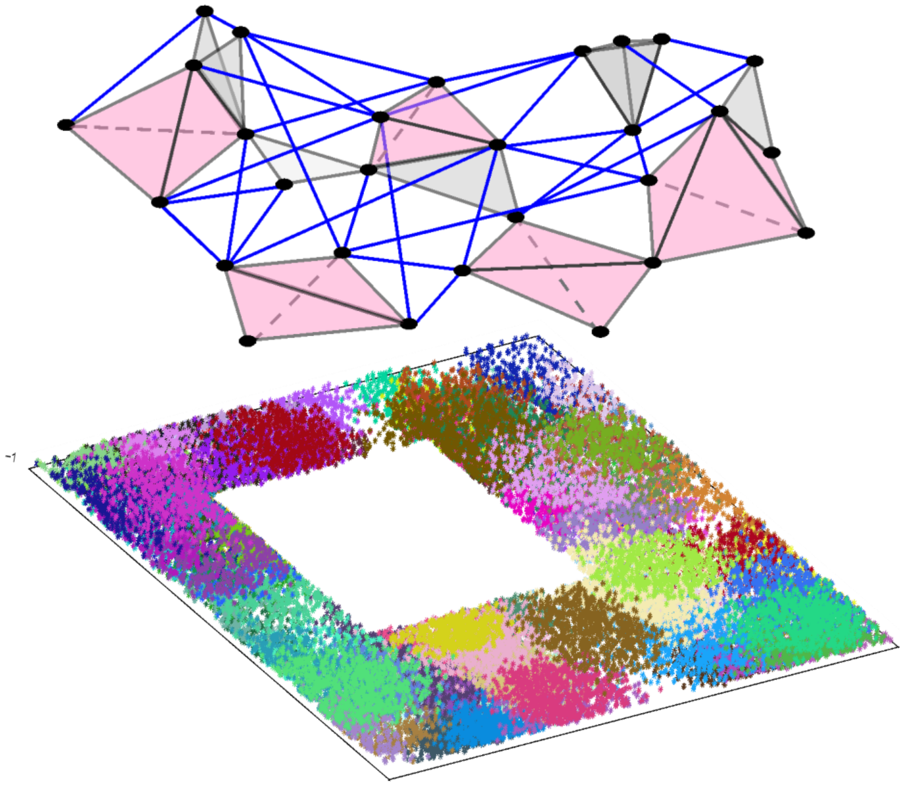


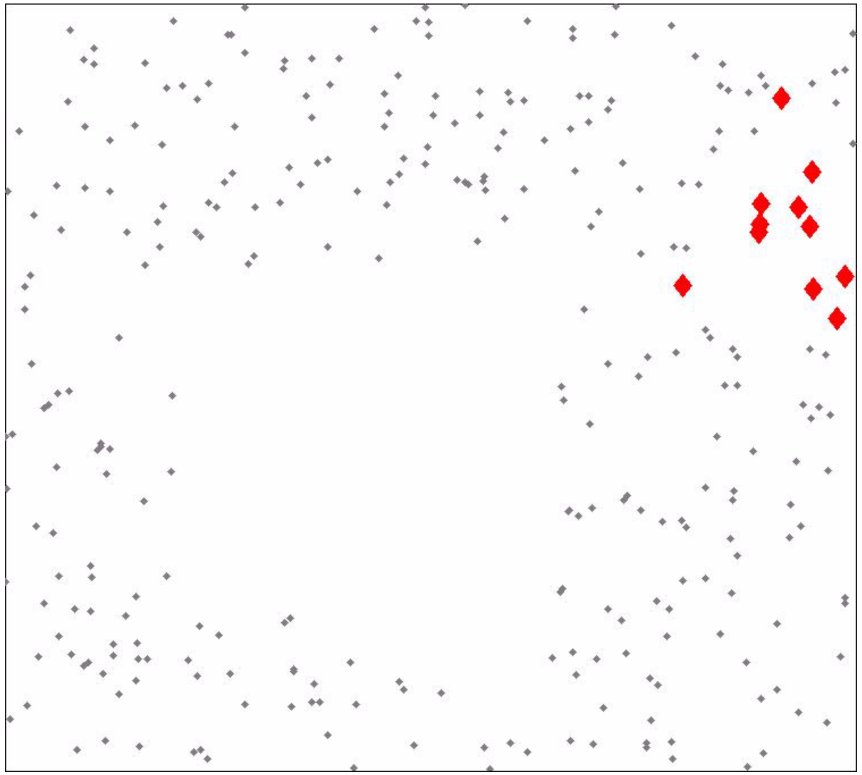
Readout neuron

The pool of coactive place cell combinations is huge, but the number of readout neuron is limited.

$$\#\,coactive\ combinations \sim \binom{\#cells}{\#coactive\ \ cells}$$

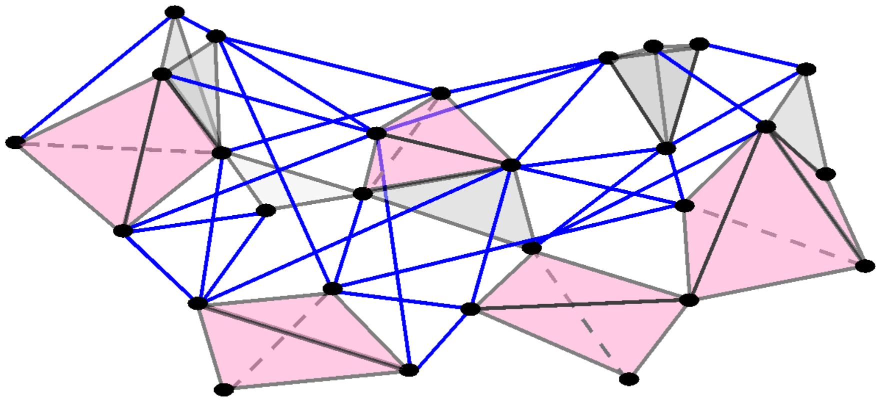
1. High dimensionality $\bar{D} \sim 20$
2. Low ignition rate f_σ
3. Irregularity
4. etc.





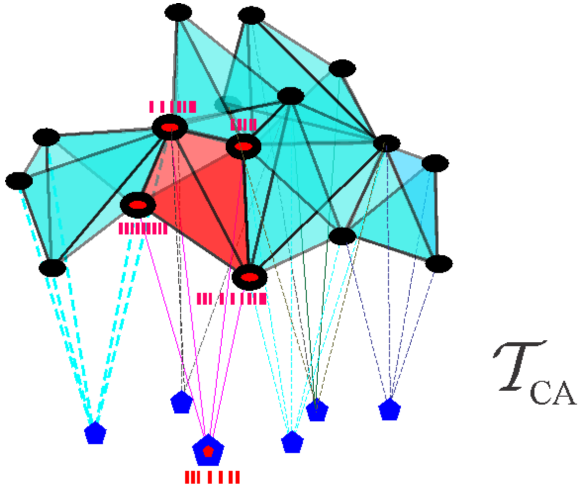
$$\#\,coactive\ combinations \sim \binom{\#cells}{\#coactive\ cells}$$

