Entanglement and coherence in quantum state merging

A. Streltsov[1,2,3,4], E. Chitambar[5], S. Rana[4], M. N. Bera[4], A. Winter[6,7], M. Lewenstein[4,7]

[1]Gdańsk University of Technology, Poland
[2]National Quantum Information Centre of Gdańsk, Poland
[3]Freie Universität Berlin, Germany
[4]ICFO, Castelldefels (Barcelona), Spain
[5]Southern Illinois University, Carbondale, USA
[6]Universitat Autònoma de Barcelona, Spain
[7]ICREA, Barcelona, Spain

Singapore
July 28, 2017

Outline

1. Resource theory of quantum coherence
   - Incoherent states and operations
   - Quantifying coherence
   - Quantum coherence in distributed scenarios
2. Quantum state merging
   - Standard quantum state merging
   - Incoherent quantum state merging

Outline

1. Resource theory of quantum coherence
   - Incoherent states and operations
   - Quantifying coherence
   - Quantum coherence in distributed scenarios

2. Quantum state merging
   - Standard quantum state merging
   - Incoherent quantum state merging

Incoherent states and operations[1][2]

- A quantum state is called *incoherent* if it is diagonal in some preferred basis:

$$\sigma = \sum_i p_i \left|i\right\rangle \left\langle i\right|, \tag{1}$$

  and any other state is called *coherent*. The set of all incoherent states will be called $\mathcal{I}$.

[1] T. Baumgratz, M. Cramer, and M. B. Plenio, PRL (2014)

[2] A. S., G. Adesso, and M. B. Plenio, arXiv (2016), to be published in RMP

Incoherent states and operations[1,2]

- A quantum state is called *incoherent* if it is diagonal in some preferred basis:

$$\sigma = \sum_i p_i |i\rangle\langle i|, \tag{1}$$

  and any other state is called *coherent*. The set of all incoherent states will be called $\mathcal{I}$.

- A quantum operation is called incoherent if it can be written as

$$\Lambda[\rho] = \sum_i K_i \rho K_i^\dagger \tag{2}$$

  with incoherent Kraus operators K_i such that

$$K_i \mathcal{I} K_i^\dagger \subseteq \mathcal{I}. \tag{3}$$

---

[1] T. Baumgratz, M. Cramer, and M. B. Plenio, PRL (2014)

[2] A. S., G. Adesso, and M. B. Plenio, arXiv (2016), to be published in RMP

Alternative frameworks of coherence

- Maximally incoherent operations (MIO)[1]: most general set, contains all operations which cannot create coherence: $\Lambda[\rho_{\rm i}] \in \mathcal{I}$, where $\mathcal{I}$ is the set of all incoherent states.

---

[1] J. Åberg, arXiv (2006)

[2] A. Winter and D. Yang, PRL (2016); B. Yadin, J. Ma, D. Girolami, M. Gu, V. Vedral, PRX (2016)

[3] G. Gour and R. W. Spekkens, NJP (2008)

[4] E. Chitambar and G. Gour, PRL (2016); I. Marvian and R. W. Spekkens, PRA (2016)

Alternative frameworks of coherence

- Maximally incoherent operations (MIO)[1]: most general set, contains all operations which cannot create coherence: $\Lambda[\rho_\mathrm{i}] \in \mathcal{I}$, where $\mathcal{I}$ is the set of all incoherent states.

- Strictly incoherent operations (SIO)[2]: Incoherent operations for which also $K_i^\dagger$ are incoherent.

[1] J. Åberg, arXiv (2006)

[2] A. Winter and D. Yang, PRL (2016); B. Yadin, J. Ma, D. Girolami, M. Gu, V. Vedral, PRX (2016)

[3] G. Gour and R. W. Spekkens, NJP (2008)

[4] E. Chitambar and G. Gour, PRL (2016); I. Marvian and R. W. Spekkens, PRA (2016)

Alternative frameworks of coherence

- Maximally incoherent operations (MIO)[1]: most general set, contains all operations which cannot create coherence: $\Lambda[\rho_{\mathrm{i}}] \in \mathcal{I}$, where $\mathcal{I}$ is the set of all incoherent states.

