Pretty good measures in QIT

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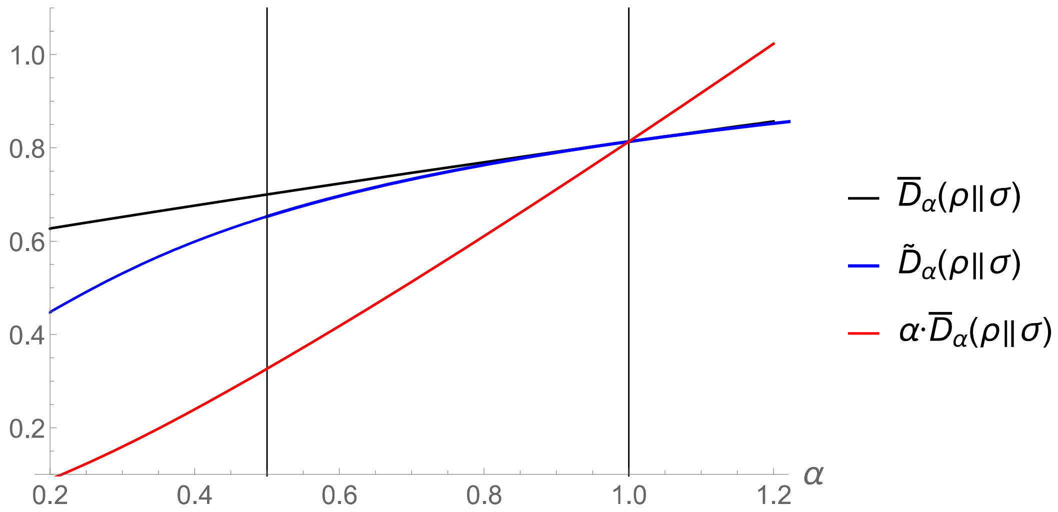

Beyond I.I.D., 24 July 2017, Singapore

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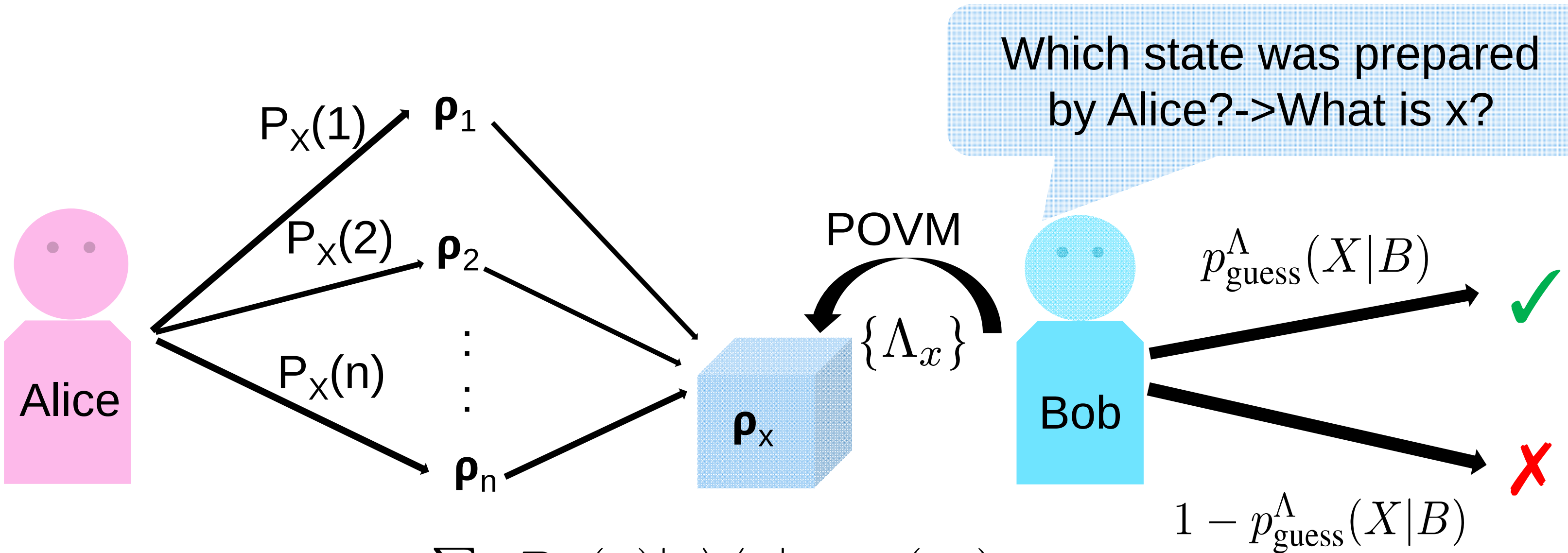
Motivation

Setup

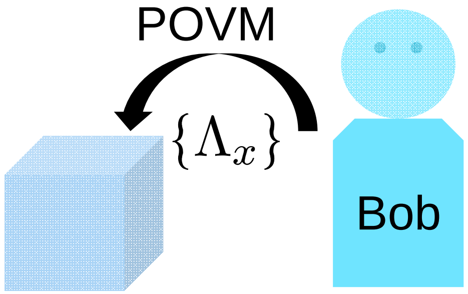


$$\rho_{XB} = \sum_x P_X(x)|x\rangle\langle x|_X \otimes (\rho_x)_B$$

$$p^{\Lambda}_{\text{guess}}(X|B) := \sum_x P_X(x) \operatorname{tr} \Lambda_x \rho_x \longrightarrow p_{\text{guess}}(X|B) := \max_{\Lambda} p^{\Lambda}_{\text{guess}}(X|B)$$

Not easy to find...