Actual temporal nerve complex $\mathcal{T}$



$\supset$

Cell assembly complex $\mathcal{T}_{CA}$



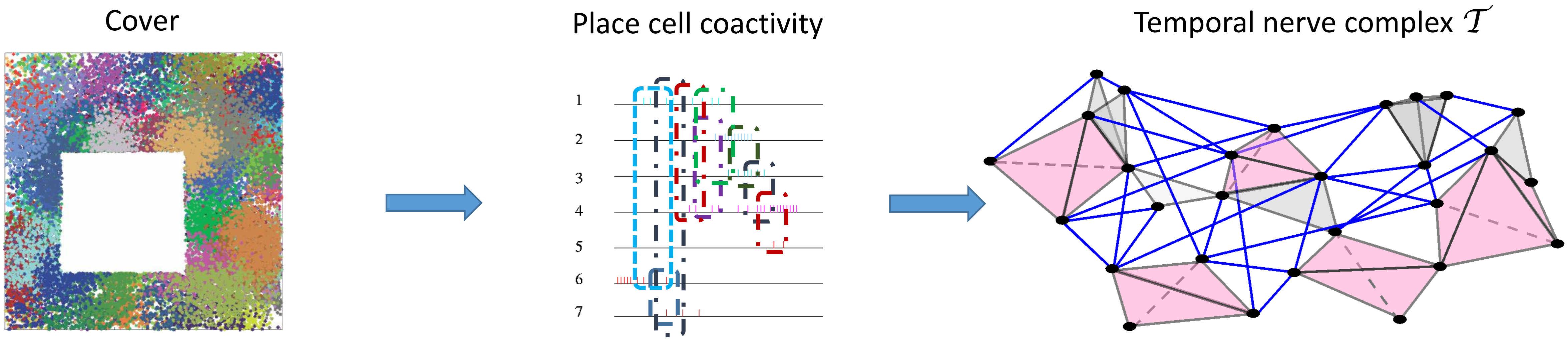
1. Cell assemblies correspond to maximal simplexes $\sigma \in \mathcal{T}_{CA}$, high ignition rate f_σ, low $\dim(\sigma)$

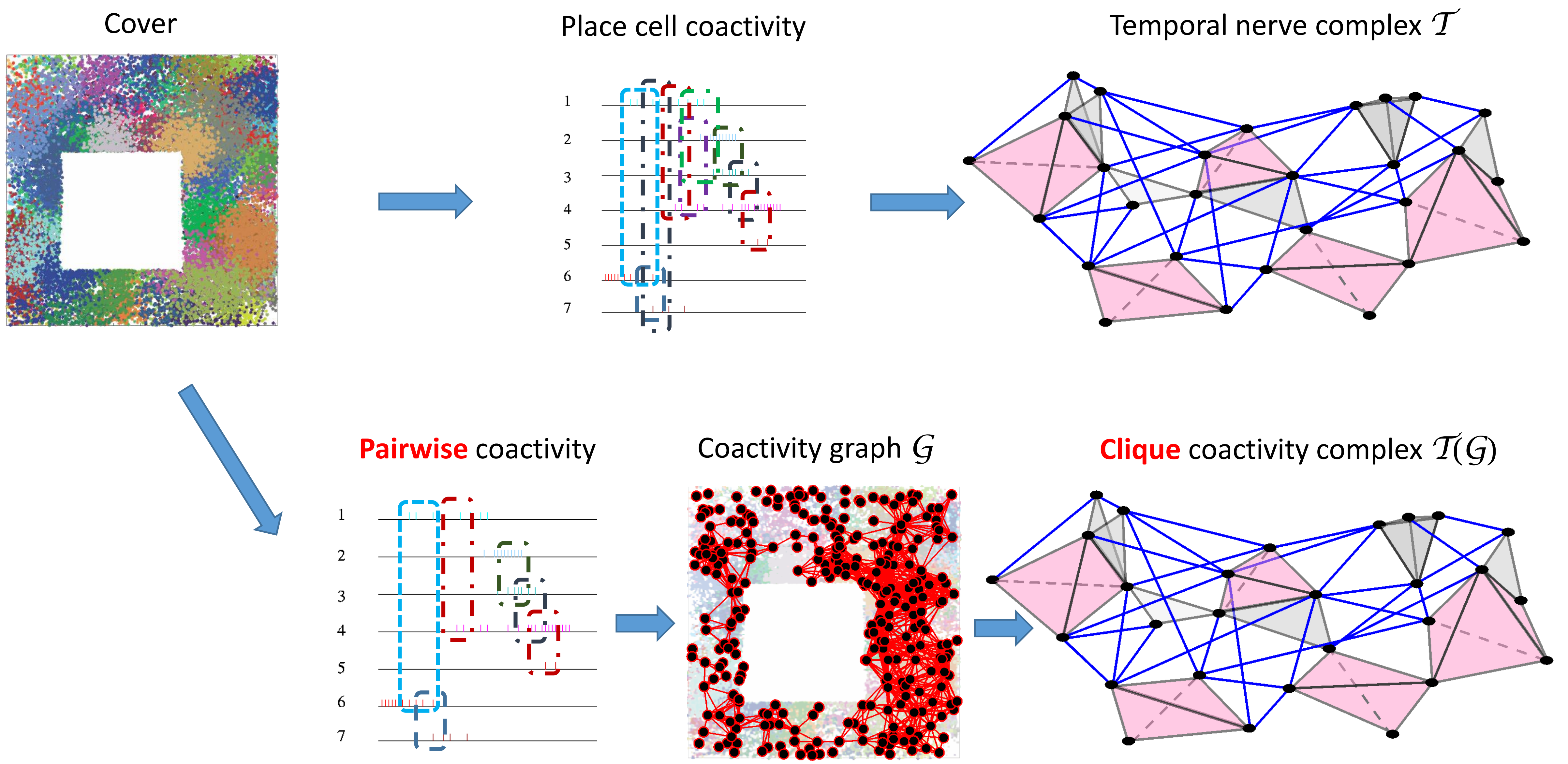
2. $N_{\text{cell assemblies}} = N_{\text{readout neurons}} \approx N_{\text{place cells}}$, hence $N_{\text{max simplexes}} \approx N_{\text{vertexes}}$

3. Maximize "contiguity of simplexes," $\xi_i = \dfrac{\dim(\sigma_i \cap \sigma_{i+1})}{\sqrt{\dim(\sigma_i)\dim(\sigma_{i+1})}}$

