Randomness in α-recursion

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ABSTRACT

Algorithmic randomness is the field that studies random reals in the recursion theoretic point of view. It defines a random real as one that has no exceptionnal but sufficiently simple property. By considering higher computability, a notion derived from descriptive set theory, higher notions of randomness as Π^1_1-randomness have been defined. However, there is an even more general notion of computation, namely α-recursion, that includes higher computability and Infinite Time Turing Machine computation. This gives a framework for defining randomness notions.

In this talk, I will first define α-recursion and give some examples for particular α. Then, I will show how to use it to define randomness notions, give some relation between them and present the problem I'm working on.

Introduction to Geometric Stability Theory

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ABSTRACT

Motivated by Morley's categoricity theorem and a conjectured generalization for the possible number of uncountable models of first-order theories, Shelah isolated the important class of stable theories and developed a rich machinery for analyzing models and definable sets for this class. Later work by Zilber, Cherlin, Hrushovski, Pillay and others demonstrated that notions and methods of stability have strong geometric content ("geometry of forking"), elucidated the fact that understanding algebraic structures such as groups and fields definable in a structure is crucial even for purely model-theoretic questions (e.g. in the study of totally categorical theories), and found multiple applications in some of the more traditional branches of mathematics such as algebra and number theory.

The aim of this course is to provide an introduction to geometric stability. We will discuss the following topics: Morley's categoricity theorem, Shelah's classification, stable theories, forking independence, strongly minimal theories and pregeometries, local modularity and related notions, Zilber trichotomy, group and field configurations, totally categorical theories, connections to combinatorics.

As a preparatory reading, I suggest "Model Theory: An Introduction" by David Marker, in particular the later chapters are relevant for some of the material we will discuss in the course. For more advanced topics, see "Geometric stability theory" by Anand Pillay.

Compositions of multi-valued functions

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ABSTRACT

In reverse mathematics, one sometimes encounters proofs which invoke some theorem multiple times in series, or invoke different theorems in series. One example is the standard proof that Ramsey's theorem for 2 colors implies Ramsey's theorem for 3 colors. A natural question is whether such repeated applications are necessary. Questions like this can be studied under the framework of Weihrauch reducibility, where one can attempt to capture the notion of P being reducible to multiple instances of Q in series, for example. We study three known ways to formalize this idea, and clarify the relationships between them.

Modal categoricity

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ABSTRACT

An isomorphism is a canonical equivalence relation preserving the structure of considered model. It is natural to relativise our questions and ask about certain facts *up to isomorphism*. Another natural equivalence relation - capturing *behavioural* (instead of structural) equivalence is *bisimulation*. Modal logic (which is precisely the bisimulation invariant fragment of FO) is then a standard tool to describe structures *up to bisimulation*. In this talk I will present a new characterization of modal theories which are bisimulationally-categorical, i.e. have a unique model up to bisimulation.

Computable Model Theory

STEFFEN LEMPP

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ABSTRACT

Detailed course description: I plan to focus on the spectra of computable models question, a problem which has driven a lot of research in computable model theory over the past decades and has led to new and exciting results even in classical model theory.

The Baldwin-Lachlan Theorem states that for an uncountably but not totally categorical first-order theory T, the countable models of T form an increasing elementary chain of models A_a (for $\alpha \leq \omega$, where A_0 is the prime model and A_ω is the countable saturated motel of T). If the language of T is computable (i.e., the arities of the language symbols can be effectively determined), then we can let SCM(T), the spectrum of computable models of T, be the set of all $\alpha \leq \omega$ such that A_α is a computable model, i.e., has a copy with universe ω such that the open diagram of the model is computable.

Clearly, the empty set and $[0, \omega]$ are possible spectra, but there are others: Goncharov (1978) was the first to show that $\{0\}$ is a possible spectrum, and a number of other possible spectra have been discovered since. However, all known spectra are unions of at most two intervals, and the best known general upper bound is not even arithmetical.