Pretty good (pg) measurement



Choose $\Lambda_x^{\mathrm{pg}} := P_X(x)\,\hat{\rho}^{-\frac{1}{2}}\,\rho_x\,\hat{\rho}^{-\frac{1}{2}}$

with $\hat{\rho} := \sum_x P_X(x)\,\rho_x$ [1,2]

How good is this choice of measurement?

$$p_{\mathrm{guess}}^{\mathrm{pg}}(X|B) \leqslant p_{\mathrm{guess}}(X|B) \leqslant \sqrt{p_{\mathrm{guess}}^{\mathrm{pg}}(X|B)}$$ [3]

with $p_{\mathrm{guess}}^{\mathrm{pg}}(X|B) := p_{\mathrm{guess}}^{\Lambda^{\mathrm{pg}}}(X|B)$

Remark:
Recently improved
by Joseph Renes

[1]: Belavkin, 1975
[2]: Hausladen and Wootters, 1994
[3]: Barnum and Knill, 2002

Structure of today's talk

Structure of today's talk

"Pure" Math

Reverse Araki-Lieb-Thirring (ALT) inequality

New relations between the Petz and the minimal divergence and between conditional Rényi entropies

Pretty good measures in QIT

- Introduction of the pretty good fidelity
- Bounds and optimality conditions for the pretty good measurement and singlet fraction

Physics

Reverse ALT inequality

ALT and reverse ALT inequality (for $r \in [0,1]$)

Theorem (ALT inequality [4,5]). *Let A and B be positive semi-definite matrices and $r \in [0,1]$. Then*

$$\operatorname{tr}(B^{\frac{r}{2}} A^r B^{\frac{r}{2}}) \leqslant \operatorname{tr}(B^{\frac{1}{2}} A B^{\frac{1}{2}})^r .$$

Theorem (Reverse ALT inequality). *Let A and B be positive semi-definite matrices. Then, for $r \in (0,1]$ and $a, b \in (0,\infty]$ such that $\frac{1}{2r} = \frac{1}{2} + \frac{1}{a} + \frac{1}{b}$, we have*

$$\operatorname{tr}(B^{\frac{1}{2}} A B^{\frac{1}{2}})^r \leqslant \left(\operatorname{tr}(B^{\frac{r}{2}} A^r B^{\frac{r}{2}})\right)^r \left\| A^{\frac{1-r}{2}} \right\|_a^{2r} \left\| B^{\frac{1-r}{2}} \right\|_b^{2r} .$$

Schatten norms

$\|M\|_p := \left(\operatorname{tr}|M|^p\right)^{\frac{1}{p}}$

with $|M| := \sqrt{M^* M}$

[4]: Lieb and Thirring, 1976
[5]: Araki, 1990

Proof of reverse ALT inequality

Theorem (Reverse ALT inequality)**.** *Let A and B be positive semi-definite matrices. Then, for $r \in (0,1]$ and $a, b \in (0,\infty]$ such that $\frac{1}{2r} = \frac{1}{2} + \frac{1}{a} + \frac{1}{b}$, we have*

$$\operatorname{tr}\left(B^{\frac{1}{2}} A B^{\frac{1}{2}}\right)^r \leqslant \left(\operatorname{tr}\left(B^{\frac{r}{2}} A^r B^{\frac{r}{2}}\right)\right)^r \left\|A^{\frac{1-r}{2}}\right\|_a^{2r} \left\|B^{\frac{1-r}{2}}\right\|_b^{2r}.$$

Proof (for $r = \frac{1}{2}$).

$$\operatorname{tr}\left(B^{\frac{1}{2}} A B^{\frac{1}{2}}\right)^{\frac{1}{2}} = \left\|B^{\frac{1}{2}} A^{\frac{1}{2}}\right\|_1 = \left\|B^{\frac{1}{4}} B^{\frac{1}{4}} A^{\frac{1}{4}} A^{\frac{1}{4}}\right\|_1 \leqslant \left\|B^{\frac{1}{4}}\right\|_b \left\|B^{\frac{1}{4}} A^{\frac{1}{4}}\right\|_2 \left\|A^{\frac{1}{4}}\right\|_a$$

Proof of reverse ALT inequality

Theorem (Generalized Hölder inequality [6]). *Let s, $s_1, \dots, s_l$ be positive real numbers (where we also allow ∞ using the convention that $\frac{1}{\infty} = 0$) and $\{A_k\}_{k=1}^{l}$ be a collection of $n \times n$ matrices. Then*

$$\left\| \prod_{k=1}^{l} A_k \right\|_s \leqslant \prod_{k=1}^{l} \|A_k\|_{s_k} , \qquad \textit{for} \quad \sum_{k=1}^{l} \frac{1}{s_k} = \frac{1}{s} .$$