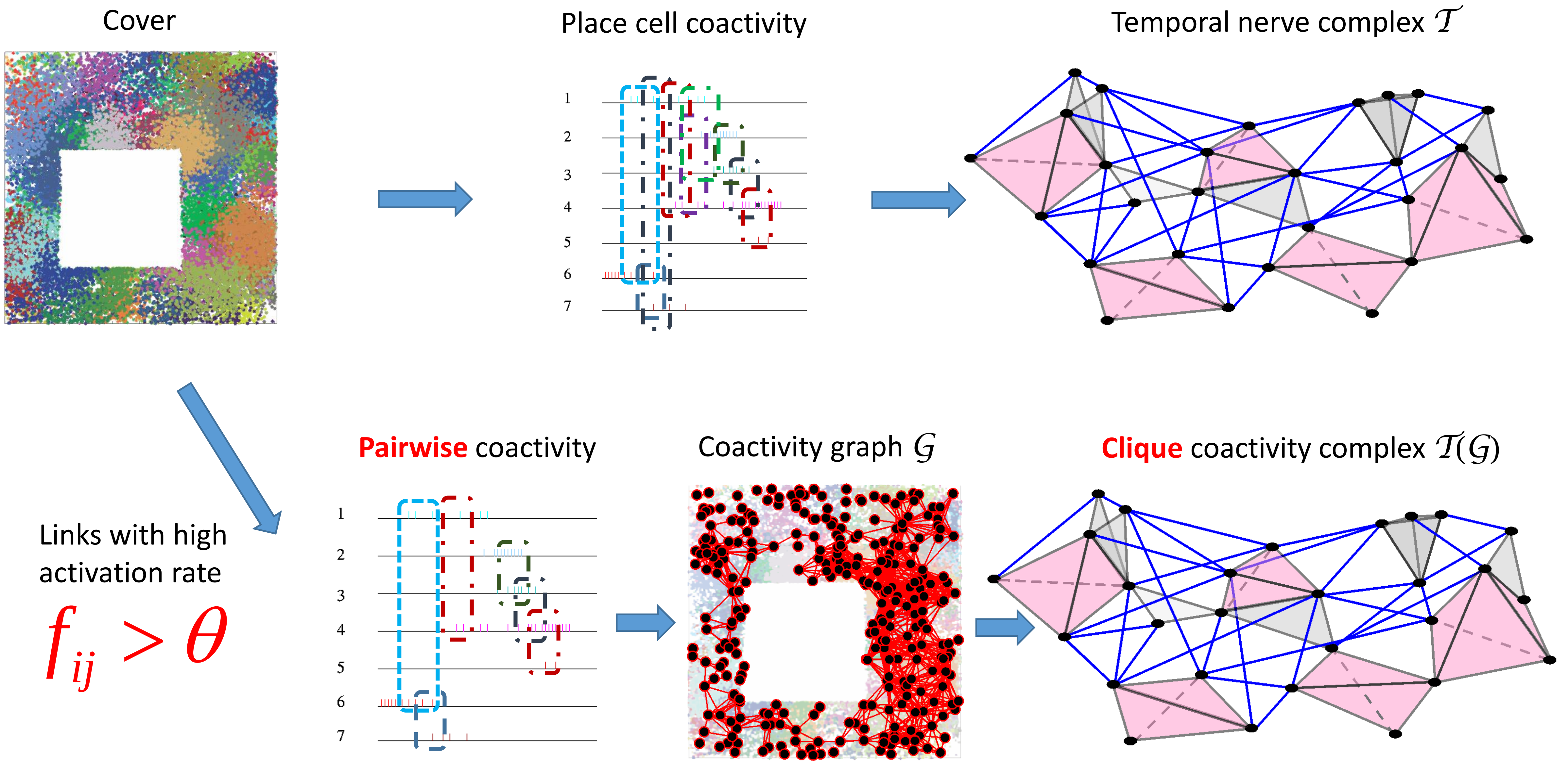
4. $\mathcal{T}_{CA}$ should correctly represent the topology of the environment, $H_*(\mathcal{T}_{CA}) = H_*(\mathcal{T}) = H_*(\mathcal{E})$

5. Learning times, T_{min}, should be reasonable

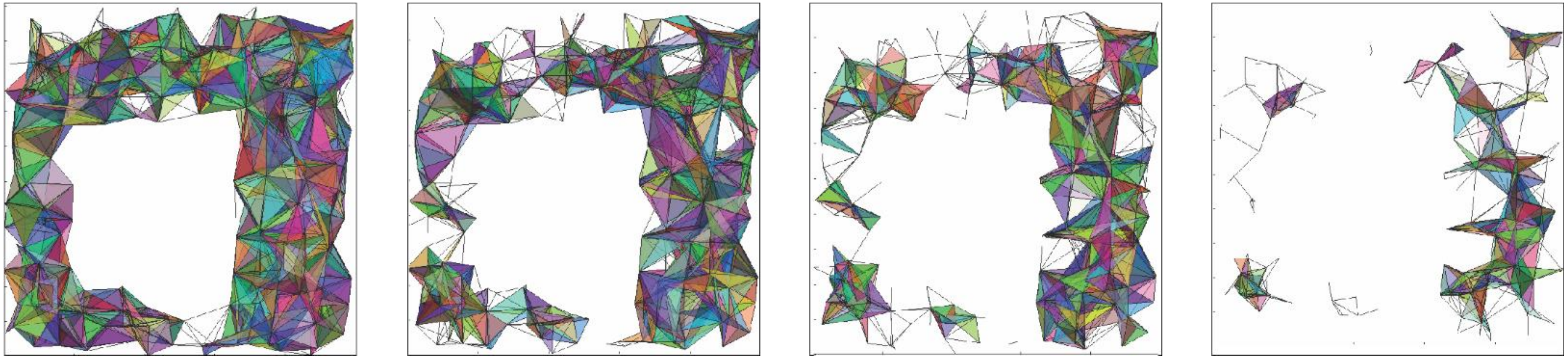

Cover
Place cell coactivity
1
2
3
4
5
6
7
Temporal nerve complex $\mathcal{T}$



Cover
Place cell coactivity
Temporal nerve complex $\mathcal{T}$
1
2
3
4
5
6
7
Pairwise coactivity
Coactivity graph $\mathcal{G}$
Clique coactivity complex $\mathcal{T}(\mathcal{G})$
1
2
3
4
5
6
7



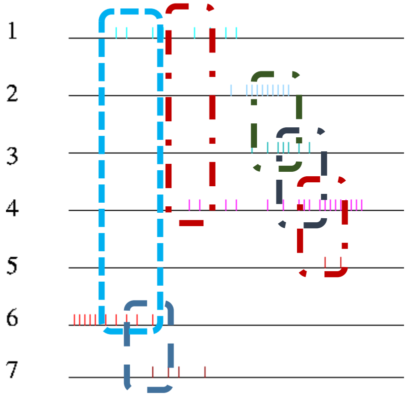
Cover
Place cell coactivity
Temporal nerve complex $\mathcal{T}$
1
2
3
4
5
6
7
Links with high activation rate
$f_{ij} > \theta$
Pairwise coactivity
1
2
3
4
5
6
7
Coactivity graph $\mathcal{G}$
Clique coactivity complex $\mathcal{T}(\mathcal{G})$



