Chern-Moser theory and Bergman kernel functions

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ABSTRACT

We give a discussion on the boundary CR invariant theory by Chern-Moser and Fefferman for a bounded strongly pseudoconvex domain D. We use the CR invriants to express the coefficients of the expansion of the Bergman kernel function near the boundary of D. We also discuss the Ramadanov conjecture and other related problems on the expansion of the Bergman kernel functions.

Distribution of zeros of random holomorphic sections

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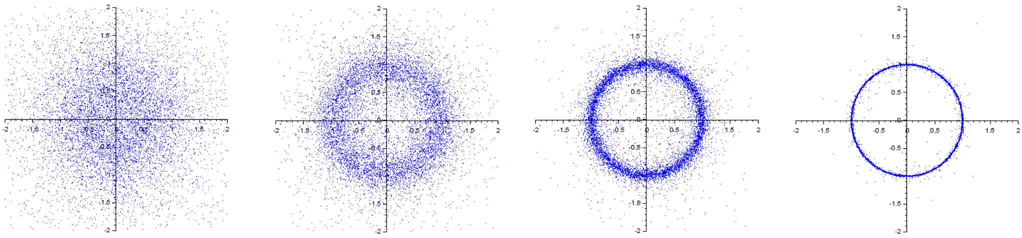
ABSTRACT

The distribution of zeros of random polynomials is a time-honoured subject, starting with the papers of Bloch-Pólya, Littlewood-Offord, Hammersley, Kac and Erdös-Turán. In order to describe the problem let us consider polynomials $\sum_{j=0}^{p} c_j z^j$ of degree p in one complex variable. Let us choose the coefficients c_j to be independent, Gaussian random variables of mean zero and variance one. Then a classical result of Hammersley states that the divisors of zeros of a sequence $(Q_p)_p$ of such polynomials tend to concentrate on the unit circle $\{|z| = 1\}$ as $p \to \infty$. This is illustrated in the following picture by zeros of several random polynomials in one variable of degree 2, 8, 32, 1024.



Polynomials of degree p can be regarded as special cases of holomorphic sections in the pth tensor power of the hyperplane line bundle over the projective space. Shiffman and Zelditch [SZ99] and Dinh–Sibony [DS06] proved the following general result: if X is a compact complex manifold endowed with an ample line bundle (L, h), then the zero sets of random sequences of holomorphic sections of L^p, viewed as currents of integration, converge almost surely to the curvature form $c_1(L, h)$ as $p \to \infty$.

In these lectures we will explain distribution results in different contexts, for non-compact manifolds, Kähler spaces, singular Hermitian bundles (L, h), sequences of bundles L_p instead of powers L^p of one bundle, and several related aspects.

We mainly follow [CM15, CMM17, CMN16, DMM16, DMS12, MM07].

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Random perturbations of nonselfadjoint operators, and the Gaussian Analytic Function

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ABSTRACT

Following works of Hager and Sjöstrand, we study the spectrum of non-selfadjoint 1-dimensional differential operators $P_h = p(x, hD_x)$, with $p(x, \xi)$ a complex-valued function, to which we add a small random perturbations δQ_ω. The selfadjointness of P_h implies that the spectrum is very sensitive to the perturbation. Since Q_ω is random, we view the spectrum of $P_h + \delta Q_\omega$ as a *random point process* on $\mathbb{C}$.

The results of Hager-Sjöstrand show that, in the semiclassical régime $h \searrow 0$, this spectral point process has an asymptotic density when $h \to 0$, depending on the symbol p (probabilistic Weyl's law). Our aim is to better understand the local statistical properties of this point process, in particular its k-point correlation functions. I will show that, after a rescaling by the mean local density, the random point process converges to a limiting process, which depends on p and the type of perturbation, but still enjoys a form of universality. This limiting point process is given by the zero set of a certain random entire function, which can be expressed in terms of a simple elementary block: the Gaussian Analytic Function (GAF).