- Strictly incoherent operations (SIO)[2]: Incoherent operations for which also $K_i^\dagger$ are incoherent.

- Translationally invariant operations (TIO)[3]: Quantum operations which commute with time translations, i.e., $e^{-iHt}\Lambda[\rho]e^{iHt} = \Lambda[e^{-iHt}\rho e^{iHt}]$ for a given Hamiltonian H.

[1] J. Åberg, arXiv (2006)

[2] A. Winter and D. Yang, PRL (2016); B. Yadin, J. Ma, D. Girolami, M. Gu, V. Vedral, PRX (2016)

[3] G. Gour and R. W. Spekkens, NJP (2008)

[4] E. Chitambar and G. Gour, PRL (2016); I. Marvian and R. W. Spekkens, PRA (2016)

Alternative frameworks of coherence

- Maximally incoherent operations (MIO)[1]: most general set, contains all operations which cannot create coherence: $\Lambda[\rho_\mathrm{i}] \in \mathcal{I}$, where $\mathcal{I}$ is the set of all incoherent states.

- Strictly incoherent operations (SIO)[2]: Incoherent operations for which also $K_i^\dagger$ are incoherent.

- Translationally invariant operations (TIO)[3]: Quantum operations which commute with time translations, i.e., $e^{-iHt}\Lambda[\rho]e^{iHt} = \Lambda[e^{-iHt}\rho e^{iHt}]$ for a given Hamiltonian H.

- Dephasing-covariant incoherent operations (DIO)[4]: Quantum operations which commute with dephasing, i.e., $\Delta[\Lambda(\rho)] = \Lambda[\Delta(\rho)]$ with the dephasing operation Δ.

---

[1] J. Åberg, arXiv (2006)

[2] A. Winter and D. Yang, PRL (2016); B. Yadin, J. Ma, D. Girolami, M. Gu, V. Vedral, PRX (2016)

[3] G. Gour and R. W. Spekkens, NJP (2008)

[4] E. Chitambar and G. Gour, PRL (2016); I. Marvian and R. W. Spekkens, PRA (2016)

Quantifying coherence

Coherence monotones have the following properties[1]:

1. $C(\rho) \geq 0$, and equality holds if and only if ρ is incoherent,
2. $C(\rho)$ is nonincreasing under incoherent operations:

$$C(\rho) \geq C(\Lambda[\rho]). \tag{4}$$

[1] T. Baumgratz, M. Cramer, and M. B. Plenio, PRL (2014).

Quantifying coherence

Coherence monotones have the following properties[1]:

1. $C(\rho) \geq 0$, and equality holds if and only if ρ is incoherent,
2. $C(\rho)$ is nonincreasing under incoherent operations:

$$C(\rho) \geq C(\Lambda[\rho]). \tag{4}$$

Many coherence monotones are additionally nonincreasing on average under selective incoherent operations:

$$C(\rho) \geq \sum_i q_i C(\sigma_i) \tag{5}$$

with $q_i = \mathrm{Tr}[K_i \rho K_i^\dagger]$ and $\sigma_i = K_i \rho K_i^\dagger / q_i$.

[1] T. Baumgratz, M. Cramer, and M. B. Plenio, PRL (2014).

Quantifying coherence

Two important coherence monotones are[1]:

- Coherence cost: quantifies the rate of maximally coherent states $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ required to create a state ρ via incoherent operations in the asymptotic limit.

$$C_c(\rho) = C_f(\rho) = \min \sum_i p_i C_r(|\psi_i\rangle). \tag{6}$$

[1] A. Winter and D. Yang, PRL (2016).

Quantifying coherence

Two important coherence monotones are[1]:

- Coherence cost: quantifies the rate of maximally coherent states $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ required to create a state ρ via incoherent operations in the asymptotic limit.

$$C_c(\rho) = C_f(\rho) = \min \sum_i p_i C_r(|\psi_i\rangle). \tag{6}$$

- Distillable coherence: quantifies the maximal rate for extracting maximally coherent states $|+\rangle$ via incoherent operations in the asymptotic limit.

$$C_d(\rho) = S(\Delta[\rho]) - S(\rho) \tag{7}$$

with the dephasing operation Δ.