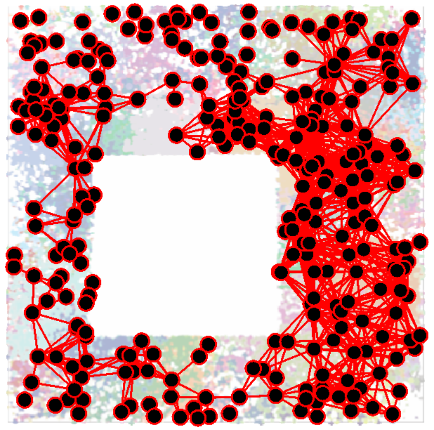
θ_1 θ_2 θ_3 θ_4

$$f_{ij} > \theta$$

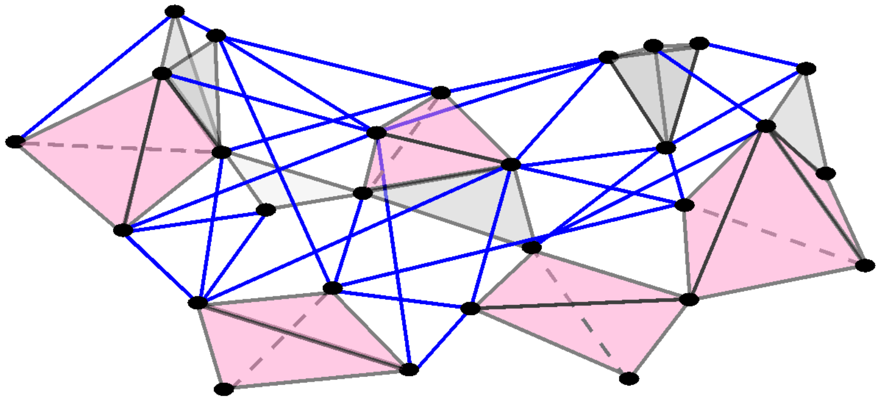
Pairwise coactivity



Coactivity graph $\mathcal{G}$

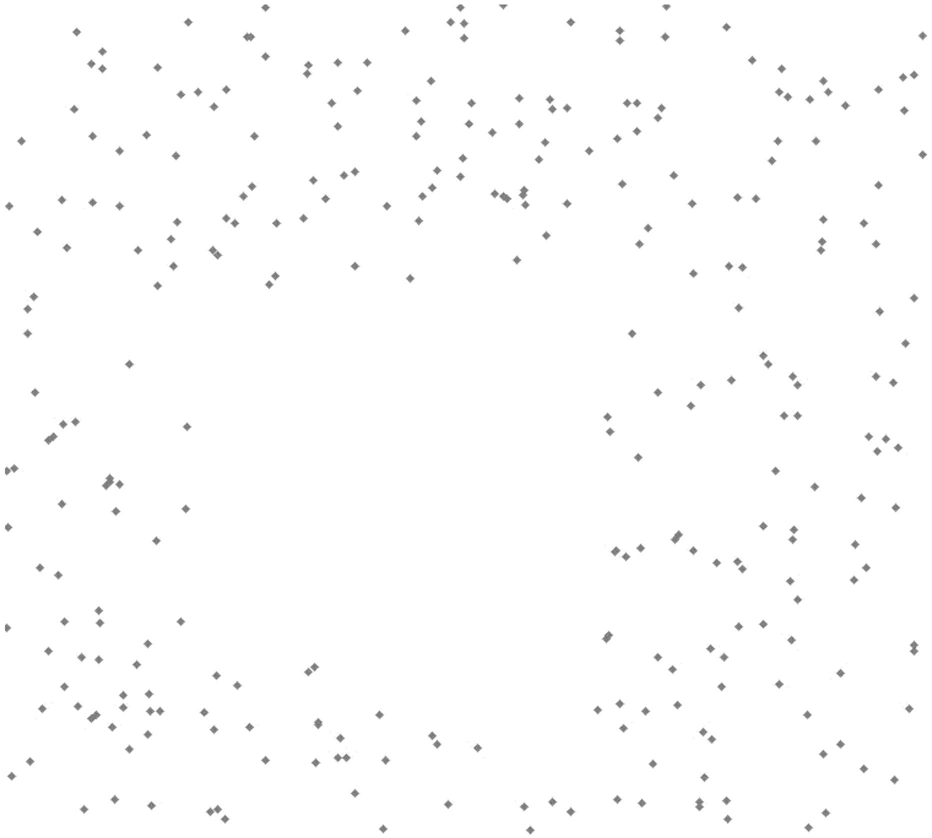


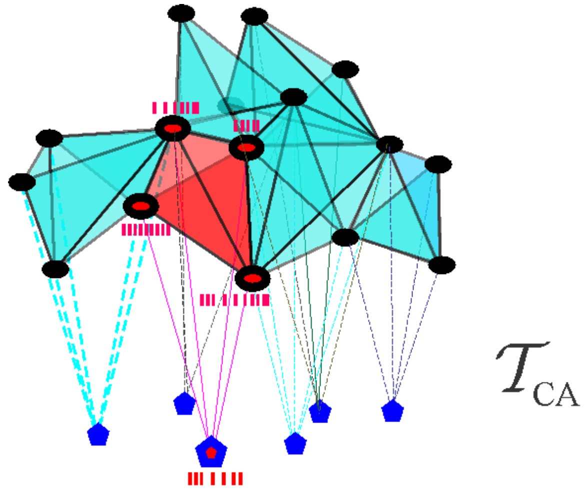
Clique coactivity complex $\mathcal{T}$

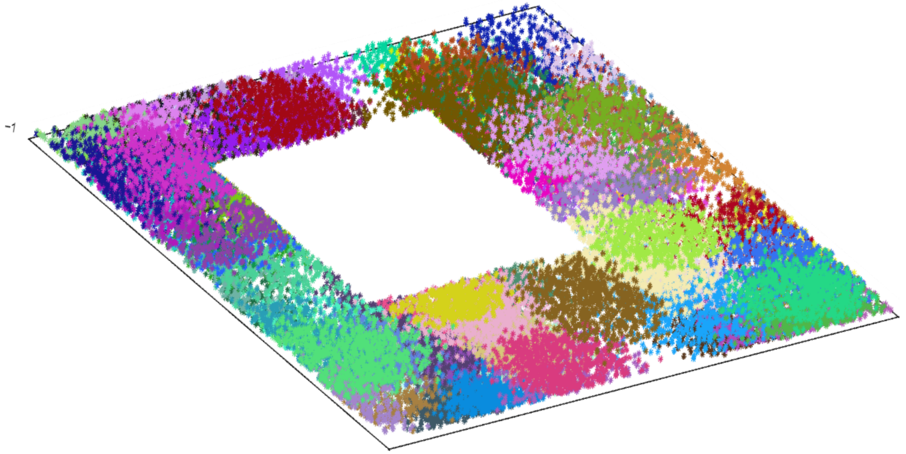


Select the most active ***combinations*** of place cells

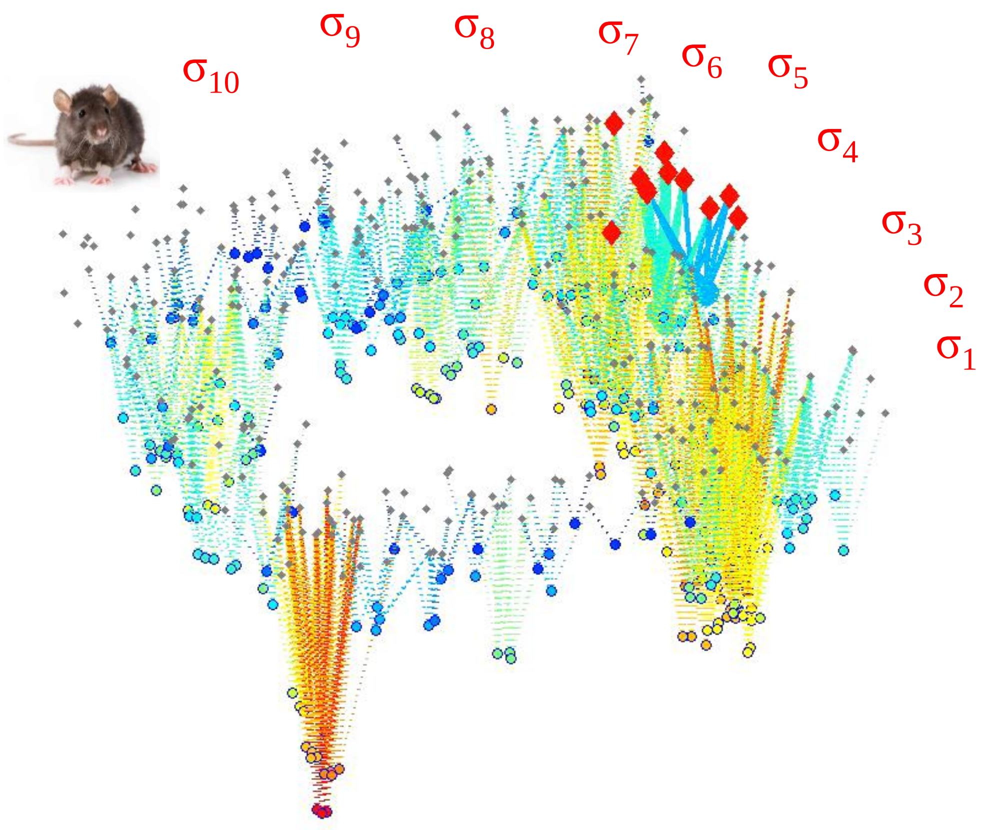
Selection of cell assemblies based on coactivity rates





