COMPRESSION OF IDENTICALLY PREPARED QUANTUM STATES

Yuxiang Yang
The University of Hong Kong

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INTRODUCTION

Why do we study identically prepared state compression

POPULATION CODING

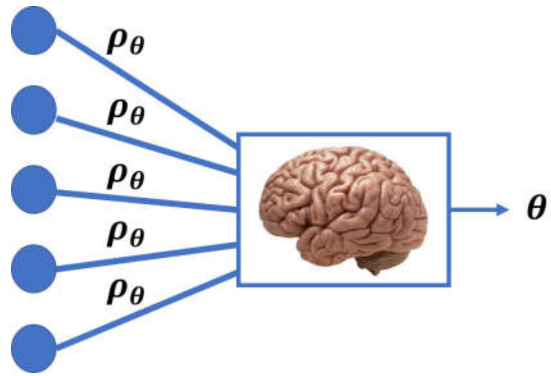
- **"States" of a neuron:** probability distributions of reactions to different stimuli.

$$\rho_{bite} = 0.9|1\rangle\langle 1| + 0.1|0\rangle\langle 0| \quad \rho_{touch} = 0.2|1\rangle\langle 1| + 0.8|0\rangle\langle 0|$$

1/0: a spike/no spike.

- A group of $n \gg 1$ neurons $\rightarrow$ tensor-power form states $\rho_\theta^{\otimes n}$ ($\theta =$ bite/touch).

- **Population coding:** the state $\rho_\theta^{\otimes n}$ of a large group of neurons is a coding for the stimulus θ.



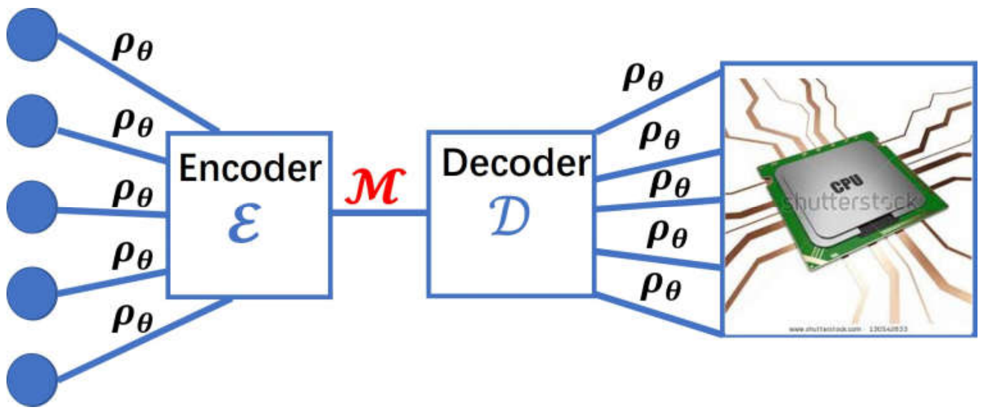
A QUANTUM EXTENSION OF POPULATION CODING

- To build a robot using quantum techniques:
  ``neurons'' releasing different quantum states for different stimuli θ.

- Population coding → A population of quantum states $\rho_\theta^{\otimes n}$ carrying θ.

- Compression: reduce the cost of population transmission.

COMPRESS A QUANTUM POPULATION CODING



- Input: a quantum population $\rho_\theta^{\otimes n}$ with θ unknown.
- Encoder/Decoder characterized by quantum channels.
- Minimize the memory cost $\mathcal{M}$ (focusing on the leading order of n).
- Output: a state that has vanishing trace distance to $\rho_\theta^{\otimes n}$ (faithfulness).