[1] A. Winter and D. Yang, PRL (2016).

Quantifying coherence

Two important coherence monotones are[1]:

- Coherence cost: quantifies the rate of maximally coherent states $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ required to create a state ρ via incoherent operations in the asymptotic limit.

$$C_c(\rho) = C_f(\rho) = \min \sum_i p_i C_r(|\psi_i\rangle). \tag{6}$$

- Distillable coherence: quantifies the maximal rate for extracting maximally coherent states $|+\rangle$ via incoherent operations in the asymptotic limit.

$$C_d(\rho) = S(\Delta[\rho]) - S(\rho) \tag{7}$$

with the dephasing operation Δ.

- The quantities differ for different frameworks of coherence.

[1] A. Winter and D. Yang, PRL (2016).

Quantifying coherence

Other important coherence monotones:

- Relative entropy of coherence[1]

$$C_r(\rho) = \min_{\sigma \in \mathcal{I}} S(\rho \| \sigma). \tag{8}$$

Note that $C_r(\rho) = C_d(\rho) = S(\Delta[\rho]) - S(\rho)$.

[1] T. Baumgratz, M. Cramer, M. B. Plenio, PRL (2014).

[2] C. Napoli, T. R. Bromley, M. Cianciaruso, M. Piani, N. Johnston, G. Adesso, PRL (2016).

[3] T. Biswas, M. García-Díaz, A. Winter, arXiv (2017).

Quantifying coherence

Other important coherence monotones:

- Relative entropy of coherence[1]

$$C_r(\rho) = \min_{\sigma \in \mathcal{I}} S(\rho \| \sigma). \tag{8}$$

Note that $C_r(\rho) = C_d(\rho) = S(\Delta[\rho]) - S(\rho)$.

- Robustness of coherence[2]

$$R_c(\rho) = \min_{\tau} \left\{ s \geq 0 \,\middle|\, \frac{\rho + s\tau}{1 + s} \in \mathcal{I} \right\}. \tag{9}$$

Operational interpretation via interferometric visibility[3].

---

[1] T. Baumgratz, M. Cramer, M. B. Plenio, PRL (2014).

[2] C. Napoli, T. R. Bromley, M. Cianciaruso, M. Piani, N. Johnston, G. Adesso, PRL (2016).

[3] T. Biswas, M. García-Díaz, A. Winter, arXiv (2017).

Quantum coherence in distributed scenarios[1,2]

Similar to LOCC operations in entanglement theory, it is possible to define local quantum-incoherent operations and classical communication (LQICC).

[1] E. Chitambar, A. S., S. Rana, M. N. Bera, G. Adesso, and M. Lewenstein, PRL 2016

[2] A. S., S. Rana, M. N. Bera, and M. Lewenstein, PRX 2017

Quantum coherence in distributed scenarios[1,2]

Similar to LOCC operations in entanglement theory, it is possible to define local quantum-incoherent operations and classical communication (LQICC).

Properties of LQICC operations:

- LQICC operations preserve the set of quantum-incoherent states (QI):

$$\rho_{\mathrm{qi}}^{AB} = \sum_i p_i \sigma_i^A \otimes |i\rangle\langle i|^B . \tag{10}$$

[1] E. Chitambar, A. S., S. Rana, M. N. Bera, G. Adesso, and M. Lewenstein, PRL 2016

[2] A. S., S. Rana, M. N. Bera, and M. Lewenstein, PRX 2017

Quantum coherence in distributed scenarios[1,2]

Similar to LOCC operations in entanglement theory, it is possible to define local quantum-incoherent operations and classical communication (LQICC).