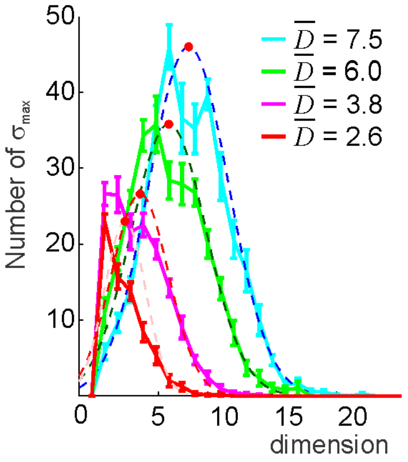


Navigation in cell assembly complex



1. # Cell Assemblies ≈ # place cells, $N_{\text{max}} \approx N_{\text{vtx}}$

2. Mean contiguity $\xi = \left\langle \dfrac{\dim(\sigma_i \cap \sigma_{i+1})}{\sqrt{\dim(\sigma_i)\dim(\sigma_{i+1})}} \right\rangle \approx 0.78$

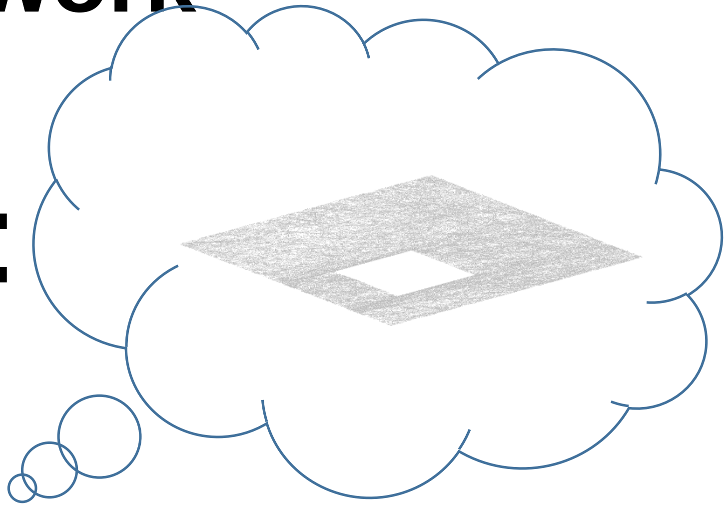
3. 

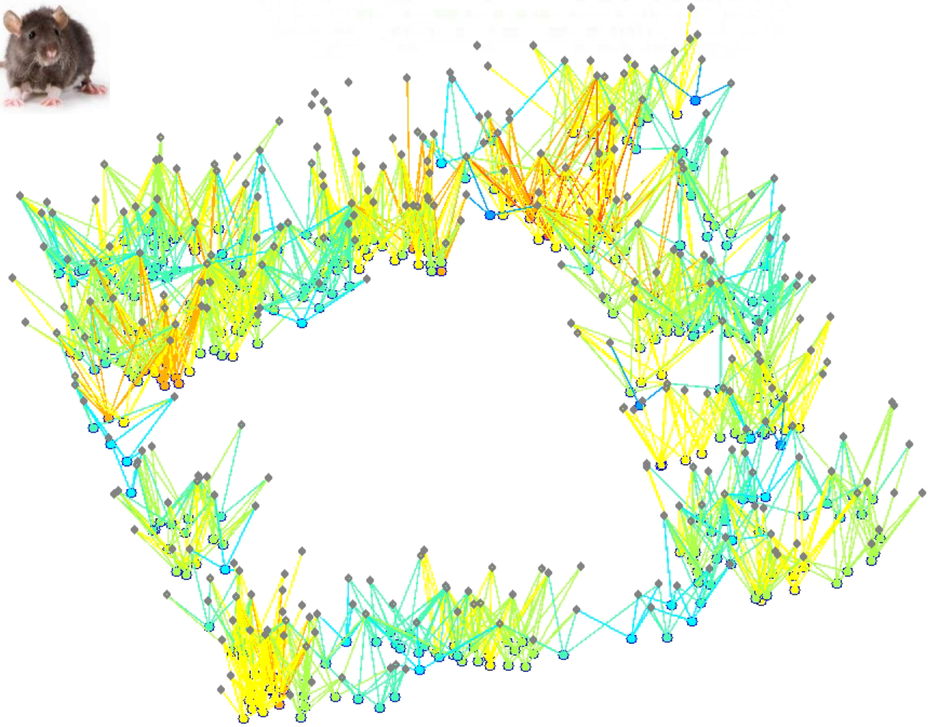
4. Correct topology, $H_*(\mathcal{T}_{CA}) = H_*(\mathcal{T}) = H_*(\mathcal{E})$