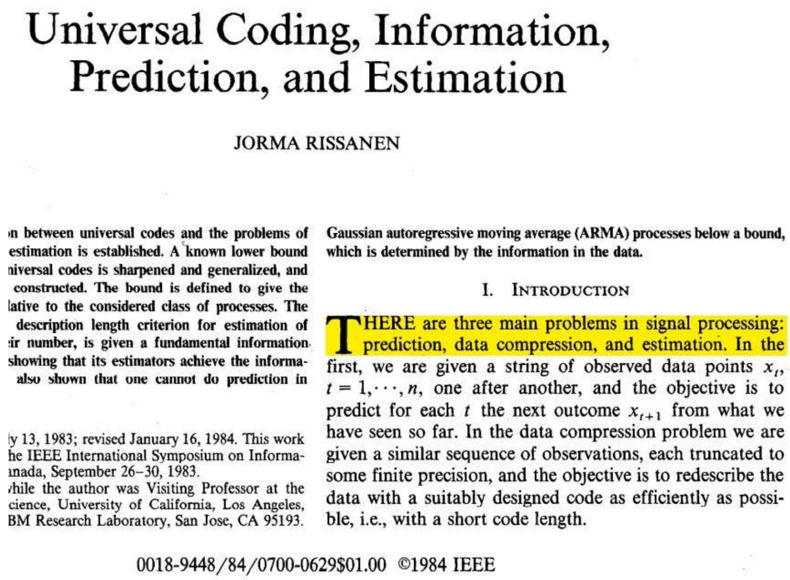
MINIMUM DESCRIPTION LENGTH

- [Rissanen 84'] The shortest length to describe a d-parametric probability distribution $P_\theta(x)$ given n samples $x_1, x_2, \ldots, x_n$:

$$-\mathbb{E}_\theta \log P_\theta(x) + \boxed{d/2 \log n}$$

Entropy (data) Distribution

- A compression task aimed at reconstructing the (unknown) distribution (not the data, and thus beyond i.i.d.) [Rissanen & followups; Hayashi, Tan 17'].
- Nontrivial to generalize to quantum.



Universal Coding, Information, Prediction, and Estimation

JORMA RISSANEN

n between universal codes and the problems of estimation is established. A known lower bound niversal codes is sharpened and generalized, and constructed. The bound is defined to give the lative to the considered class of processes. The description length criterion for estimation of ir number, is given a fundamental information showing that its estimators achieve the informa- also shown that one cannot do prediction in Gaussian autoregressive moving average (ARMA) processes below a bound, which is determined by the information in the data.

ly 13, 1983; revised January 16, 1984. This work he IEEE International Symposium on Informa- nada, September 26–30, 1983. hile the author was Visiting Professor at the cience, University of California, Los Angeles, BM Research Laboratory, San Jose, CA 95193.

I. INTRODUCTION

THERE are three main problems in signal processing: prediction, data compression, and estimation. In the first, we are given a string of observed data points x_t, $t = 1, \cdots, n$, one after another, and the objective is to predict for each t the next outcome x_{t+1} from what we have seen so far. In the data compression problem we are given a similar sequence of observations, each truncated to some finite precision, and the objective is to redescribe the data with a suitably designed code as efficiently as possible, i.e., with a short code length.

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RELATED WORKS

- Identically prepared pure qubit states [Plesch, Buzek; PRA 10']: $\log n$ qubits. Demonstrated (3 copies → 2 copies) by Rozema et al.
- Mixed qubit states [YY, GC, MH; PRL 16']: $\log n$ qubits + $1/2 \log n$ bits (necessary only when the state's mixedness is unknown).
- Clock states [YY, GC, MH arxiv:1703.05876]: $1/2 \log n$ qubits.
- General finite dimensional systems [YY,GC, Ebler; PRL 16']: a protocol requiring $O(\log n)$-size memory. Not optimal in general.
- A general compression protocol, requiring the minimum total memory and less quantum memory?

MEMORY COST OF COMPRESSION

How many bits and qubits do we need to encode $\rho_\theta^{\otimes n}$

CLASSICAL AND QUANTUM PARAMETERS

A non-degenerate state family $\{\rho_\theta^{\otimes n}: \theta = (\mu, \xi) \in \Theta\}$ is characterized by two kinds of parameters:

$$\rho_\theta = U_\xi \rho_\mu U_\xi^\dagger$$

- **Classical** (free) parameters μ: determining the spectrum
- **Quantum** (free) parameters ξ: determining the eigenbasis

EXAMPLES

- Full qudit state family: $f_c = d - 1$ and $f_q = d^2 - d$.

- Phase-covariant state family: $f_c = 0$ and $f_q = d - 1$

$$\rho_\theta = U_\theta \rho_0 U_\theta^\dagger \quad U_\theta = \sum_k e^{i\theta_k} |k\rangle\langle k|.$$

- Classical distribution family: $f_c = d - 1$ and $f_q = 0$.

