Deconstruction and Conditional Erasure of Correlations

Joint work with Mario Berta, Fernando Brandao, and Mark Wilde

(arXiv:1609.06994)

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Introduction:
Decoupling and Erasure

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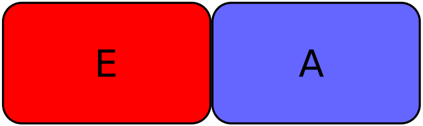

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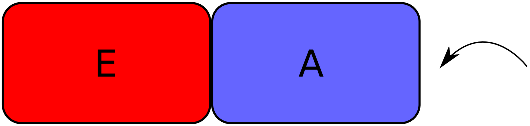

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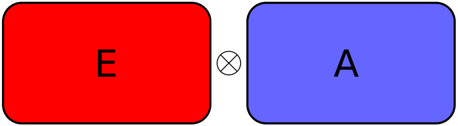

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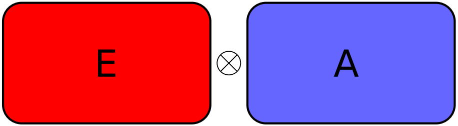

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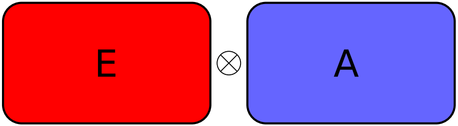

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$\Rightarrow$ Operational interpretation of the quantum mutual information!

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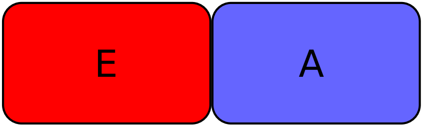

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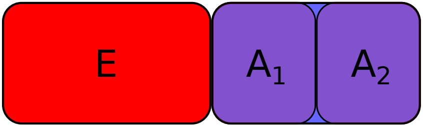

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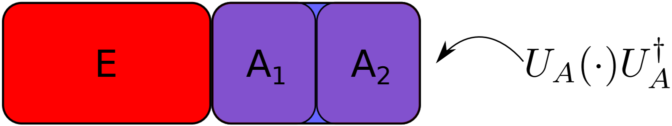

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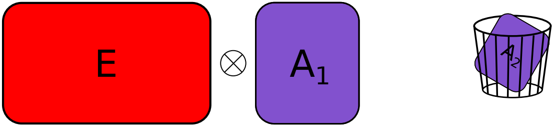

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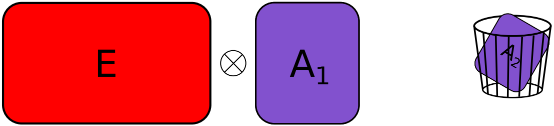

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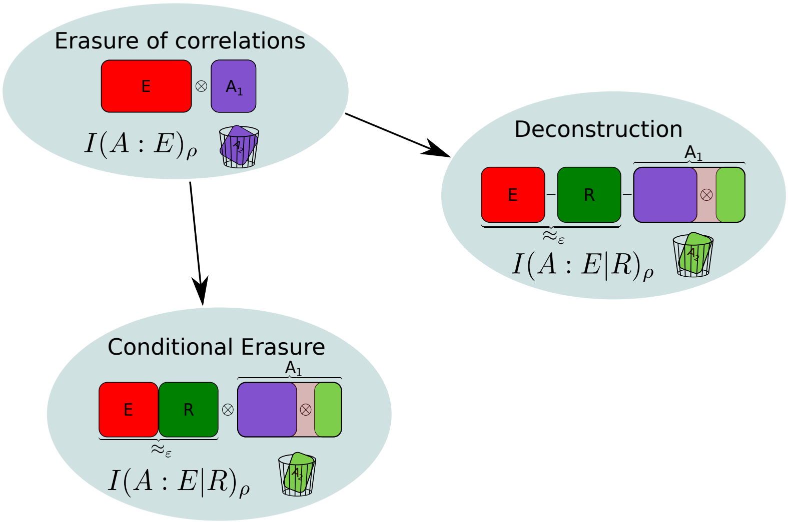
! Erasure models ar related, exact one shot equivalence if ancillary states are allowed