5. Learning times T_{min} are the approximately the same

Rewiring cell assembly network

=

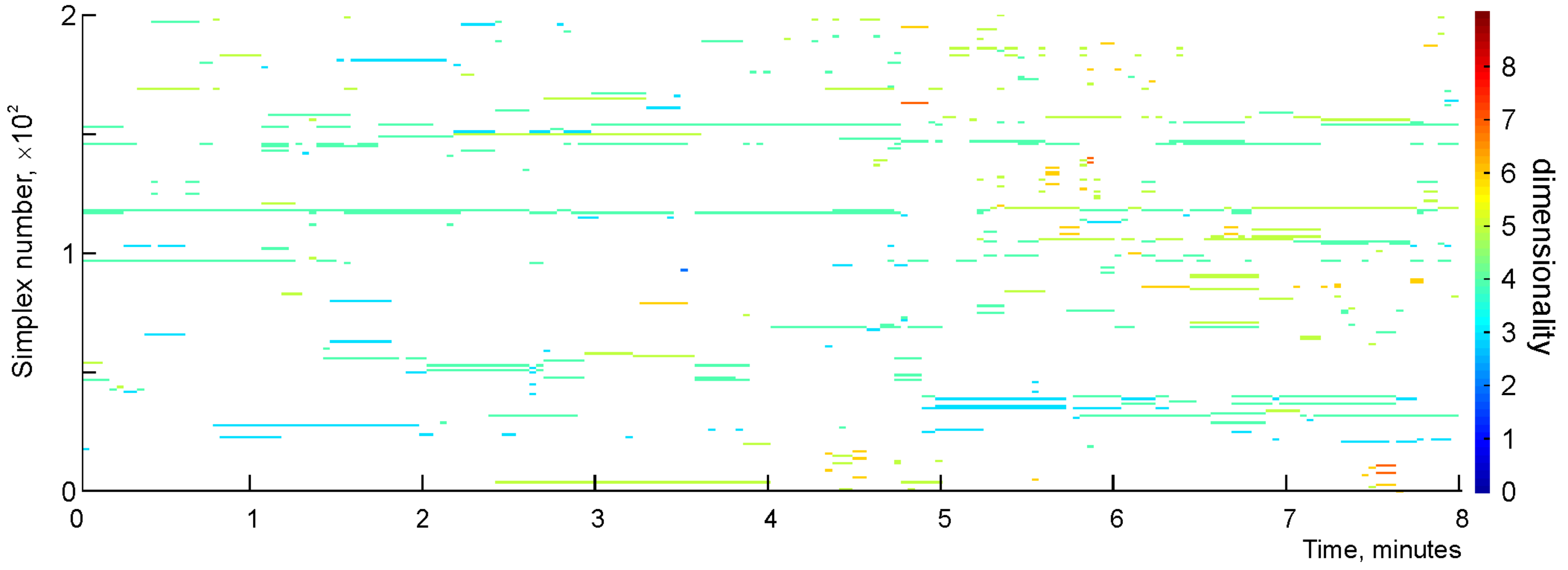


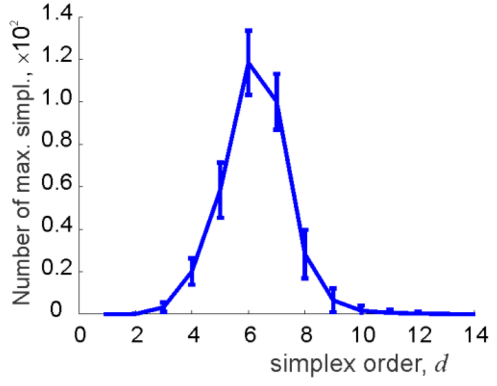


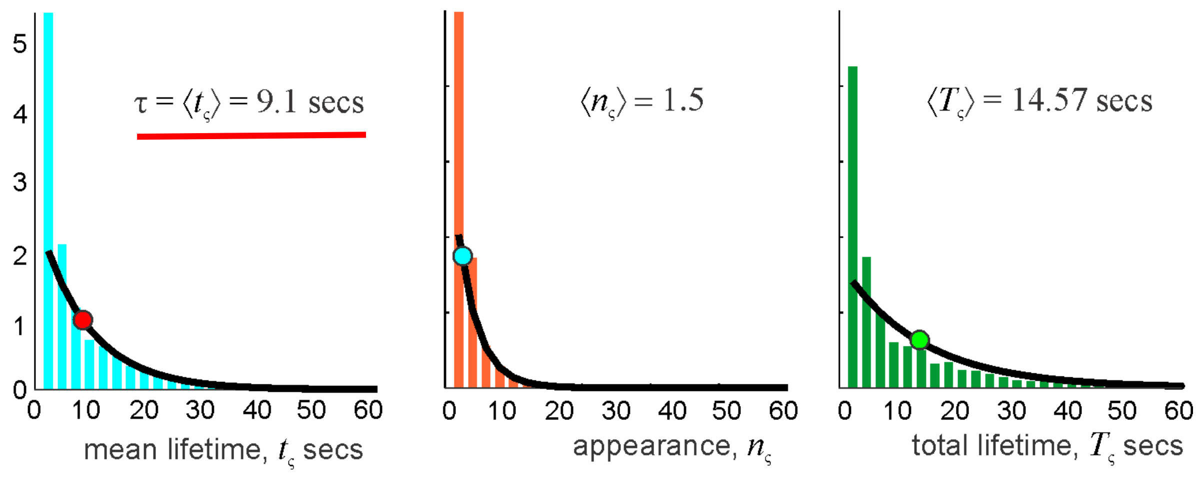
1. Cell assemblies are unstable
2. Cognitive maps are stable

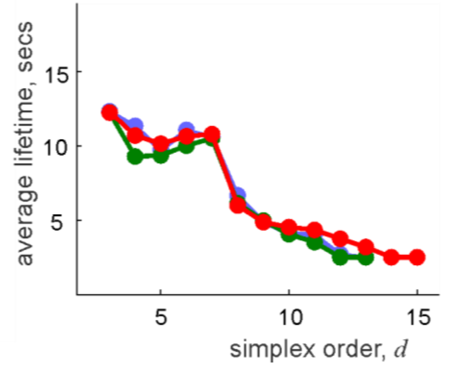
How can that work?

