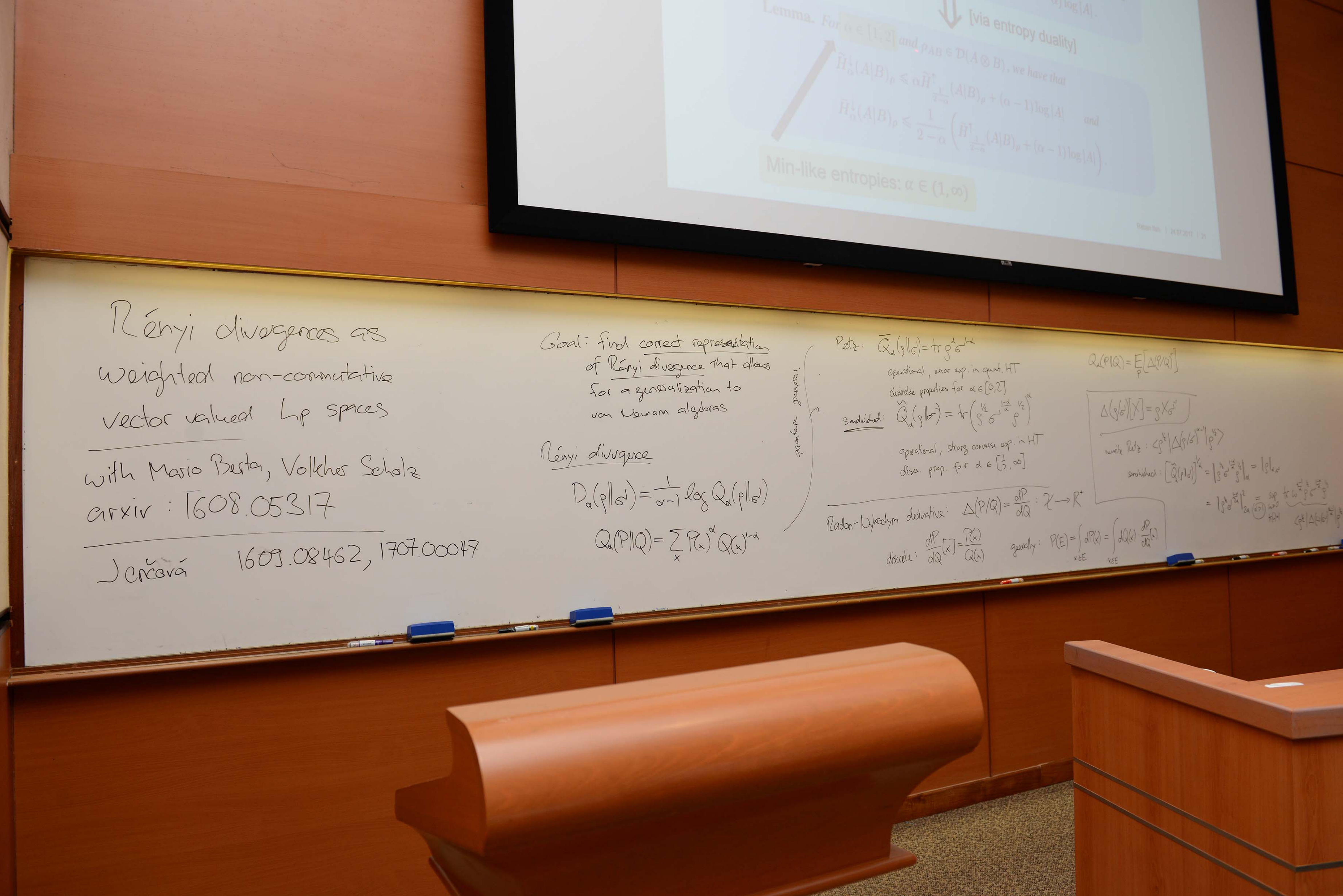
[via entropy duality]
Min-like entropies: $\alpha \in (1,\infty)$
Rényi divergences as weighted non-commutative vector valued Lp spaces
with Mario Berta, Volkher Scholz
arxiv: 1608.05317
Jenčová 1609.08462, 1707.00047
Goal: find correct representation of Rényi divergence that allows for a generalization to von Neumann algebras
Rényi divergence
$D_\alpha(\rho\|\sigma) = \frac{1}{\alpha-1}\log Q_\alpha(\rho\|\sigma)$
$Q_\alpha(P\|Q) = \sum_x P(x)^\alpha Q(x)^{1-\alpha}$
quantum general
Petz: $\bar{Q}_\alpha(\rho\|\sigma) = \mathrm{tr}\,\rho^\alpha\sigma^{1-\alpha}$
operational, error exp. in quant. HT
desirable properties for $\alpha \in [0,2]$
sandwiched: $\hat{Q}_\alpha(\rho\|\sigma) = \mathrm{tr}\left(\rho^{1/2}\sigma^{\frac{1-\alpha}{\alpha}}\rho^{1/2}\right)^\alpha$
operational, strong converse exp. in HT
desir. prop. for $\alpha \in [\frac{1}{2},\infty]$
Radon-Nykodym derivative: $\Delta(P/Q) = \frac{dP}{dQ} : \mathcal{X} \to \mathbb{R}^+$
discrete: $\frac{dP}{dQ}[x] = \frac{P(x)}{Q(x)}$
generally: $P(E) = \int_{x\in E} dP(x) = \int_{x\in E} dQ(x)\,\frac{dP}{dQ}[x]$
$Q_\alpha(P\|Q) = \mathop{E}_P\left[\Delta(P/Q)^{\alpha-1}\right]$
$\Delta(\rho/\sigma)[X] = \rho X \sigma^{-1}$
rewrite Petz: $\langle \rho^{1/2} | \Delta(\rho/\sigma)^{\alpha-1} | \rho^{1/2} \rangle$