Choose $s = 1$, and $s_1 = b$, $s_2 = 2$, and $s_3 = a$ for some $a, b \in (0, \infty]$ with $\frac{1}{1} = \frac{1}{2} + \frac{1}{a} + \frac{1}{b}$

Proof (for $r = \frac{1}{2}$).

$$\operatorname{tr}\left(B^{\frac{1}{2}} A B^{\frac{1}{2}}\right)^{\frac{1}{2}} = \left\| B^{\frac{1}{2}} A^{\frac{1}{2}} \right\|_1 = \left\| B^{\frac{1}{4}} B^{\frac{1}{4}} A^{\frac{1}{4}} A^{\frac{1}{4}} \right\|_1 \leqslant \left\| B^{\frac{1}{4}} \right\|_b \left\| B^{\frac{1}{4}} A^{\frac{1}{4}} \right\|_2 \left\| A^{\frac{1}{4}} \right\|_a$$

[6]: Bhatia, Matrix Analysis, Exercise IV.2.7

Proof of reverse ALT inequality

Theorem (Reverse ALT inequality). *Let A and B be positive semi-definite matrices. Then, for $r \in (0,1]$ and $a, b \in (0,\infty]$ such that $\frac{1}{2r} = \frac{1}{2} + \frac{1}{a} + \frac{1}{b}$, we have*

$$\operatorname{tr}\left(B^{\frac{1}{2}} A B^{\frac{1}{2}}\right)^{r} \leqslant \left(\operatorname{tr}\left(B^{\frac{r}{2}} A^{r} B^{\frac{r}{2}}\right)\right)^{r} \left\| A^{\frac{1-r}{2}} \right\|_{a}^{2r} \left\| B^{\frac{1-r}{2}} \right\|_{b}^{2r}.$$

Choose $s = 1$, and $s_1 = b$, $s_2 = 2$, and $s_3 = a$ for some $a, b \in (0,\infty]$ with $\frac{1}{1} = \frac{1}{2} + \frac{1}{a} + \frac{1}{b}$

Proof (for $r = \frac{1}{2}$).

$$\operatorname{tr}\left(B^{\frac{1}{2}} A B^{\frac{1}{2}}\right)^{\frac{1}{2}} = \left\| B^{\frac{1}{2}} A^{\frac{1}{2}} \right\|_{1} = \left\| B^{\frac{1}{4}} B^{\frac{1}{4}} A^{\frac{1}{4}} A^{\frac{1}{4}} \right\|_{1} \leqslant \left\| B^{\frac{1}{4}} \right\|_{b} \left\| B^{\frac{1}{4}} A^{\frac{1}{4}} \right\|_{2} \left\| A^{\frac{1}{4}} \right\|_{a}$$

$$\left\| B^{\frac{1}{4}} A^{\frac{1}{4}} \right\|_{2} = \left(\operatorname{tr} B^{\frac{1}{4}} A^{\frac{1}{2}} B^{\frac{1}{4}}\right)^{\frac{1}{2}}$$

Relations between the Petz and the minimal divergence

Families of quantum Rényi divergences

Let $\alpha \in (0,1) \cup (1,\infty)$, and ρ, σ be density matrices. We define:

Petz quantum Rényi divergence

$$\bar{D}_\alpha(\rho\|\sigma) := \frac{1}{\alpha-1}\log \mathrm{tr}\rho^\alpha\sigma^{1-\alpha}$$ [7]

Minimal quantum Rényi divergence

$$\widetilde{D}_\alpha(\rho\|\sigma) := \frac{1}{\alpha-1}\log \mathrm{tr}\left(\sigma^{\frac{1-\alpha}{2\alpha}}\rho\sigma^{\frac{1-\alpha}{2\alpha}}\right)^\alpha$$ [8,9]