Transient (finite time) cell assemblies

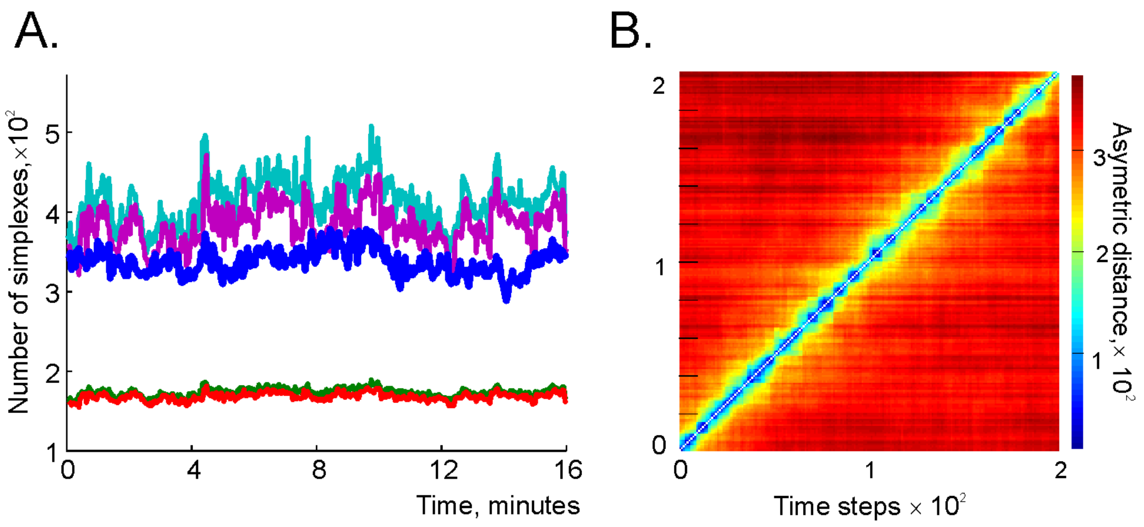




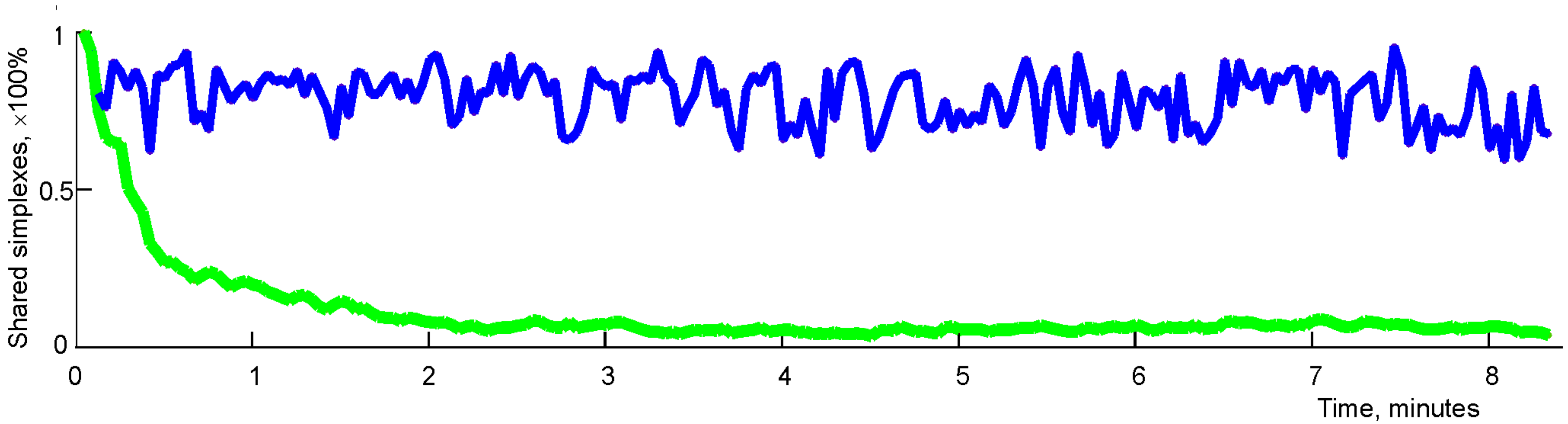




Transient (flickering) cell assembly complex $\mathcal{F}(t)$

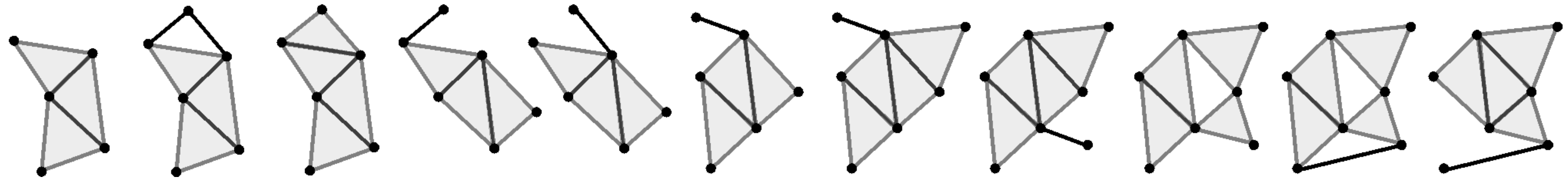


$$d_{ij} = \langle \mathcal{F}(t_i), \mathcal{F}(t_j) \rangle$$

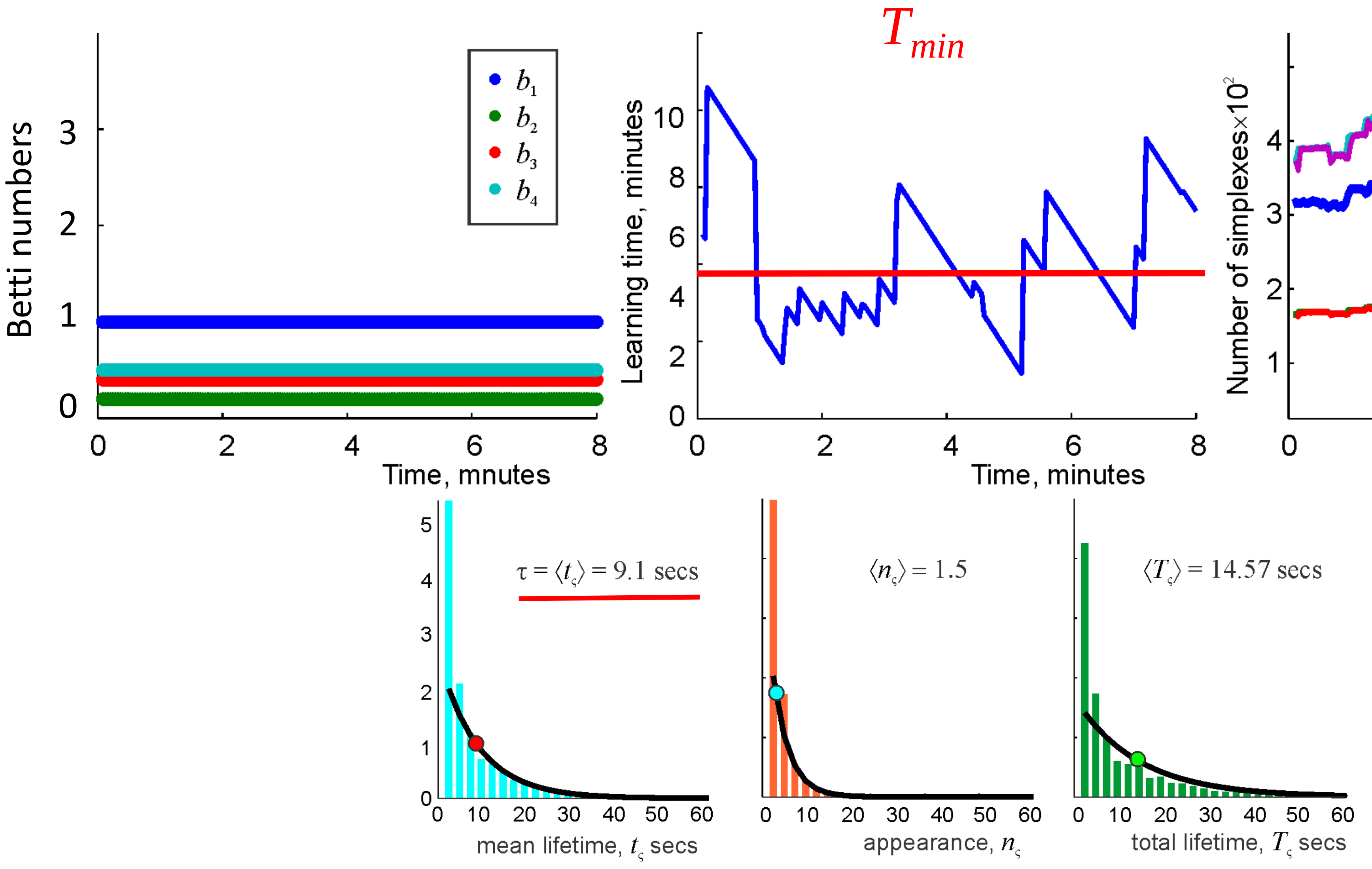


$$C(t) = \langle \mathcal{F}(t+1), \mathcal{F}(t) \rangle$$

