Bifurcations within holomorphic families of endomorphisms on $\mathbb{P}^k$

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ABSTRACT

Bifurcations within holomorphic families of rational maps on the Riemann sphere are well understood since the works of Mañé-Sad-Sullivan and, independently, Lyubich in the eighties.

Their techniques are however totally inoperant in a higher dimensional setting and one has therefore to adopt a different approach. For holomorphic families of endomorphisms on $\mathbb{P}^k$ this has been done in a joint work with Fabrizio Bianchi and Christophe Dupont. Our point of view on bifurcation phenomena combines ergodic and pluripotential-theoretic tools. In particular the Lyapunov exponents of the maximal entropy measure play an important role.

In this talk we will both describe this approach and discuss some recent related results.

Complexification of real manifolds and complex Hamiltonians

Dan Burns

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ABSTRACT

We present two problems in the complexification of real manifolds where complex Hamiltonians play an important role. The first is a geometric construction for the Exponential map of the formal complexification of the Hamiltonian diffeomorphism group proposed by Donaldson. Our construction is motivated by semi-classical analysis and is related to recent work on non-unitary quantization. It is joint work with Ernesto Lupercio and Alejandro Uribe [2].
The second example is about the determination of new invariants for the global properties of a Grauert tube complexification beyond the known curvature conditions. Invariant metrics on $SU(2)$ admit an invariant holomorphic extension to the complex group $SL(2,\mathbb{C})$. We study the geodesic flow on $SL(2,\mathbb{C})$ for this holomorphic metric. The key role is played by the complete integrability of this flow, coming from classical mechanics, as it is equivalent to the complexification of rigid body rotation in three dimensions. This is joint work with Vaqaas Aslam and Daniel Irvine [1].

References

[1] V. Aslam, D. Burns, D. Irvine, *Left-invariant Grauert tubes on SU(2)*, in preparation.

[2] D.Burns, E. Lupercio, A. Uribie, *The Exponential map of the complexification of Ham in the real-analytic case*, arXiv 1307.0493.

The deformed Hermitian-Yang-Mills equation

Tristan Collins[a], Adam Jacob[b], and Shing-Tung Yau[c]

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ABSTRACT

Mirror symmetry predicts that the moduli space of complex structures/ special Lagrangians on one Calabi-Yau is dual to the moduli space of complexified forms/ stable bundles on the mirror Calabi-Yau. However, the precise definition of a complexified Kahler form/stable bundle has remained mysterious. I will discuss these notions in the setting of Strominger-Yau-Zaslow mirror symmetry, the connection to fully nonlinear PDEs and algebro-geometric stability.

On the first order asymptotics of partial Bergman kernels

Dan Coman[a] and George Marinescu[b]

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ABSTRACT

Let L be a holomorphic line bundle over a compact complex manifold X, and h be a singular Hermitian metric on L. We discuss the asymptotics of the partial Bergman kernel function of the space of holomorphic sections of $L^{\otimes p}$ vanishing to high order along an analytic hypersurface Σ of X. We show that under very general assumptions it has exponential decay in a neighborhood of Σ. In the case when (L,h) is positive we obtain a uniform estimate of the Bergman kernel function associated to a singular metric along Σ. We use this to study the asymptotics of the partial Bergman kernel function on compact sets away from Σ and near the vanishing locus.

Holomorphic Legendrian curves

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ABSTRACT

I will present recent results on the existence and behaviour of noncompact holomorphic Legendrian curves in complex contact manifolds. While these curves are ubiquitous in the model contact space $(\mathbb{C}^{2n+1},\alpha)$ with $\alpha = dz + \sum_{j=1}^n x_j dy_j$ (see [1]), there also exist Kobayashi hyperbolic complex contact structures on $\mathbb{C}^{2n+1}$ which do not admit any nonconstant Legendrian complex lines [2]. Further, we construct a holomorphic Darboux chart around any noncompact holomorphic Legendrian curve in an arbitrary complex contact manifold; see [2]. As an application, we show that every bordered Legendrian curve can be uniformly approximated by complete bounded Legendrian curves.

References

[1] A. Alarcón, A., Forstnerič, F., and López, F.J.: Holomorphic Legendrian curves. Preprint (2016).