Properties of LQICC operations:

- LQICC operations preserve the set of quantum-incoherent states (QI):

$$\rho_{\mathrm{qi}}^{AB} = \sum_i p_i \sigma_i^A \otimes |i\rangle \langle i|^B . \tag{10}$$

- LQICC operations cannot increase the QI relative entropy

$$C_r^{A|B}(\rho^{AB}) = \min_{\sigma^{AB} \in \mathcal{QI}} S(\rho^{AB} \| \sigma^{AB}) = S(\Delta^B[\rho^{AB}]) - S(\rho^{AB}). \tag{11}$$

QI relative entropy is additive in the input state.

---

[1] E. Chitambar, A. S., S. Rana, M. N. Bera, G. Adesso, and M. Lewenstein, PRL 2016

[2] A. S., S. Rana, M. N. Bera, and M. Lewenstein, PRX 2017

Application: Assisted distillation of quantum coherence[1,2]

Alice and Bob share many copies a bipartite state ρ^{AB} and can perform bipartite LQICC operations.

Aim of the task: asymptotic distillation of maximally coherent states $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ on Bob's side.

[1] E. Chitambar, A. S., S. Rana, M. N. Bera, G. Adesso, and M. Lewenstein, PRL 2016

[2] A. S., S. Rana, M. N. Bera, and M. Lewenstein, PRX 2017

Application: Assisted distillation of quantum coherence[1,2]

Alice and Bob share many copies a bipartite state ρ^{AB} and can perform bipartite LQICC operations.

Aim of the task: asymptotic distillation of maximally coherent states $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ on Bob's side.

- The figure of merit is the distillable coherence of collaboration which is bounded as follows:

$$C_d^{A|B}(\rho^{AB}) \leq C_r^{A|B}(\rho^{AB}) = S(\Delta^B[\rho^{AB}]) - S(\rho^{AB}). \qquad (12)$$

[1] E. Chitambar, A. S., S. Rana, M. N. Bera, G. Adesso, and M. Lewenstein, PRL 2016

[2] A. S., S. Rana, M. N. Bera, and M. Lewenstein, PRX 2017

Application: Assisted distillation of quantum coherence[1,2]

Alice and Bob share many copies a bipartite state ρ^{AB} and can perform bipartite LQICC operations.

Aim of the task: asymptotic distillation of maximally coherent states $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ on Bob's side.

- The figure of merit is the distillable coherence of collaboration which is bounded as follows:

$$C_d^{A|B}(\rho^{AB}) \leq C_r^{A|B}(\rho^{AB}) = S(\Delta^B[\rho^{AB}]) - S(\rho^{AB}). \qquad (12)$$

- For pure states $|\psi\rangle^{AB}$ we have

$$C_d^{A|B}(|\psi\rangle^{AB}) = C_r^{A|B}(|\psi\rangle^{AB}) = S(\Delta[\rho^B]). \qquad (13)$$

[1] E. Chitambar, A. S., S. Rana, M. N. Bera, G. Adesso, and M. Lewenstein, PRL 2016

[2] A. S., S. Rana, M. N. Bera, and M. Lewenstein, PRX 2017

Application: Assisted distillation of quantum coherence[1,2]

Alice and Bob share many copies a bipartite state ρ^{AB} and can perform bipartite LQICC operations.

Aim of the task: asymptotic distillation of maximally coherent states $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ on Bob's side.

- The figure of merit is the distillable coherence of collaboration which is bounded as follows:

$$C_d^{A|B}(\rho^{AB}) \leq C_r^{A|B}(\rho^{AB}) = S(\Delta^B[\rho^{AB}]) - S(\rho^{AB}). \qquad (12)$$

- For pure states $|\psi\rangle^{AB}$ we have

$$C_d^{A|B}(|\psi\rangle^{AB}) = C_r^{A|B}(|\psi\rangle^{AB}) = S(\Delta[\rho^B]). \qquad (13)$$

- Without assistance Bob can distill coherence at rate

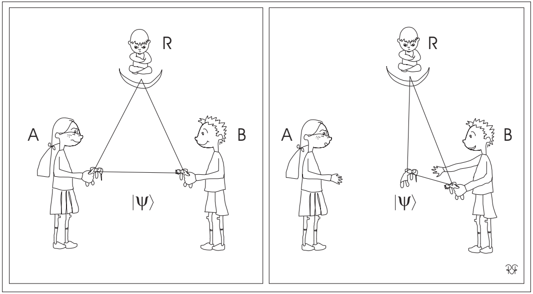
$$C_d(\rho^B) = S(\Delta[\rho^B]) - S(\rho^B). \qquad (14)$$