If one imposes stronger conditions on T, one can get better results. E.g., Andrews and Medvedev have shown that if the language is finite and the models are strongly minimal (i.e., the subsets of the model definable with parameters are all either finite or cofinite), then in "many" cases, there are only three possible spectra. Andrews and I have extended this result in on-going work to infinite languages of bounded arity.

Recommended preparatory reading: While no knowledge of logic beyond basics assumed for the summer school is needed, you will get more out of the course if you familiarize yourself with the material (but not necessarily all the proofs!) on uncountably categorical models (e.g., in Tent/Ziegler, chapter 5) and on computability theory and computable model theory (e.g., in Ash/Knight, chapters 1 and 3).

Please refer to updates on my webpage at http://www.math.wisc.edu/ lempp/teach/IMS17.html

Infinite time Turing machines, an introduction to gaps

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ABSTRACT

I will present you a part of my PhD thesis entitled "From algorithmics to logics through infinite time computation" in which I have studied the infinite time Turing machines model of computation from a computer scientist point of view. In particular I focused on the structure of gaps in the clockable ordinals, that is to say, ordinal times at which no infinite time program halts.

So in this talk I will present infinite time Turing machines (ITTM), from the original definition of the model to some new infinite time algorithms. I will show you some information about the structure of gaps and the properties of the ITTM-computable ordinals, most of them obtained by such algorithmic techniques.

Introduction to Ramsey Algebras

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Abstract

The study of Ramsey algebras can be thought of as a Ramseyan type of study on algebras (in the sense of universal algebra). Ramsey algebra has its roots in Carlson's (topological) Ramsey space [1]. In this talk, I will introduce the notion of a Ramsey algebra with a focus on examples. The relation between Ramsey algebras and Ramsey spaces will be briefly given. We will see examples and nonexamples of Ramsey algebras as well as some selected results. Finally, I will present some open problems.

References

[1] Carlson, T.J.: Some unifying principles in Ramsey theory. Discrete Mathematics **68**(2), 117–169 (1988)

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Definability of ω in models of n-definable elements

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ABSTRACT

Let M be any model of Peano Arithmetic, let a be a nonstandard element in M, and let K be the Σ_n Skolem hull of a in M. One can check that in K, its standard part ω can defined with a Σ_{n+1} formula. During IMS 2017 summer school a question has been asked by Theodore Slaman whether in K, ω can be also defined with a Π_{n+1} formula (which would make omega Δ_{n+1} definable).

We answer that question in negative. I.e., we show that one can find a model of PA in which omega is not Δ_{n+1} definable. The proof essentially relies on some standard arguments in the theory of models of Peano Arithmetic.

Reverse Mathematics and Divisible Abelian Groups

Huishan Wu

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ABSTRACT

In reverse algebra, we study the logical strength of theorems of countable algebra in second order arithmetic. In this talk, I will present several old and new results in reverse algebra; in particular, I will focus on several classical theorems related to countable divisible abelian groups.

Reductions on Equivalence Relations Generated by Universal Sets

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ABSTRACT

There is interest in reductions on equivalence relations generated by universal sets, i.e. given Polish spaces X, Y and a universal set $\mathrm{A} \subseteq \mathrm{X} \times \mathrm{Y}$, $xE_Ax' \leftrightarrow \{y: \mathrm{A}(x, y)\} = \{y: \mathrm{A}(x', y)\}$. If A is universal for the closed subset of Y, $E_A \leq_{\sigma(\boldsymbol{\Sigma}_1^1)} id(2^\omega)$. If A is a Borel set universal for the countable subset of Y, $E_A \nleq_{\boldsymbol{\Delta}_1^1} id(2^\omega)$. If A is an analytic set universal for the countable subset of Y, whether $E_A \leq_{\boldsymbol{\Delta}_2^1} id(2^\omega)$ holds or not is independent.

In this talk, I plan to present some results we have achieved, the obstacles we are facing and some closely related open problems.

Suggestions, questions and interest are all welcome!