This talk



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E
A1
I(A : E)ρ
Deconstruction
A1
E
R
≈ε
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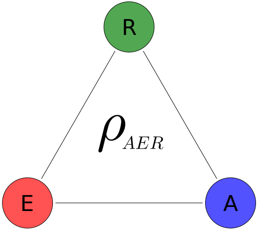

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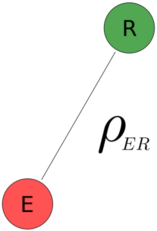

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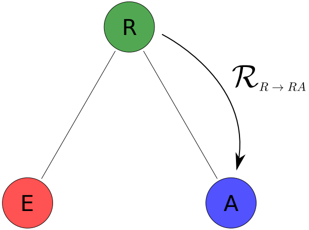

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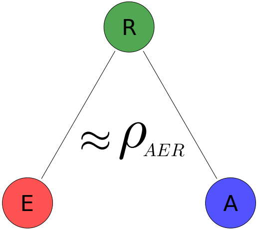

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- $I(A:E|R)$ measures *conditional correlations*

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! No, as shown by Wakakuwa et al. (2016, Poster at BIID2016)

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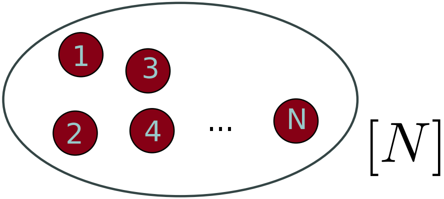

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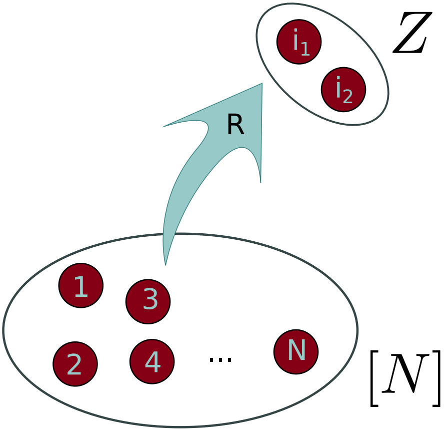

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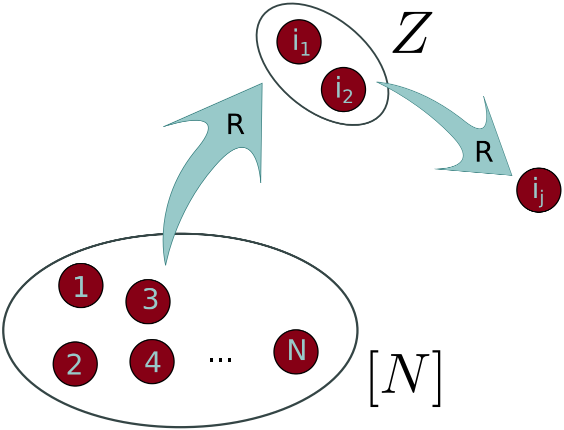

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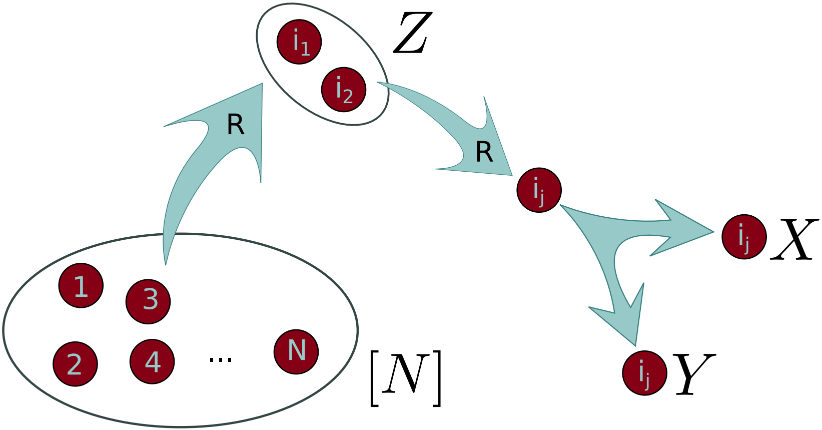