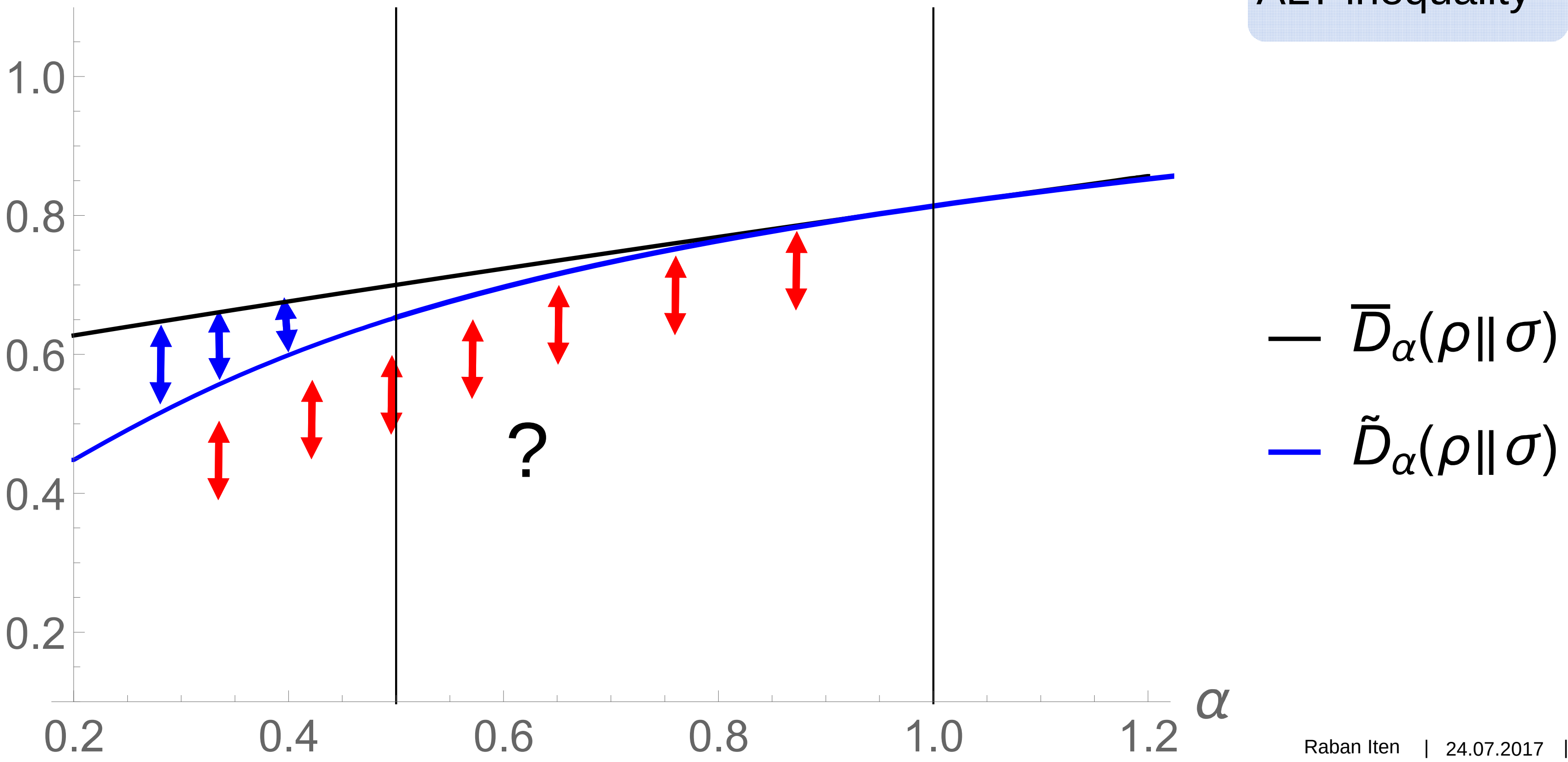
Is indeed minimal (see e.g. [10] for an overview)

[7]: Petz, 1986
[8]: Müller-Lennert et al., 2013
[9]: Wilde, Winter and Yang, 2014
[10]: Marco Tomamichel, Quantum Rényi divergences

Reversed relation between the Petz and the minimal divergence

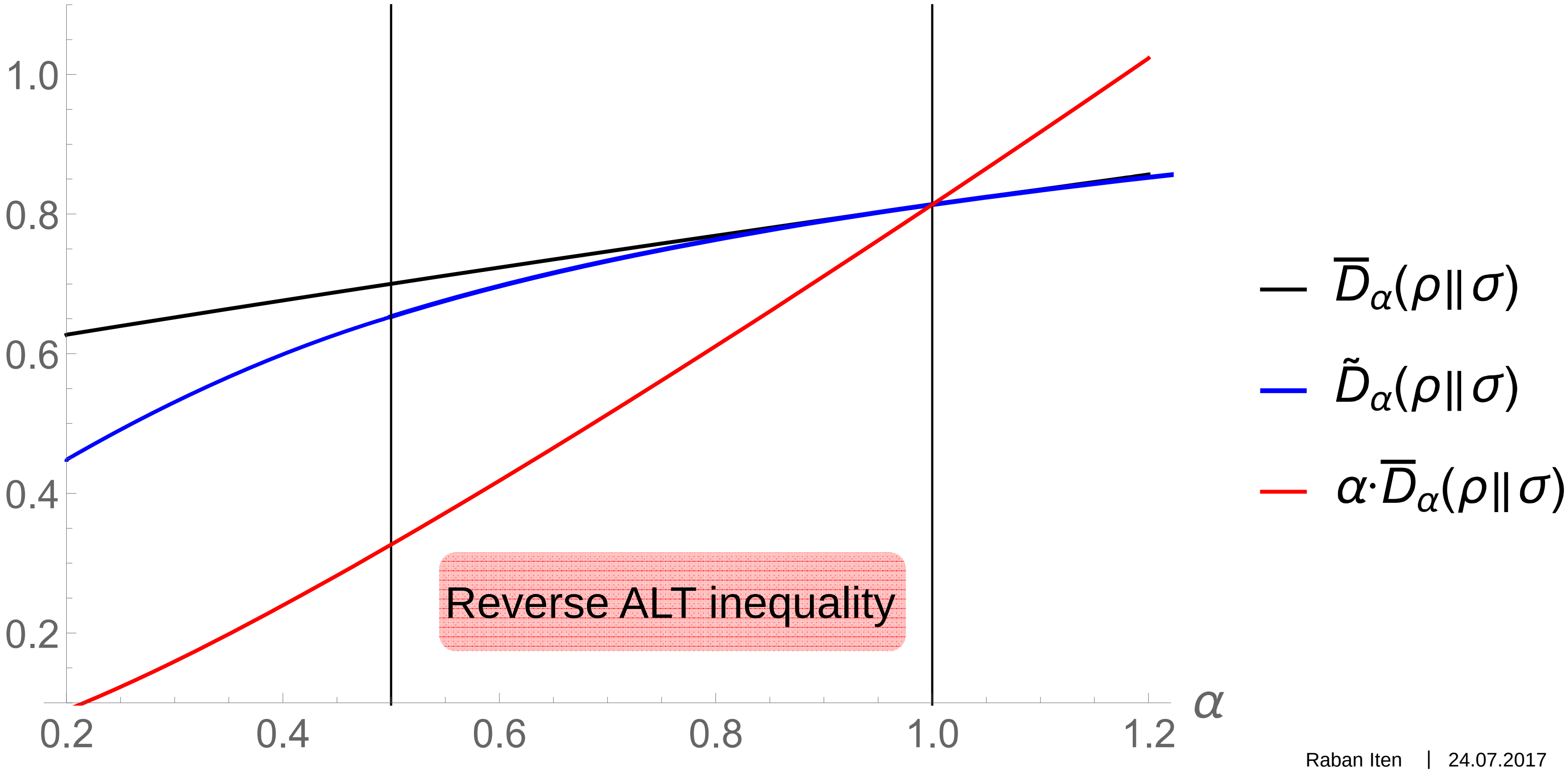
$$\widetilde{D}_\alpha(\rho\|\sigma) \leqslant \bar{D}_\alpha(\rho\|\sigma)$$