[1] E. Chitambar, A. S., S. Rana, M. N. Bera, G. Adesso, and M. Lewenstein, PRL 2016

[2] A. S., S. Rana, M. N. Bera, and M. Lewenstein, PRX 2017

Outline

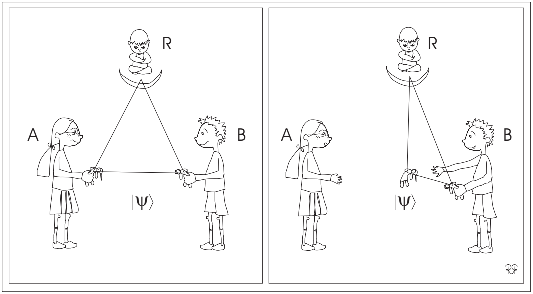
Standard quantum state merging



M. Horodecki, J. Oppenheim, and A. Winter, Nature (2005).

- Alice, Bob, and a referee share many copies of a pure state $|\psi\rangle^{RAB}$.

Standard quantum state merging



M. Horodecki, J. Oppenheim, and A. Winter, Nature (2005).

- Alice, Bob, and a referee share many copies of a pure state $|\psi\rangle^{RAB}$.
- Aim of Alice and Bob: merge their systems on Bob's side while preserving the total state, i.e., the final state $|\psi\rangle^{RBB'}$ is the same as $|\psi\rangle^{RAB}$ up to relabeling A and B'.

Standard quantum state merging[1]

- For this purpose, Alice and Bob have access to shared singlets and a classical channel.

[1] M. Horodecki, J. Oppenheim, and A. Winter, Nature 2005.

Standard quantum state merging[1]

- For this purpose, Alice and Bob have access to shared singlets and a classical channel.
- The minimal number of singlets, asymptotically needed per copy of the state $|\psi\rangle^{RAB}$, is given by the conditional entropy:

$$S(A|B) = S(\rho^{AB}) - S(\rho^{B}). \quad (15)$$

- Recall that in quantum theory the conditional entropy can be either positive or negative.

[1] M. Horodecki, J. Oppenheim, and A. Winter, Nature 2005.

Standard quantum state merging[1]

- For this purpose, Alice and Bob have access to shared singlets and a classical channel.

- The minimal number of singlets, asymptotically needed per copy of the state $|\psi\rangle^{RAB}$, is given by the conditional entropy:

$$S(A|B) = S(\rho^{AB}) - S(\rho^{B}). \quad (15)$$

- Recall that in quantum theory the conditional entropy can be either positive or negative.

- If $S(A|B)$ is positive, merging is possible with singlets at rate $S(A|B)$, and merging is not possible if less singlets are available.
- If $S(A|B)$ is negative, merging is possible without singlets, and Alice and Bob can additionally gain singlets at rate $-S(A|B)$.

[1] M. Horodecki, J. Oppenheim, and A. Winter, Nature 2005.

Incoherent quantum state merging[1]

- In standard quantum state merging, shared entanglement is considered as an expensive resource, while local coherence is available at no additional cost.

[1] A. S., E. Chitambar, S. Rana, M. N. Bera, A. Winter, and M. Lewenstein, PRL 2016.

Incoherent quantum state merging[1]

- In standard quantum state merging, shared entanglement is considered as an expensive resource, while local coherence is available at no additional cost.
- In incoherent quantum state merging we also take Bob's local coherence into account.

[1] A. S., E. Chitambar, S. Rana, M. N. Bera, A. Winter, and M. Lewenstein, PRL 2016.

Incoherent quantum state merging[1]

- In standard quantum state merging, shared entanglement is considered as an expensive resource, while local coherence is available at no additional cost.
- In incoherent quantum state merging we also take Bob's local coherence into account.
- In particular, for a tripartite state ρ^{RAB}, we consider state merging via LQICC operations, where additional singlets and maximally coherent states on Bob's side are provided at rates E and C.