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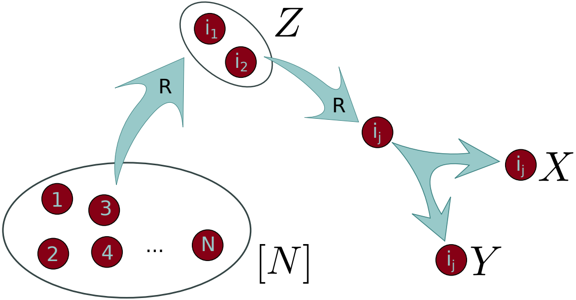

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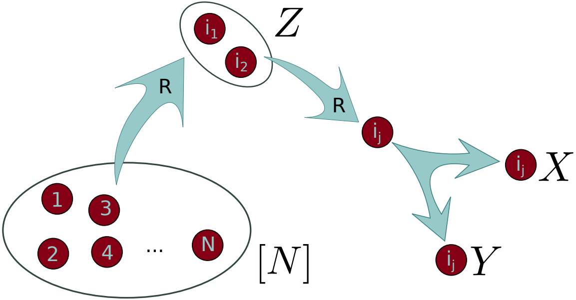

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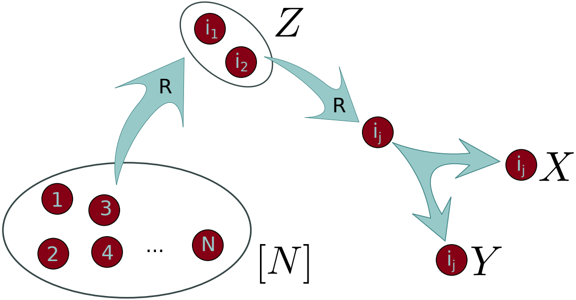

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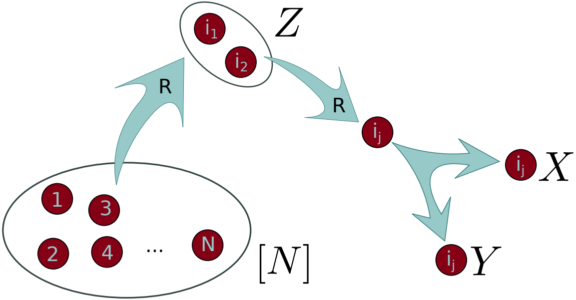

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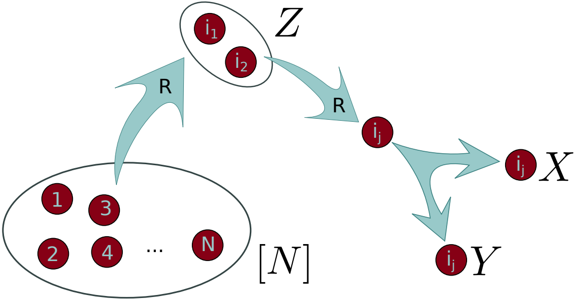

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- need this for most pairs $(i_1, i_2) \Rightarrow M$ small

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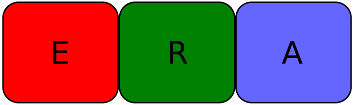

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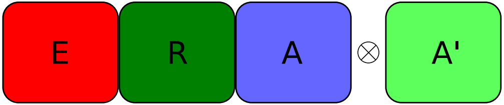

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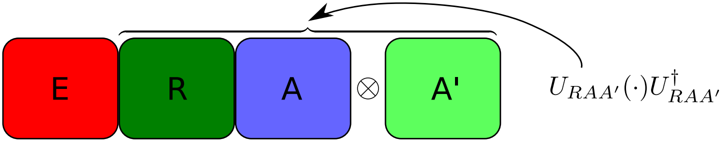