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[2] Forstnerič, F.: *Hyperbolic complex contact structures on* $\mathbb{C}^{2n+1}$. J. Geom. Anal., `http://dx.doi.org/10.1007/s12220-017-9800-9`

[2] Alarcón, A. and F. Forstnerič: *Darboux charts around holomorphic Legendrian curves and applications.* Preprint (2017). `arxiv.org/abs/1702.00704`

G-invariant Szegö kernel asymptotics and CR reduction

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ABSTRACT

Let $(X, T^{1,0}X)$ be a compact connected orientable CR manifold of dimension $2n+1$ with non-degenerate Levi curvature. Assume that X admits a connected compact Lie group action G. Under certain natural assumptions about the group action G, we show that the G-invariant Szegö kernel for $(0,q)$ forms is a complex Fourier integral operator, smoothing away $\mu^{-1}(0)$ and there is a precise description of the singularity near $\mu^{-1}(0)$, where μ denotes the CR moment map. We apply our result to the case when X admits a transversal CR S^1 action and deduce an asymptotic expansion for the m-th Fourier component of the G-invariant Szegö kernel for $(0,q)$ forms as $m \to +\infty$. As an application, we show that if m large enough, quantization commutes with reduction. This is a joint work with Rung-Tzung Huang.

Recognizing symplectic Grassmannians by their varieties of minimal rational tangents

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ABSTRACT

Let G/P be a rational homogeneous space of Picard number 1 and let X be a Fano manifold of Picard number 1. The question we are interested in is: if the variety of minimal rational tangents at a general point of X is isomorphic to that of G/P, is X biholomorphic to G/P? Ngaiming Mok gave an affirmative answer when G/P is a Hermitian symmetric space or a homogeneous contact manifold. Mok's argument was generalized by Hong-Hwang to any G/P associated with a long root. In these cases, the underlying geometric structure modeled on G/P is a parabolic geometry, for which local differential geometric machinery is well-developed in the classical works of Kobayashi, Nagano, Ochiai and Tanaka. The question has been open for the other G/P, i.e., symplectic Grassmannians or two cases of F_4-homogeneous spaces. The main difficulty in the remaining cases is that the underlying geometric structure is no longer a parabolic geometry and certain degenerate structures may occur in a neighborhood of a minimal rational curve. Because of the potential degeneracy, standard local differential geometric machinery is not available. In a collaboration with Qifeng Li, we overcome this difficulty by constructing a Cartan connection associated to the geometric structure in a neighborhood of a minimal rational curve, under the assumption that certain vector bundles arising from Spencer complexes do not have nonzero sections. Using this construction, we settle the case of symplectic Grassmannians. Our argument also works for odd-symplectic Grassmannians.

Poincaré-Bendixson theory for parabolic holomorphic foliations by curves

Serguei Ivachkovitch

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ABSTRACT

The classical Poincaré - Bendixson theory describes the way a trajectory of a vector field on the real plane behaves when accumulating to the singular locus of a vector field in question. In this talk we shall describe the way a leaf with contracting holonomy of a parabolic holomorphic foliation by curves on a compact complex manifold approaches the singular locus of the foliation.

Singular Yang-Mills connections on cylindrical Kahler manifolds

Adam Jacob

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ABSTRACT

Given a complete Kahler manifold X with a cylindrical end, in this talk I will prove existence of a singular Hermitian-Yang-Mills connection over X. Furthermore, I will demonstrate how to achieve exponential decay of a solution along the cylindrical end, and how to understand the structure of the singularity in certain cases. This work has applications to the study of G2-instantons, and is joint with T. Walpuski.

Weak solutions of complex Hessian equations on compact Hermitian manifolds

Sławomir Kołodziej

Jagiellonian University, Poland

ABSTRACT

This is joint work with Cuong Ngoc Nguyen. Let (X, ω) be a compact Hermitian manifold of complex dimension n. We prove the existence of weak solutions to the complex Hessian equation

$$(\omega + dd^c\varphi)^k \wedge \omega^{n-k} = cf\omega^n,$$

where $0 \leq f \in L^p(X, \omega^n)$, $p > n/k$, and u so called (ω, k) subharmonic function. For smooth, positive data the equation has recently been solved by Szekelyhidi and Zhang. We also obtain stability estimates for weak solutions.

An equivariant parametric Oka principle for bundles of homogeneous spaces

Finnur Lárusson

University of Adelaide, Australia

ABSTRACT

I will report on new joint work with Frank Kutzschebauch and Gerald Schwarz (arXiv:1612.07372). Under certain conditions, every continuous section of a holomorphic fibre bundle can be deformed to a holomorphic section. In fact, the inclusion of the space of holomorphic sections into the space of continuous sections is a weak homotopy equivalence. What if a complex Lie group acts on the bundle and its sections? We have proved a parametric Oka principle for equivariant sections of a holomorphic fibre bundle E with a structure group bundle $\mathcal{G}$ on a reduced Stein space X, such that the fibre of E is a homogeneous space of the fibre of $\mathcal{G}$, with the complexification $K^{\mathbb{C}}$ of a compact real Lie group K acting on X, $\mathcal{G}$, and E. Our main result is that the inclusion of the space of $K^{\mathbb{C}}$-equivariant holomorphic sections of E over X into the space of K- equivariant continuous sections is a weak homotopy equivalence. The result has a wide scope; if time permits, I will describe one or two special cases. We have applied the result to strengthen Heinzner and Kutzschebauch's classification of equivariant principal bundles, and to strengthen an Oka principle for equivariant isomorphisms proved by us in a previous paper.