[1] A. S., E. Chitambar, S. Rana, M. N. Bera, A. Winter, and M. Lewenstein, PRL 2016.

Incoherent quantum state merging[1]

- In standard quantum state merging, shared entanglement is considered as an expensive resource, while local coherence is available at no additional cost.
- In incoherent quantum state merging we also take Bob's local coherence into account.
- In particular, for a tripartite state ρ^{RAB}, we consider state merging via LQICC operations, where additional singlets and maximally coherent states on Bob's side are provided at rates E and C.
- We consider optimal entanglement-coherence pairs (E, C): these are pairs of entanglement and coherence rate for which merging is possible, but neither E nor C can be reduced.

[1] A. S., E. Chitambar, S. Rana, M. N. Bera, A. Winter, and M. Lewenstein, PRL 2016.

Incoherent quantum state merging[1]

- In standard quantum state merging, shared entanglement is considered as an expensive resource, while local coherence is available at no additional cost.
- In incoherent quantum state merging we also take Bob's local coherence into account.
- In particular, for a tripartite state ρ^{RAB}, we consider state merging via LQICC operations, where additional singlets and maximally coherent states on Bob's side are provided at rates E and C.
- We consider optimal entanglement-coherence pairs (E, C): these are pairs of entanglement and coherence rate for which merging is possible, but neither E nor C can be reduced.
- The main problem is to determine all such optimal pairs (E, C) for a given state ρ^{RAB}.

[1] A. S., E. Chitambar, S. Rana, M. N. Bera, A. Winter, and M. Lewenstein, PRL 2016.

Incoherent quantum state merging[1]

Theorem

Given a tripartite quantum state ρ^{RAB}, any achievable pair (E, C) fulfills the following inequality:

$$E + C \geq S\left(\Delta^{AB}[\rho^{RAB}]\right) - S\left(\Delta^{B}[\rho^{RAB}]\right). \quad (16)$$

[1] A. S., E. Chitambar, S. Rana, M. N. Bera, A. Winter, and M. Lewenstein, PRL 2016.

Incoherent quantum state merging[1]

Theorem

Given a tripartite quantum state ρ^{RAB}, any achievable pair (E, C) fulfills the following inequality:

$$E + C \geq S\left(\Delta^{AB}[\rho^{RAB}]\right) - S\left(\Delta^{B}[\rho^{RAB}]\right). \tag{16}$$

Here, S is the von Neumann entropy and Δ^X denotes full decoherence of a (possibly multipartite) subsystem X:

$$\Delta^X[\rho] = \sum_i |i\rangle\langle i|^X \rho\, |i\rangle\langle i|^X . \tag{17}$$

[1] A. S., E. Chitambar, S. Rana, M. N. Bera, A. Winter, and M. Lewenstein, PRL 2016.

Incoherent quantum state merging[1]

Theorem

Given a tripartite quantum state ρ^{RAB}, any achievable pair (E, C) fulfills the following inequality:

$$E + C \geq S\left(\Delta^{AB}[\rho^{RAB}]\right) - S\left(\Delta^{B}[\rho^{RAB}]\right). \tag{16}$$

Here, S is the von Neumann entropy and Δ^X denotes full decoherence of a (possibly multipartite) subsystem X:

$$\Delta^X[\rho] = \sum_i |i\rangle\langle i|^X \rho \, |i\rangle\langle i|^X . \tag{17}$$

Since the right-hand side of Eq. (16) is nonnegative, the sum $E + C$ is also nonnegative: **no merging procedure can gain coherence and entanglement at the same time**.

[1] A. S., E. Chitambar, S. Rana, M. N. Bera, A. Winter, and M. Lewenstein, PRL 2016.