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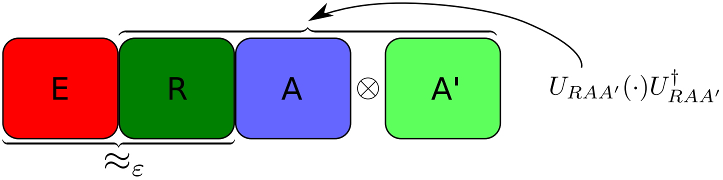

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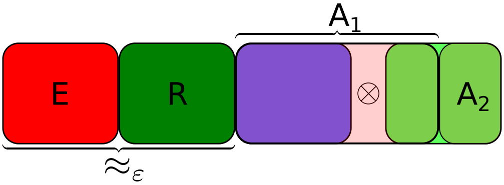

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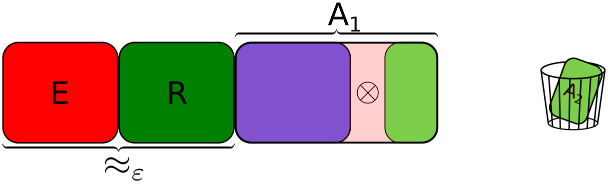
- trace out A_2



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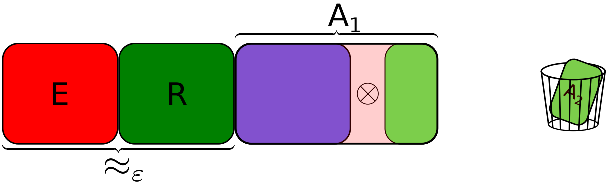

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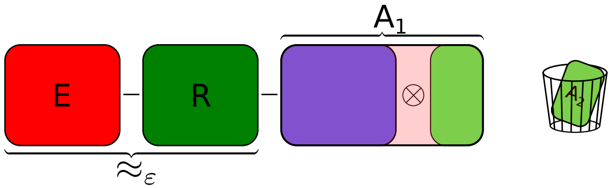
- Different goals:
- make $E - R - A_1$ an approximate quantum Markov chain, *deconstruction* of correlations



Deconstruction, conditional erasure II

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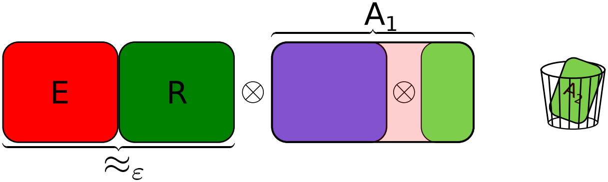
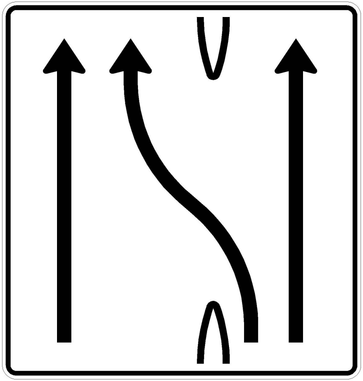
- make A_1 product with ER, *conditional erasure* of correlations ($\Rightarrow$ deconstruction of correlations)



State redistribution



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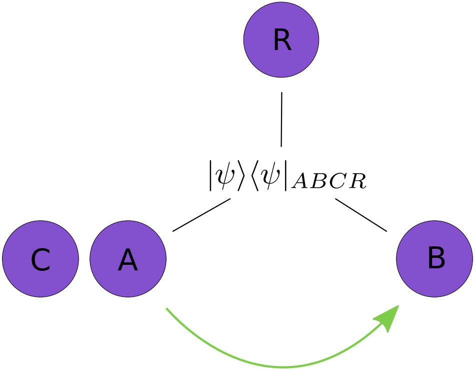

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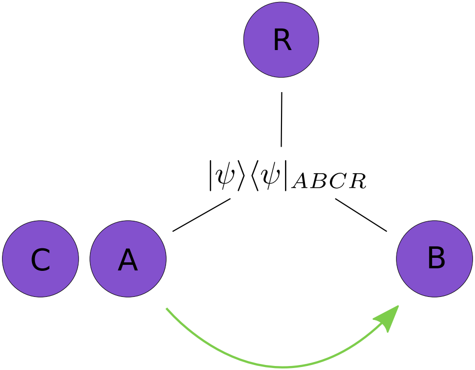