ALT inequality



Reversed relation between the Petz and the minimal divergence

$$\alpha \bar{D}_\alpha(\rho\|\sigma) \leqslant \tilde{D}_\alpha(\rho\|\sigma) \leqslant \bar{D}_\alpha(\rho\|\sigma) \quad for\ \alpha \leq 1$$



Reverse bound between the Petz and the minimal divergence

Corollary (Corollary of the reverse ALT inequality). *Let ρ and σ be two density matrices and $\alpha \in [0,1]$. Then*

$$\alpha \bar{D}_\alpha(\rho\|\sigma) \leqslant \widetilde{D}_\alpha(\rho\|\sigma) \leqslant \bar{D}_\alpha(\rho\|\sigma)\,.$$

Theorem (Reverse ALT inequality). *Let A and B be positive semi-definite matrices. Then, for $r \in (0,1]$ and $a, b \in (0,\infty]$ such that $\frac{1}{2r} = \frac{1}{2} + \frac{1}{a} + \frac{1}{b}$, we have*

$$\operatorname{tr}\left(B^{\frac{1}{2}} A B^{\frac{1}{2}}\right)^r \leqslant \Big(\operatorname{tr}\left(B^{\frac{r}{2}} A^r B^{\frac{r}{2}}\right)\Big)^r \left\|A^{\frac{1-r}{2}}\right\|_a^{2r} \left\|B^{\frac{1-r}{2}}\right\|_b^{2r}\,.$$

Proof of the reverse bound between the Petz and the minimal divergence

Corollary (Corollary of the reverse ALT inequality). *Let ρ and σ be two density matrices and $\alpha \in [0,1]$. Then*

$$\alpha \bar{D}_\alpha(\rho\|\sigma) \leqslant \widetilde{D}_\alpha(\rho\|\sigma) \leqslant \bar{D}_\alpha(\rho\|\sigma)\,.$$

Proof of the first inequality (**for $\alpha = 1/2$**):

By definition

$$-\log \operatorname{tr} \rho^{\frac{1}{2}}\sigma^{\frac{1}{2}} \leqslant -2\log \operatorname{tr}\left(\sigma^{\frac{1}{2}}\rho\sigma^{\frac{1}{2}}\right)^{\frac{1}{2}} \Leftrightarrow$$

$$\log\left(\operatorname{tr} \rho^{\frac{1}{2}}\sigma^{\frac{1}{2}}\right)^{\frac{1}{2}} \geqslant \log \operatorname{tr}\left(\sigma^{\frac{1}{2}}\rho\sigma^{\frac{1}{2}}\right)^{\frac{1}{2}} \Leftrightarrow$$

$$\left(\operatorname{tr} \rho^{\frac{1}{2}}\sigma^{\frac{1}{2}}\right)^{\frac{1}{2}} \geqslant \operatorname{tr}\left(\sigma^{\frac{1}{2}}\rho\sigma^{\frac{1}{2}}\right)^{\frac{1}{2}}$$

Reverse ALT

$$\operatorname{tr}\left(B^{\frac{1}{2}}AB^{\frac{1}{2}}\right)^{\frac{1}{2}} \leqslant \left(\operatorname{tr}\left(B^{\frac{1}{2}}A^{\frac{1}{2}}\right)\right)^{\frac{1}{2}} \left\|A^{\frac{1}{4}}\right\|_a \left\|B^{\frac{1}{4}}\right\|_b$$