- Displaced thermal states at known/unknown temperature: $f_c = 0/1$ and $f_q = 2$

$$\rho_{\alpha,\beta} = D_\alpha \rho_\beta D_\alpha^\dagger$$
$$\rho_\beta = \sum_m (1-\beta)\beta^m |m\rangle\langle m| \quad D_\alpha = e^{\alpha \hat{a}^\dagger - \bar{\alpha}\hat{a}}$$

MEMORY COST OF THE COMPRESSION

- [Recall] Pure qubits:

$\cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i\varphi}|1\rangle$ has two quantum parameters (θ, φ) and requires $\log n$ qubits.

- **[Main result.]** For each free parameter t, it takes:

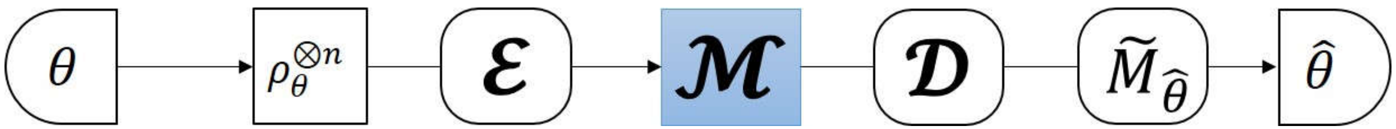
1. $(1/2 + \delta)\log n$ bits for t classical
2. $1/2 \log n$ bits + $\delta \log n$ qubits for t quantum

to encode faithfully the n-copy state.

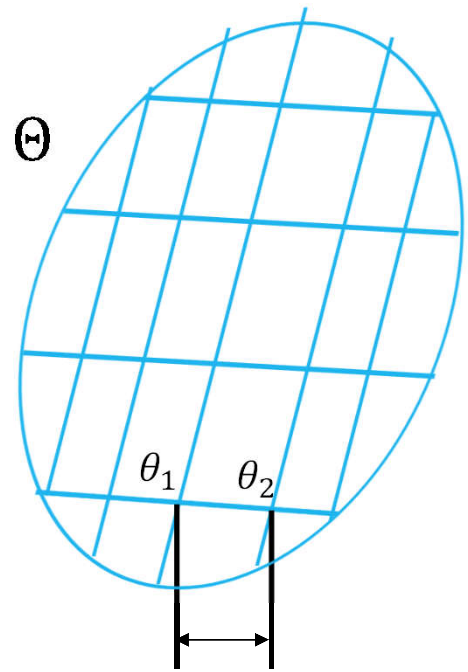
- $\delta > 0$ is a parameter independent of n (also affects the error), which can be arbitrarily close to zero.

OPTIMALITY

- Construct a mesh on Θ containing $\mathrm{n}^{f/2-\delta}$ mutually distinguishable states for any $\delta > 0$. $f = f_c + f_q$.
- Consider any faithful compression protocol $(\mathcal{E}, \mathcal{D})$:



- Can faithfully communicate $(f/2 - \delta)\log n$ bits of messages.
- The communication cost $\log|\mathcal{M}|$ cannot be smaller than the amount of messages.



$O(n^{-1/2+\alpha}), \alpha > 0$

$\Rightarrow \rho_{\theta_1}^{\otimes n}$ distinguishable from $\rho_{\theta_2}^{\otimes n}$

COMPRESSION PROTOCOL

How to achieve the minimal memory cost

PROTOCOL FOR DISPLACED THERMAL STATES

- Displaced thermal states $\rho_{\alpha,\beta} = D_\alpha \rho_\beta D_\alpha^\dagger \quad \alpha \in \mathbb{C}$, where D_α is the displace operator and ρ_β is a fixed thermal state (state of a system in equilibrium).
- Concentration of the displacement

$$\rho_{\alpha,\beta}^{\otimes n} \xrightarrow{Beam\ Splitters} \rho_{\sqrt{n}\alpha,\beta} \otimes \rho_\beta^{\otimes(n-1)}$$

- Photon number distribution of $\rho_{\sqrt{n}\alpha,\beta}$: concentrated in an-$O(n)$ window.
- Photon number truncation $\rho_{\sqrt{n}\alpha,\beta} \rightarrow (1+\delta)\log n$ qubits for any $\delta > 0$.
- Two free parameters ($\alpha \in \mathbb{C}$), each around $1/2\log n$ qubits.

PROTOCOL FOR QUDIT STATES

- Localization.

- Local asymptotic equivalence of n-tensor power qudit states and Gaussian (displaced thermal $\otimes$ classical Gaussian) states.