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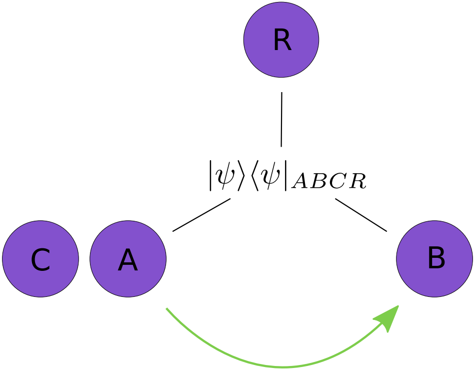

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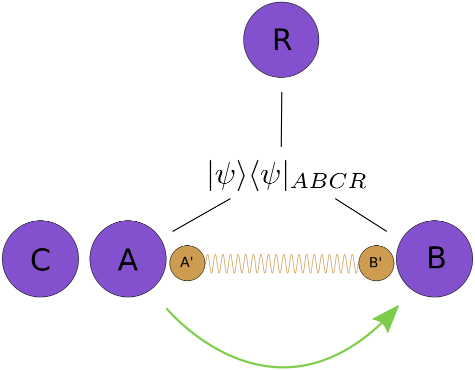

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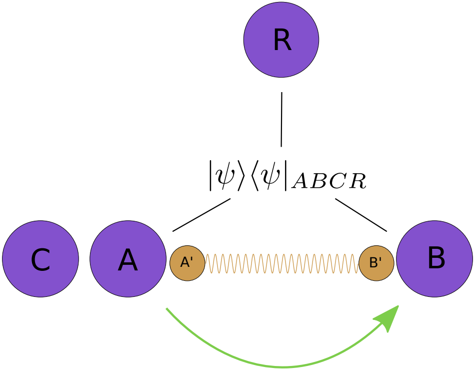
- optimal comunication rate $\frac{1}{2}I(A:R|C)$ (Devetak & Yard '06)



Characterization theorem

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- Operational interpretation of quantum conditional mutual information!

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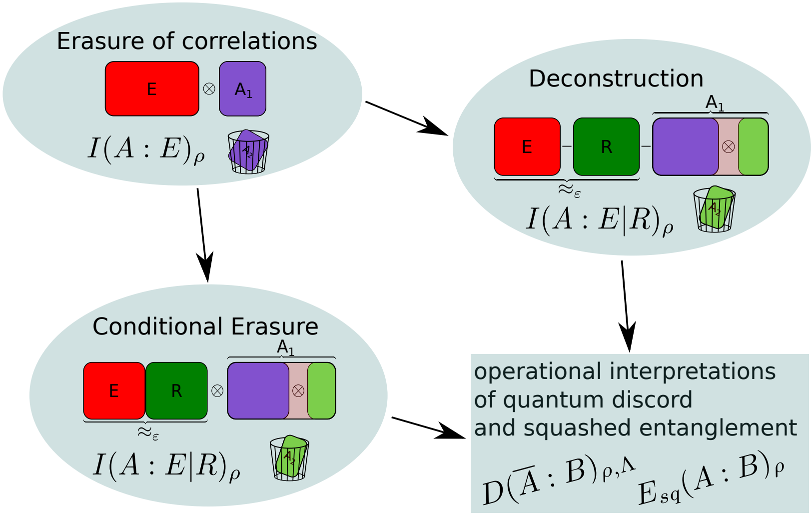
- Other application related to Squashed entanglement:
  $E_{sq}(A:B)_\rho = \inf_\sigma I(A:B|E)_\sigma$, inf over all σ_{ABE} with $\mathrm{tr}_E\, \sigma_{ABE} = \rho_{AB}$

The End


Erasure of correlations
E
⊗
A1
$I(A:E)_\rho$
Deconstruction
A1
E
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Conditional Erasure
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operational interpretations
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backup slide

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"$\Leftarrow$":

- Replace the decoupling protocol in the standard state merging protocol by the conditional erasure protocol at hand