Incoherent quantum state merging[1]

For pure states $|\psi\rangle^{RAB}$ we have the following bounds:

$$E \geq E_{\min} = S(\rho^{AB}) - S(\rho^{B}), \tag{18}$$

$$E + C \geq S(\Delta^{AB}[\rho^{AB}]) - S(\Delta^{B}[\rho^{B}]). \tag{19}$$

Crucially, the bound in Eq. (19) is achievable for all pure states.

[1] A. S., E. Chitambar, S. Rana, M. N. Bera, A. Winter, and M. Lewenstein, PRL 2016.

Incoherent quantum state merging[1]

For pure states $|\psi\rangle^{RAB}$ we have the following bounds:

$$E \geq E_{\min} = S(\rho^{AB}) - S(\rho^{B}), \tag{18}$$

$$E + C \geq S(\Delta^{AB}[\rho^{AB}]) - S(\Delta^{B}[\rho^{B}]). \tag{19}$$

Crucially, the bound in Eq. (19) is achievable for all pure states.

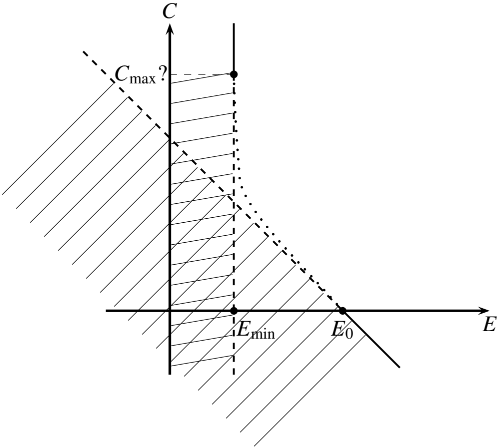
Theorem

Any pure state $|\psi\rangle^{RAB}$ can be merged without local coherence by using singlets at rate

$$E_0 = S(\Delta^{AB}[\rho^{AB}]) - S(\Delta^{B}[\rho^{B}]). \tag{20}$$

[1] A. S., E. Chitambar, S. Rana, M. N. Bera, A. Winter, and M. Lewenstein, PRL 2016.

Incoherent quantum state merging[1]



$$E_{\min} = S(\rho^{AB}) - S(\rho^{B}), \quad (21)$$

$$E_0 = S(\Delta^{AB}[\rho^{AB}]) - S(\Delta^{B}[\rho^{B}]). \quad (22)$$

1 A. S., E. Chitambar, S. Rana, M. N. Bera, A. Winter, and M. Lewenstein, PRL 2016.

Incoherent quantum state merging[1]

For some mixed states ρ^{RAB} it is possible to find all optimal pairs.
Example: states of the form

$$\rho^{RAB} = \sum_{i,j} p_{ij} \left|ij\right\rangle\left\langle ij\right|^{R} \otimes \left|\psi_{ij}\right\rangle\left\langle\psi_{ij}\right|^{A} \otimes \left|i\right\rangle\left\langle i\right|^{B}, \tag{23}$$

where $|\psi_{ij}\rangle$ are mutually orthogonal for different j. Note that this state can be merged without entanglement.

[1] A. S., E. Chitambar, S. Rana, M. N. Bera, A. Winter, and M. Lewenstein, PRL 2016.

Incoherent quantum state merging[1]

For some mixed states ρ^{RAB} it is possible to find all optimal pairs.
Example: states of the form

$$\rho^{RAB} = \sum_{i,j} p_{ij} \left|ij\right\rangle \left\langle ij\right|^{R} \otimes \left|\psi_{ij}\right\rangle \left\langle \psi_{ij}\right|^{A} \otimes \left|i\right\rangle \left\langle i\right|^{B}, \tag{23}$$

where $|\psi_{ij}\rangle$ are mutually orthogonal for different j. Note that this state can be merged without entanglement.
All optimal pairs are given by

$$(E, C) = (aC_{\max}, [1 - a]C_{\max}) \tag{24}$$

with $a \geq 0$ and $C_{\max} = \sum_{i,j} p_{ij} S(\Delta[\psi_{ij}])$.

[1] A. S., E. Chitambar, S. Rana, M. N. Bera, A. Winter, and M. Lewenstein, PRL 2016.