- Compression of Gaussian states (Solved already!).

LOCALIZATION

- Take out a negligible portion of $n^{1-\delta/2}$ copies and use them for tomography.
- Tomography pins θ to a neighborhood Θ_L of size $O(n^{-\frac{1}{2}+\frac{\delta}{3}})$ with exponentially vanishing error.
- Encode the tomography outcome into a classical memory, so that the overall quantum memory cost can be reduced.
- The same strategy can be used in the displaced thermal state case.
- Left with $n - n^{1-\delta/2}$ copies (the lost copies can be retrieved later by amplification).

QUANTUM LOCAL ASYMPTOTIC NORMALITY (Q-LAN)

- Q-LAN [Kahn, Guta; CMP 09']

In the neighborhood Θ_L, $\rho_\theta^{\otimes n}$ is asymptotically equivalent to a classical-quantum Gaussian state:

$$\rho_{\theta/\sqrt{n}}^{\otimes n} \overset{Q-LAN}{\Longleftrightarrow} \gamma_\theta = \gamma_\mu^{class} \otimes \gamma_\xi^{quant}$$

- Classical mode γ_μ^{class} : a Gaussian distribution with f_c variates;
- Quantum mode γ_ξ^{quant}: a multimode (number of modes depending on f_q) displaced thermal state.
- Problem reduced to compression of Gaussian distributions and displaced thermal states.

ERROR BOUND

- The compression error is upper bounded as

$$\epsilon_n = O(n^{-\delta/2}) + O\left(n^{-\kappa(\delta)}\right),$$

where the latter is the error of Q-LAN. Especially, $\kappa(\delta) > 0$ for $\delta \in (0,2/9)$.

- Faithfulness $\lim_{n\to\infty} \epsilon_n = 0$ is guaranteed as long as $\delta > 0$.
- The error vanishes slower when less quantum memory is used.

QUANTUM MEMORY IS ESSENTIAL

Why fully classical memory doesn't work

FULLY CLASSICAL MEMORY DOESN'T WORK

- Is it possible to reversibly convert $\rho_\theta^{\otimes n}$ into classical bits with an error vanishing in n ?

- Fact: a state family can be perfectly compressed into classical memory if and only if it is classical, i.e. $[\rho_1, \rho_2] = 0$ for any ρ_1, ρ_2 from the family.

- Compression is only approximately perfect. Cannot directly apply the fact.

FULLY CLASSICAL MEMORY DOESN'T WORK

- Consider $\rho_{\theta_0}^{\otimes n}$ and $\rho_{\theta_0+t/\sqrt{n}}^{\otimes n}$; $t > 0$ is a vector of quantum parameters.
  - Approximation by Gaussian states:

$$\rho_{\theta_0}^{\otimes n} \overset{Q-LAN}{\Longleftrightarrow} \gamma_0$$

$$\rho_{\theta_0+t/\sqrt{n}}^{\otimes n} \overset{Q-LAN}{\Longleftrightarrow} \gamma_t := D_t \gamma_0 D_t^{\dagger}$$

  - $\|[\gamma_0, \gamma_t]\| > 0$ (independent of n). $||\cdot||$: operator norm.
  - A compression protocol for $\rho_{\theta_0}^{\otimes n}, \rho_{\theta_0+t/\sqrt{n}}^{\otimes n} \rightarrow$ a protocol for γ_0, γ_t.
  - State families containing both $\rho_{\theta_0}^{\otimes n}$ and $\rho_{\theta_0+t/\sqrt{n}}^{\otimes n}$ cannot be faithfully encoded in a classical memory.
- For state families with quantum parameters, compression with only classical memory cannot have vanishing error.

SUMMARY AND FUTURE WORKS

- Compression of $\rho_\theta^{\otimes n}$:
  I. minimal memory cost: approximately $1/2 \log n$ for each degree of freedom;
  II. the required memory is mainly classical;
  III. a fully classical memory is not OK

- Extension to non-product states with symmetry; e. g. states of bosonic systems.
- A second-order theory?

AUTHORS OF THIS WORK

Yuxiang Yang

Ge Bai

Giulio Chiribella

Masahito Hayashi

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CIFAR
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THANKS FOR YOUR ATTENTION!

See arXiv 1701.03372
for more details

And enjoy the workshop and Singapore ~