On the dynamics of dissipative complex Henon maps

Misha Lyubich

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ABSTRACT

Dissipative Henon maps are polynomial automorphisms of C^2 which can be viewed as perturbations of one-dimensional quadratic polynomials. We will discuss several themes in this area: classification of periodic Fatou components, "critical points" in the Fatou components, problem of existence of wandering domains, and hedgehogs for Cremer maps.

Finite type pseudoconvex domains in Stein manifolds Grauert tubes, Hyperbolic surfaces, Monge Ampère determinants, and Cartan CR-curvatures

JOËL MERKER

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ABSTRACT

The first moment of the talk will present an effective result of Wei-Guo Foo (Ph. D., Orsay) based on achievements of Y-T. Siu. Specifically, consider finite type peudoconvex domains of the special shape:

$$\Big\{(z_1,z_2,z_3)\in\mathbb{C}^3\colon\ 2\,\mathsf{Re}\,z_3+\sum_{i=1}^N\big|F_i(z_1,z_2)\big|^2<0\Big\},$$

with holomorphic function germs $F_1,\ldots,F_N\in\mathcal{O}_{\mathbb{C}^2,0}$ having finite intersection multiplicity:

$$s\ :=\ \mathsf{dim}\,\mathcal{O}_{\mathbb{C}^2,0}\Big/\big\langle F_1,\ldots,F_N\big\rangle\ <\ \infty.$$

Theorem 1. *An ε-subelliptic regularity for $(0,1)$-forms holds whenever:*

$$\varepsilon\ \geqslant\ \frac{1}{2^{(4s^2-1)s+3}s^2(4s^2-1)^4\binom{8s+1}{8s-1}}.$$ □

The second moment of the talk will present a counterexample due to Zhangchi Chen (M2, Orsay) to Hartogs extension of holomorphic line bundles, namely for domains $\Omega\subset\mathbb{C}^n$, for compact sets $K\subset\subset\Omega$ with $\Omega\backslash K$ connected, to surjectivity of:

$$H^1\big(\Omega,\mathcal{O}^*\big)\ \longrightarrow\ H^1\big(\Omega\backslash K,\mathcal{O}^*\big).$$

In $\mathbb{C}^n$ $(n\geqslant 1)$, for $0<\epsilon<n$, introduce the domain:

$$G_\epsilon\ :=\ \Big\{z\in\mathbb{C}^n\colon\ \sum_{j=1}^n\big(\log\,|z_j|\big)^2<\epsilon\Big\},$$

which contains the n-dimensional standard totally real torus:

$$\mathbb{T}^n\ =\ \big\{|z_1|=\cdots=|z_n|=1\big\}\ \cong\ (S^1)^n.$$

For $0<\epsilon\ll n$ small, G_ϵ is a thin Grauert tube around $\mathbb{T}^n$, and it is always relatively compact in the large ball:

$$\Omega\ :=\ B\big(2\sqrt{n}\,e^{\sqrt{\epsilon}}\big).$$

Perforate a small ball-hole $p\subset B_p\subset\mathbb{C}^n$ centered at the point:

$$p\ :=\ \big(e^{\sqrt{\epsilon/n}},\ldots,e^{\sqrt{\epsilon/n}}\big)\ \in\ \partial G_\epsilon.$$

Theorem 2. *With the compact:*

$$K \,:=\, \partial G_\epsilon \big\backslash B_p \,\subset\, \Omega,$$

the open set $\Omega\backslash K$ is connected, and there exists a (nontrivial) holomorphic line bundle L on $\Omega\backslash K$ having the property that there exists no *holomorphic line bundle $\widetilde{L}$ on Ω that would extend $L = \widetilde{L}\big|_{\Omega\backslash K}$.* □