Incoherent quantum state merging[1]

There is evidence that a large amount of local coherence can be saved by using little extra entanglement, i.e., that for some states the pairs $(E, C \gg 0)$ and $(E' = E + \varepsilon, C' \ll C)$ are both optimal for small ε.

[1] A. S., E. Chitambar, S. Rana, M. N. Bera, A. Winter, and M. Lewenstein, PRL 2016.

Incoherent quantum state merging[1]

There is evidence that a large amount of local coherence can be saved by using little extra entanglement, i.e., that for some states the pairs $(E, C \gg 0)$ and $(E' = E + \varepsilon, C' \ll C)$ are both optimal for small ε.

Possible candidate for such states:

$$\rho = \frac{1}{d_B} \sum_{i=0}^{d_B - 1} |i\rangle\langle i|^R \otimes |\phi_i\rangle\langle\phi_i|^A \otimes |\psi_i\rangle\langle\psi_i|^B , \qquad (25)$$

where $|\psi_i\rangle$ are mutually orthogonal maximally coherent states of arbitrary dimension d_B, and $|\phi_i\rangle$ are single-qubit states.

[1] A. S., E. Chitambar, S. Rana, M. N. Bera, A. Winter, and M. Lewenstein, PRL 2016.

Incoherent quantum state merging[1]

There is evidence that a large amount of local coherence can be saved by using little extra entanglement, i.e., that for some states the pairs $(E, C \gg 0)$ and $(E' = E + \varepsilon, C' \ll C)$ are both optimal for small ε.

Possible candidate for such states:

$$\rho = \frac{1}{d_B} \sum_{i=0}^{d_B-1} |i\rangle\langle i|^R \otimes |\phi_i\rangle\langle\phi_i|^A \otimes |\psi_i\rangle\langle\psi_i|^B , \qquad (25)$$

where $|\psi_i\rangle$ are mutually orthogonal maximally coherent states of arbitrary dimension d_B, and $|\phi_i\rangle$ are single-qubit states.

- This state can be merged without entanglement if Bob performs a local von Neumann measurement in the basis $\{|\psi_i\rangle\}$, and conditionally prepares the states $|\phi_i\rangle$. However, this process requires a large amount of coherence.
- Alice and Bob can use one singlet for teleporting Alice's system to Bob, in which case no coherence is needed.

[1] A. S., E. Chitambar, S. Rana, M. N. Bera, A. Winter, and M. Lewenstein, PRL 2016.

Summary

- We introduced the task of incoherent quantum state merging, in which both entanglement and local coherence are considered as a resource.

Summary

- We introduced the task of incoherent quantum state merging, in which both entanglement and local coherence are considered as a resource.
- We showed that the entanglement-coherence sum in this procedure are bounded below as

$$E + C \geq S(\Delta^{AB}[\rho^{RAB}]) - S(\Delta^{B}[\rho^{RAB}]). \tag{26}$$

- This implies that no merging procedure can gain entanglement and coherence at the same time.

Summary

- We introduced the task of incoherent quantum state merging, in which both entanglement and local coherence are considered as a resource.
- We showed that the entanglement-coherence sum in this procedure are bounded below as
$$E + C \geq S(\Delta^{AB}[\rho^{RAB}]) - S(\Delta^{B}[\rho^{RAB}]). \quad (26)$$
- This implies that no merging procedure can gain entanglement and coherence at the same time.
- The bound is tight for all pure states: any pure state can be merged without coherence by using singlets at rate $E_0 = S(\Delta^{AB}[\rho^{AB}]) - S(\Delta^{B}[\rho^{B}])$.
- Our results imply an incoherent version of Schumacher compression: $S(\Delta[\rho])$ is the optimal compression rate for a state ρ if the decompression has to be performed via incoherent operations only.

For more details see
A. Streltsov, E. Chitambar, S. Rana,
M. N. Bera, A. Winter, and M. Lewenstein,
Phys. Rev. Lett. **116**, 240405 (2016)

and

A. Streltsov, G. Adesso, and M. B. Plenio,
arXiv:1609.02439,
to be published in Rev. Mod. Phys.