Set $A = \rho, B = \sigma$

Choose $a = 4, b = 4$ $\qquad \left\|A^{\frac{1}{4}}\right\|_4 = (\operatorname{tr} A)^{\frac{1}{4}} = 1 \quad \text{if } \operatorname{tr} A = 1$

Relations between conditional Rényi entropies

Families of quantum conditional Rényi entropies

Let ρ_{AB} be a density matrix on the system $A \otimes B$, i.e. $\rho_{AB} \in \mathcal{D}(A \otimes B)$, and $\alpha \in (0,1) \cup (1,\infty)$. We define the following *quantum conditional Rényi entropies* of A given B as

$$\bar{H}^{\downarrow}_{\alpha}(A|B)_{\rho} := -\bar{D}_{\alpha}(\rho_{AB} \| \mathrm{id}_A \otimes \rho_B)\,,$$

$$\bar{H}^{\uparrow}_{\alpha}(A|B)_{\rho} := \sup_{\sigma_B \in \mathcal{D}(B)} -\bar{D}_{\alpha}(\rho_{AB} \| \mathrm{id}_A \otimes \sigma_B)\,,$$

$$\widetilde{H}^{\downarrow}_{\alpha}(A|B)_{\rho} := -\widetilde{D}_{\alpha}(\rho_{AB} \| \mathrm{id}_A \otimes \rho_B) \quad \text{and}$$

$$\widetilde{H}^{\uparrow}_{\alpha}(A|B)_{\rho} := \sup_{\sigma_B \in \mathcal{D}(B)} -\widetilde{D}_{\alpha}(\rho_{AB} \| \mathrm{id}_A \otimes \sigma_B)\;.$$

Duality relations for conditional entropies

Lemma (Duality relations [13,14,15,8,16,17]). *Let ρ_{ABC} be a pure state on $A \otimes B \otimes C$. Then*

$$\bar{H}^{\downarrow}_{\alpha}(A|B)_{\rho} + \bar{H}^{\downarrow}_{\beta}(A|C)_{\rho} = 0 \quad \textit{when} \quad \alpha + \beta = 2 \textit{ for } \alpha, \beta \in [0,2] \quad \textit{and}$$

$$\widetilde{H}^{\uparrow}_{\alpha}(A|B)_{\rho} + \widetilde{H}^{\uparrow}_{\beta}(A|C)_{\rho} = 0 \quad \textit{when} \quad \frac{1}{\alpha} + \frac{1}{\beta} = 2 \textit{ for } \alpha, \beta \in [\frac{1}{2}, \infty] \quad \textit{and}$$

$$\bar{H}^{\uparrow}_{\alpha}(A|B)_{\rho} + \widetilde{H}^{\downarrow}_{\beta}(A|C)_{\rho} = 0 \quad \textit{when} \quad \alpha\beta = 1 \textit{ for } \alpha, \beta \in [0, \infty]\,,$$

where we use the convention that $\frac{1}{\infty} = 0$ *and* $\infty \cdot 0 = 1$.

[13]: Tomamichel, Colbeck and Renner, 2009
[14]: Tomamichel, Berta and Hayashi, 2014
[15]: Beigi, 2013
[8]: Müller-Lennert et al., 2013
[16]: König, Renner and Schaffner, 2009
[17]: Berta, Diplom Thesis, 2008

Relations between conditional entropies

Max-like entropies: $\alpha \in (0,1)$

Lemma. *For* $\alpha \in [0,1]$ *and* $\rho_{AB} \in \mathcal{D}(A \otimes B)$, *we have that*

$$\bar{H}^{\downarrow}_{\alpha}(A|B)_\rho \leqslant \widetilde{H}^{\downarrow}_{\alpha}(A|B)_\rho \leqslant \alpha \bar{H}^{\downarrow}_{\alpha}(A|B)_\rho + (1-\alpha)\log|A| \qquad \textit{and}$$

$$\bar{H}^{\uparrow}_{\alpha}(A|B)_\rho \leqslant \widetilde{H}^{\uparrow}_{\alpha}(A|B)_\rho \leqslant \alpha \bar{H}^{\uparrow}_{\alpha}(A|B)_\rho + (1-\alpha)\log|A|\,.$$

$\Downarrow$ [via entropy duality]

Lemma. *For* $\alpha \in [1,2]$ *and* $\rho_{AB} \in \mathcal{D}(A \otimes B)$, *we have that*

$$\widetilde{H}^{\downarrow}_{\alpha}(A|B)_\rho \leqslant \alpha \widetilde{H}^{\uparrow}_{\frac{1}{2-\alpha}}(A|B)_\rho + (\alpha-1)\log|A| \qquad \textit{and}$$

$$\bar{H}^{\downarrow}_{\alpha}(A|B)_\rho \leqslant \frac{1}{2-\alpha}\left(\bar{H}^{\uparrow}_{\frac{1}{2-\alpha}}(A|B)_\rho + (\alpha-1)\log|A|\right).$$