The third moment of the talk will present a result of The-Anh Ta (M2, Orsay). Consider the Poincaré upper half-plane:

$$\mathbb{H} \,:=\, \big\{x_1 + \sqrt{-1}\, x_2 \in \mathbb{C}\colon\ x_2 > 0\big\}$$

having the hyperbolic metric $ds^2 = \frac{dx_1^2+dx_2^2}{x_2^2}$. Equip its Bruhat-Whitney complexification:

$$\mathbb{H}^c \,:=\, \big\{(z_1, z_2) \in \mathbb{C}^2\colon\ \mathsf{Re}\, z_2 > 0\big\}$$

with the associated Guillemin-Stenzel-Lempert-Szöke Kähler metric, whose complexified distance function $\sqrt{\rho}$ enjoys Monge-Ampère vanishing:

$$0 \,\equiv\, \det\, \left(\frac{\partial^2 \sqrt{\rho}}{\partial z_i \partial w_j}(z, w)\right).$$

Let S be an abstract $\mathcal{C}^\omega$ compact genus $g \geqslant 2$ hyperbolic surface. By Poincaré-Köbe:

$$S \,\cong\, \mathbb{H}\big/\pi_1(S),$$

hence the hyperbolic metric on $\mathbb{H}$ descends to S.

Theorem 3. *Thin Grauert $\mathcal{C}^\omega$ tubes around S have no CR-umbilic points.* □

The proof utilizes an expanded explicit formula ($\sim 1\,500\,000$ terms) due to Merker-Sabzevari 2014 for the (single) complex-valued principal Cartan curvature $\mathfrak{I}$ of any $\mathcal{C}^\omega$ locally graphed hypersurface in $\mathbb{C}^2$.

The outcome is that in coordinates:

$$(z, w) \,=\, \big(x + \sqrt{-1}\, y,\, u + \sqrt{-1}\, v\big),$$

for the concerned graph:

$$v \,=\, \sqrt{\epsilon^2\, x^2 - y^2} \qquad (x > 0)$$

of the Grauert ϵ-tube around $\mathbb{H}^c$ with any $0 < \epsilon < 1$, the invariant compactifies strikingly as:

$$\mathfrak{I} \,=\, -\,\frac{9}{16}\, \frac{1-\varepsilon^4}{\big(\epsilon^2 x^2 - y^2\big)^2}\, \frac{\big(x + \sqrt{-1}\, y\big)^2}{\big(x - \sqrt{-1}\, y\big)^2} \,\neq\, 0.$$

The epilogue of the talk will sketch how a number of old results in CR geometry can nowadays be somehow instantly re-obtained thanks to a clever use of formal computational softwares.

Coble surfaces whose automorphism groups are virtually free

Shigeru Mukai

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ABSTRACT

The blow-up of the projective plane $\mathbb{P}^2$ at the nodes of a nodal rational plane sextic, allowed to be reducible, is a typical example of a *Coble* surface ([1]). One can define the twisted Picard lattice and root system of a Coble surface using the double $\mathbb{P}^2$ with branch the sextic, similarly to Enriques surfaces ([2, §5]). The automorphism group of a Coble surface is a subgroup of the 2-dimensional Cremona group Cr_2. It is always discrete, and shrinks from a very infinite group to a mildly infinite group and finally to a finite group under specialization, just like the Mordell-Weil group of an elliptic surface, *e.g.*, the blow-up of $\mathbb{P}^2$ at intersection of two cubics. After reviewing basic material, I will present some examples of Coble surfaces whose automorphism groups are very infinite, virtually free ([3]), and finite, respectively. This is a joint work with H. Ohashi.

References

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[3] Mukai, S. and Ohashi, S.: The automorphism groups of Enriques surfaces covered by symmetric quartic surfaces, in "Recent Advances in Algebraic Geometry" (eds. Hacon, Mustata and Popa), Cambridge Univ. Press, 2015, pp. 307-320.

Growth of the number of periodic points for meromorphic maps

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ABSTRACT

Let f be a dominant meromorphic self-map on a compact Kähler manifold X and $P_n(f)$ the number of its isolated periodic points of period n (counting multiplicity). We deal with the problem of finding a good upper bound for $P_n(f)$. Our method relies on the theory of densities and the theory of super-potentials for positive closed currents. This talk grows out of my recent joint-works with Tien-Cuong Dinh, Tuyen Trung Truong and Duc-Viet Vu.

Log canonical thresholds and Monge-Ampère masses

Hoang Hiep Pham

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ABSTRACT

In this talk, we prove an inequality for log canonical thresholds and Monge-Ampère masses. We also discuss open questions concerning to log canonical thresholds and Monge-Ampère masses.