Petz: $\bar{Q}_\alpha(\rho\|\sigma) = \operatorname{tr} \rho^\alpha \sigma^{1-\alpha}$

operational, error exp. in quant. HT

desirable properties for $\alpha \in [0,2]$

Sandwiched: $\hat{Q}_\alpha(\rho\|\sigma) = \operatorname{tr}\left(\rho^{1/2} \sigma^{\frac{1-\alpha}{\alpha}} \rho^{1/2}\right)^\alpha$

operational, strong converse exp. in HT

des. prop. for $\alpha \in [\frac{1}{2}, \infty]$

$Q_\alpha(P\|Q) = \mathop{E}_{P}\left[\Delta(P/Q)^{\alpha-1}\right]$

$\boxed{\Delta(\rho/\sigma)[X] = \rho X \sigma^{-1}}$

rewrite Petz: $\langle \rho^{1/2} | \Delta(\rho/\sigma)^{\alpha-1} | \rho^{1/2} \rangle$

sandwiched: $\left[\hat{Q}_\alpha(\rho\|\sigma)\right]^{1/\alpha} = \left\|\rho^{1/2} \sigma^{\frac{1-\alpha}{\alpha}} \rho^{1/2}\right\|_\alpha = \|\rho\|_{\alpha,\sigma}$

$= \left\|\rho^{1/2} \sigma^{\frac{1-\alpha}{2\alpha}}\right\|_{2\alpha}^2 \underset{\alpha>1}{=} \sup_{\substack{\omega \geq 0 \\ \operatorname{tr}(\omega)=1}} \underbrace{\operatorname{tr} \omega^{\frac{\alpha-1}{\alpha}} \rho^{1/2} \sigma^{\frac{1-\alpha}{\alpha}} \rho^{1/2}}_{\langle \rho^{1/2} | \Delta(\omega/\sigma)^{\frac{\alpha-1}{\alpha}} | \rho^{1/2} \rangle}$

Radon-Nykodym derivative: $\Delta(P/Q) = \frac{dP}{dQ} : \mathcal{X} \to \mathbb{R}^+$

discrete: $\frac{dP}{dQ}[x] = \frac{P(x)}{Q(x)}$

generally: $P(E) = \int_{x \in E} dP(x) = \int_{x \in E} dQ(x) \cdot \frac{dP}{dQ}[x]$