Min-like entropies: $\alpha \in (1,\infty)$

Pretty good measures in QIT

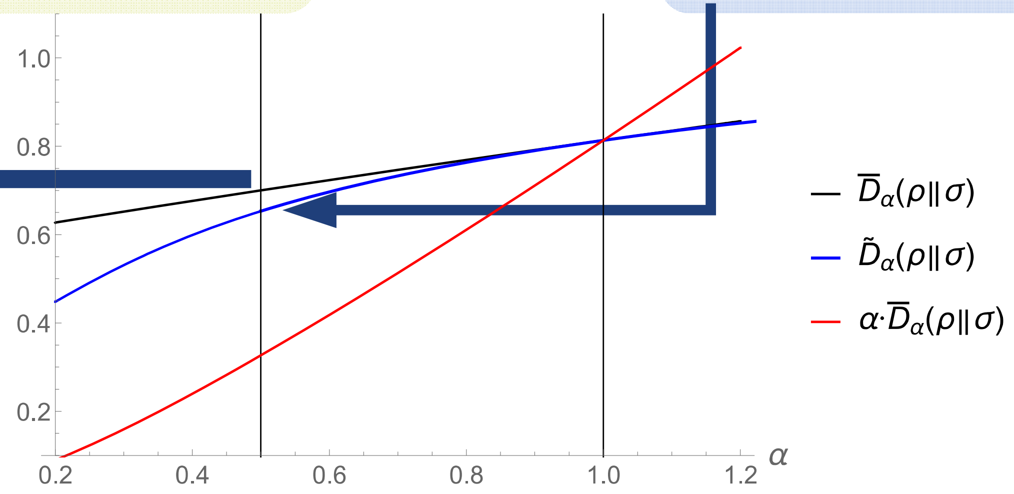
Pretty good fidelity

Fidelity

$$F(\rho,\sigma) \;:=\; \mathrm{tr}\left(\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}\right)^{1/2}$$

$\Rightarrow$

$$\tilde{D}_{\frac{1}{2}}(\rho\|\sigma) = -2\log F(\rho,\sigma)$$



$$\bar{D}_{\frac{1}{2}}(\rho\|\sigma) = -2\log F_{\mathrm{pg}}(\rho,\sigma)$$

$\Rightarrow$

Pretty good fidelity

$$F_{\mathrm{pg}}(\rho,\sigma) := \mathrm{tr}\sqrt{\rho}\sqrt{\sigma}$$

Bounds for pretty good measures

The pretty good fidelity is indeed pretty good

$$F_{\text{pg}}(\rho,\sigma) \leqslant F(\rho,\sigma) \leqslant \sqrt{F_{\text{pg}}(\rho,\sigma)}$$

(via entropy duality)

Pretty good measurement [3]

$$p_{\text{guess}}^{\text{pg}}(X|B) \leqslant p_{\text{guess}}(X|B) \leqslant \sqrt{p_{\text{guess}}^{\text{pg}}(X|B)}$$

Pretty good singlet fraction [18]

$$R_{\text{pg}}(A|B)_\rho \leqslant R(A|B)_\rho \leqslant \sqrt{R_{\text{pg}}(A|B)_\rho}$$

Measure for the largest achievable overlap with the maximally entangled state one can obtain from ρ_{AB} by applying a quantum channel on system B.

[3]: Barnum and Knill, 2002
[18]: Dupuis, Fawzi and Wehner, 2013

Derivation of the bound for the pretty good measurement

$\widetilde{H}^{\uparrow}_{\infty}(X|B)_\rho = -\log p_{\mathrm{guess}}(X|B)$ [16]

$\widetilde{H}^{\downarrow}_{2}(X|B)_\rho = -\log p^{\mathrm{pg}}_{\mathrm{guess}}(X|B)$ [19]

$$p^{\mathrm{pg}}_{\mathrm{guess}}(X|B) \leqslant p_{\mathrm{guess}}(X|B) \leqslant \sqrt{p^{\mathrm{pg}}_{\mathrm{guess}}(X|B)}$$

$\alpha = 2$

Lemma. *Let $\alpha \in [1,2]$ and ρ_{XB} be a cq state on $X \otimes B$, i.e., $\rho_{XB} = \sum_x p_x |x\rangle\langle x|_X \otimes (\rho_x)_B$ where $(\rho_x)_B$ are density operators and $p_x \in [0,1]$, such that $\sum_x p_x = 1$. Then*

$$\widetilde{H}^{\downarrow}_{\alpha}(X|B)_\rho \leqslant \alpha \widetilde{H}^{\uparrow}_{\frac{1}{2-\alpha}}(X|B)_\rho \quad \text{and}$$

$$\bar{H}^{\downarrow}_{\alpha}(X|B)_\rho \leqslant \frac{1}{2-\alpha} \bar{H}^{\uparrow}_{\frac{1}{2-\alpha}}(X|B)_\rho .$$