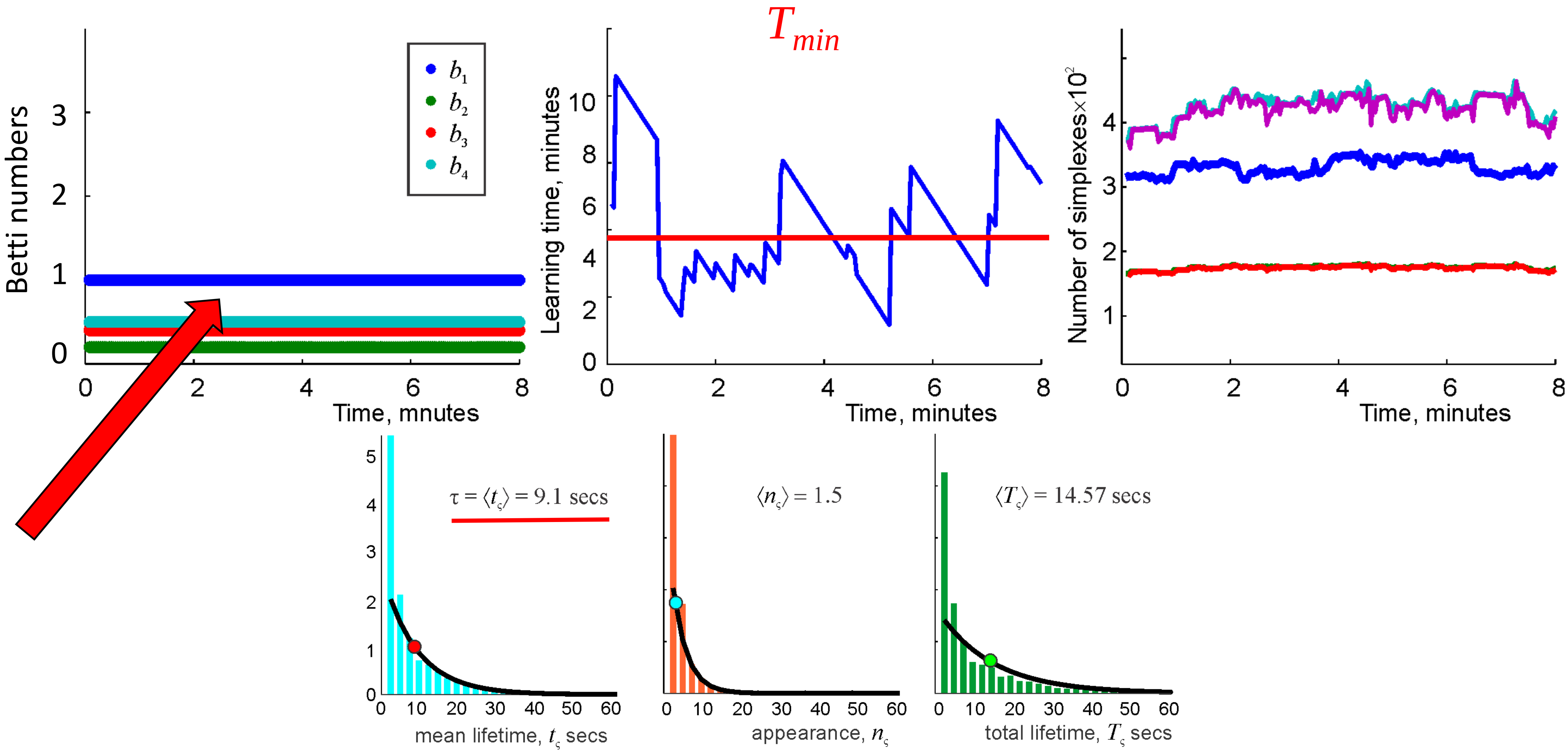
$$D(t) = \langle \mathcal{F}(1), \mathcal{F}(t) \rangle$$



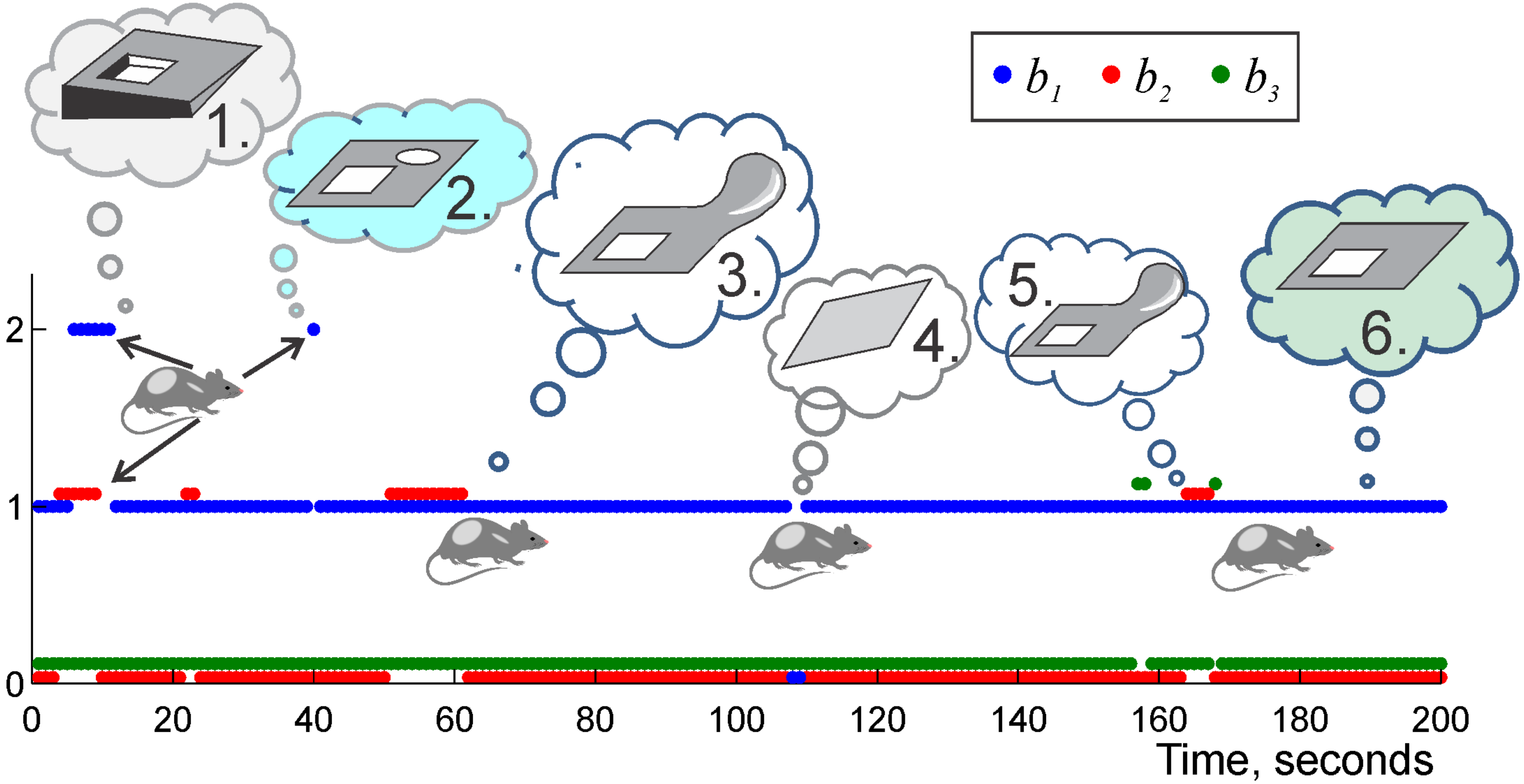
Rewiring cell assembly network encodes a stable map



Rewiring cell assembly network encodes a stable map



Rewiring cell assembly network encodes a stable map



Andrey Babichev (Rice University/BCM)

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