Supersymmetric string vacua and complex geometry

Duong Hong Phong

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ABSTRACT

Superstring theory requires a 10-dimensional space-time, and thus a fundamental issue in this theory is to find compactifications to an effective 4-dimensional theory which preserve supersymmetry. This problem from high energy physics turns out to be one of complex geometry, because the infinitesimal generator of supersymmetry transformations produces a complex structure for the compactification space. We discuss the case of the heterotic string, and a system of equations proposed by C. Hull and A. Strominger which generalizes the classical Calabi-Yau solutions found in 1985 by P. Candelas, G. Horowitz, A. Strominger, and Witten. These equations allow metrics with torsion, so they may also be interesting from the point of view of non-Kähler geometry and the theory of partial differential equations. We discuss a geometric flow approach, proposed recently in joint work with S. Picard and X.W. Zhang.

Unique ergodicity for foliations and for discrete dynamical systems

Nessim Sibony

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ABSTRACT

Consider the polynomial differential equation in $\mathbb{C}^2$

$$\frac{dz}{dt} = P(z,w), \qquad \frac{dw}{dt} = Q(z,w).$$

The polynomials P and Q are holomorphic, the time is complex. In order to study the global behavior of the solutions, it is convenient to consider the extension as a foliation in the projective plane P^2.

Assume that the foliation is generic in the sense that its singular points are hyperbolic and that the line at infinity is the unique invariant algebraic curve. In this context, Khudai-Veronov has shown that except for the line at infinity all leaves are dense. This follows from the study of the holonomy on the invariant line. We show that there is a unique positive dd^c-closed $(1,1)$-current of mass 1 directed by the foliation and this is the current of integration on the invariant line. In a point of view from Nevanlinna's theory, every leaf of the foliation is concentrated near the invariant line. Although leaves are dense. A unique ergodicity theorem for the distribution of leaves follows: for any leaf L, appropriate averages of L converge to the current of integration on the invariant line. Similar results hold for foliations onKähler surfaces. The results are based on a new geometric method, the density of a current along a curve.

We will compare the previous result to analogue rigidity results for discrete dynamical systems. Rigidity results for Hénon type maps or for automorphisms of positive entropy on compact Kähler manifolds. It turns out that in this context many averages converge to the same limit.

The lecture is based on joint work with T.C. Dinh.

Analytic Methods of Constructing Bundle Sections and their Geometric Applications

Yum-Tong Siu

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ABSTRACT

Many important problems in complex geometry involve the construction of holomorphic sections of bundles with special properties. For example, the deformational invariance of plurigenera, the finite generation of the canonical ring, the abundance conjecture, effective results in algebraic geometry such as the Fujita conjecture, hyperbolicity problems, nonexistence problem for smooth Levi-flat hypersurface, the splitting of unstable plane bundle over complex projective space, algebraic-geometric techniques in the complex Neumann problem for weakly pseudoconvex domains. In these two talks, we will discuss from scratch the background and the motivation of the methods of constructing holomorphic sections of bundles. Then we will focus on some recent results and unsolved problems and promising approaches to them.

Identities for hyperbolic surfaces

Ser Peow Tan

National University of Singapore

ABSTRACT

We will give a general talk about identities for hyperbolic surfaces due variously to Basmajian, McShane, Bridgeman and Luo and Tan, and the general principles behind the derivation of these identities.

Weil-Petersson metric and hyperbolicity problems of some families of polarized manifolds.

Sai-Kee Yeung

Purdue University, USA

ABSTRACT

We would report on some joint work with Wing-Keung To on hyperbolicity of moduli spaces of polarized algebraic manifolds using the method of Weil-Petersson metric. We study families or moduli spaces of Kaehler-Einstein manifolds of negative scalar curvature or trivial scalar curvature, and some log-general type manifolds for quasi-projective versions. Classically, results on moduli space of Riemann surfaces of genus at least two have been obtained by Ahlfors, Royden and Wolpert with Weil-Petersson metric. Study of moduli of higher dimensional manifolds in terms of Weil-Petersson metrics began with a work of Siu thirty years ago. We would explain the curvature of some augmented Weil-Petersson metrics and their applications to Kobayashi hyperbolicity and log-general type properties of such families.

Positive scalar curvature on foliations

Weiping Zhang

Nankai University, China

ABSTRACT

We present generalizations of some classical results on the existence and/or nonexistence of metrics of positive scalar curvature on spin manifolds to the case of foliations.

Recent results on multiplier ideal sheaves

Xiangyu Zhou

Chinese Academy of Sciences, China

ABSTRACT

In this talk, we'll review some basic properties of the multplier ideal sheaves, and then present our recent solution of Demailly's strong openness conjecture and the related applications.