Classical quantum state

Lemma. *For $\alpha \in [1,2]$ and $\rho_{AB} \in \mathcal{D}(A \otimes B)$, we have that*

$$\widetilde{H}^{\downarrow}_{\alpha}(A|B)_\rho \leqslant \alpha \widetilde{H}^{\uparrow}_{\frac{1}{2-\alpha}}(A|B)_\rho + (\alpha - 1)\log|A| \quad \text{and}$$

$$\bar{H}^{\downarrow}_{\alpha}(A|B)_\rho \leqslant \frac{1}{2-\alpha}\left(\bar{H}^{\uparrow}_{\frac{1}{2-\alpha}}(A|B)_\rho + (\alpha - 1)\log|A|\right) .$$

[16]: König, Renner and Schaffner, 2009
[19]: Buhrman et al., 2008

Optimality conditions for pretty good measures

Equality condition for max-like entropies

Lemma (Equality condition for entropies)**.** *Let* $\alpha \in [\frac{1}{2}, 1)$*,* ρ_{AB} *be a density operator and* $\hat{\sigma}^\star_B := \mathrm{tr}_A\, \rho^\alpha_{AB}$*. Then, the following are equivalent*

1. $\bar{H}^\uparrow_\alpha(A|B)_\rho = \widetilde{H}^\uparrow_\alpha(A|B)_\rho$
2. $[\rho_{AB}, \mathrm{id}_A \otimes \hat{\sigma}^\star_B] = 0\,.$

Proof [Sketch]: $\bar{H}^\uparrow_\alpha(A|B)_\rho = \sup_{\sigma_B \in \mathcal{D}(B)} -\bar{D}_\alpha(\rho_{AB} \| \mathrm{id}_A \otimes \sigma_B)$

Optimizer is known [20]: $\sigma^\star_B = \frac{(\mathrm{tr}_A\, \rho^\alpha_{AB})^{\frac{1}{\alpha}}}{\mathrm{tr}\, \left(\mathrm{tr}_A \rho^\alpha_{AB}\right)^{\frac{1}{\alpha}}}$

ALT inequality

$\Longrightarrow$ $\bar{H}^\uparrow_\alpha(A|B)_\rho = -\bar{D}_\alpha(\rho_{AB} \| \mathrm{id}_A \otimes \sigma^\star_B) \leqslant \sup_{\sigma_B \in \mathcal{D}(B)} -\widetilde{D}_\alpha(\rho_{AB} \| \mathrm{id}_A \otimes \sigma_B)$

$\Longrightarrow$ Necessary condition [21]: $[\rho_{AB}, \mathrm{id}_A \otimes \sigma^\star_B] = 0$

Enough to show: $\sigma_B \mapsto -\widetilde{D}_\alpha(\rho_{AB} \| \mathrm{id}_A \otimes \sigma_B)$ attains its global maximum at $\sigma_B = \sigma^\star_B$ if $[\rho_{AB}, \mathrm{id}_A \otimes \sigma^\star_B] = 0.$ → Cf. our arXiv version ...

[20]: Tomamichel, Berta, and Hayashi, 2014 [21]: F. Hiai, 1994

Optimality conditions for the pretty good measurement

$\widetilde{H}^{\uparrow}_{\infty}(X|B)_\rho = -\log p_{\text{guess}}(X|B)$ [16]

$\widetilde{H}^{\downarrow}_{2}(X|B)_\rho = -\log p^{\text{pg}}_{\text{guess}}(X|B)$ [19]

Let τ_{XBC} be a purification of ρ_{XB}. Then, the duality relations for Rényi entropies imply

$$\widetilde{H}^{\downarrow}_{2}(X|B)_\tau = \widetilde{H}^{\uparrow}_{\infty}(X|B)_\tau \iff \bar{H}^{\uparrow}_{1/2}(X|C)_\tau = \widetilde{H}^{\uparrow}_{1/2}(X|C)_\tau \,.$$

Lemma (Optimality condition for the pretty good measurement)**.** *The pretty good measurement is optimal for distinguishing states in the ensemble* $\{p_x, \rho_x\}$ *if and only if* $[G_{X'B'}, \hat{\sigma}^{\star}_{X'B'}] = 0$.

Generalized Gram matrix
[cf. our arXiv version for the definition]

$$\hat{\sigma}^{\star}_{X'B'} := \sum_x |x\rangle\langle x|_{X'} \otimes \langle x| \sqrt{G_{X'B'}}\, |x\rangle_{X'}$$

[16]: König, Renner and Schaffner, 2009
[19]: Buhrman et al., 2008

Conclusion

Mathematical results

- Reverse Araki-Lieb-Thirring (ALT) inequality
- Introducing a reverse relation between the Petz and the minimal divergence:

$$\alpha \bar{D}_\alpha(\rho\|\sigma) \leqslant \widetilde{D}_\alpha(\rho\|\sigma) \leqslant \bar{D}_\alpha(\rho\|\sigma) \qquad for\ \alpha \leq 1$$

- Inequalities and equality conditions between conditional entropies

Unified picture for pretty good measures in QIT

- Introducing a **pretty good fidelity**
- Showing that the pretty good fidelity is indeed pretty good
- Bounds between the fidelity and the pretty good fidelity **lead to known bounds for pretty good measures** via duality of quantum entropies
- Introducing necessary and sufficient **optimality conditions for the pretty good measurement and singlet fraction**

Thanks for your attention!

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