The GAF is a random entire series with Gaussian parameters. It appeared in the field of Quantum Chaos as a model for chaotic eigenfunctions (in a holomorphic representation), and in Kähler geometry as a local model for random holomorphic sections of positive line bundles. Its zero set has been described in detail in the literature. In the present problem, the GAF will arise through the spectral determinant of our randomly perturbed operator.

Symplectic topology, Hamiltonian dynamics and pseudoholomorphic curves

Yong-Guen Oh

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ABSTRACT

In this mini-course, I will introduce basic ingredients entering in the study of symplectic topology, Hamiltonian dynamics and techniques of pseudoholomorphic curves. I will in particular explain how these two ingredients are used in the analytic proof of nondegeneracy of the so called Hofer norm of the Hamiltonian diffeomorphism group.

Recent Development in Nevanlinna theory and Diophantine approximation

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ABSTRACT

In the talks, I'll give a survey with details on the recent development about the quantitive results (in the spirit of the Second Main type Theorem) for holomorphic mappings from the complex plane into algebraic varieties intersecting divisors. Our method is to use the classical H. Cartan's Cecond Main Theorem. Corresponding results in Diophantine approximation will also be discussed if time permits.

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Projective manifolds with nef anticanonical bundles

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ABSTRACT

Let X be a projective manifold with nef anticanonical bundle. We prove that the Albanese map is a submersion. We study also the universal cover of it.

Monge-Ampere operator for plurisubharmonic functions with analytic singularities

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ABSTRACT

W say that a plurisubharmonic function u has analytic singularities if locally it can be written in the form log|F|+B, where F is a tuple of holomorphic functions and B is bounded.

We discuss the Monge-Ampere operator for such functions defined earlier by Andersson and Wulcan, in particular continuity for decreasing approximationg sequences, maximality, and a total mass of such functions on compact Kahler manifolds. This is joint work with Mats Andersson and Elizabeth Wulcan.

Lelong numbers for singular metrics on vector bundles

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ABSTRACT

Singular hermitean metrics on vector bundles can be defined in analogy with singular metrics on line bundles. In general, such metrics do not have a well defined curvature tensor (in the sense of currents with measure coefficients), but one can nevertheless define what it means for them to be positively or negatively curved. We introduce a notion of Lelong numbers for such metrics that are negatively curved, and study it in a very special situation. We also give applications to the strong openness problem (now a theorem of Guan-Zhou) and the Ohsawa-Takegoshi extension theorem for singular varieties.

Curvature of higher direct image bundles

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(joint work with Mihai Paun and Xu Wang)

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ABSTRACT

The (0:th) direct image of the relative canonical bundle twisted with a positive line bundle has positive curvature. We discuss generalizations of this result to higher direct images. This generalizes previous work of Siu, Schumacher and To-Yeung.

Strictly plurisubharmonic functions and their minimum sets

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ABSTRACT

A set $K \subset \mathbb{C}^n$ is called the minimum set of a real valued function f if $f_{|K} = \min\{f(x) : x \in D_f\}$ and $f(x) > f_{|K}$ for any $x \in D_f \setminus K$. In this talk we will try to address the problem how the minimum sets of plurisubharmonic functions look like under certain additional assumptions.

A classical theorem by Harvey and Wells ([HW73]) states that the zero set of a nonnegative strongly plurisubharmonic function (of class $\mathcal{C}^2$) is contained in a totally-real submanifold of class $\mathcal{C}^1$ of the domain of definition of the function. In particular it follows that the Hausdorff dimension of this set is small (literally: does not exceed n) and that this set does not have an analytic structure.

We study how the assertion changes when we drop the regularity assumption or assume just that the Monge-Ampère measure is positive. The main findings are: K may contain analytic subsets, and its Hausdorff dimension can be (a bit) greater than n. I will also show examples of fractal sets which are minimum sets.

This is a joint work with Sławomir Dinew.

References

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