Rearrangements

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Authors

joint work with Andreas Blass, Will Brian, Joel Hamkins, Michael Hardy, Paul Larson, and Jonathan Verner

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(set theory)

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work of Iván Ongay-Valverde and Paul Tveite

(computability theory)

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Notes: (1) $\sum a_n$ convergent $\implies a_n \to 0$

(2) If $\sum a_n$ is conditionally convergent then

$$\sum_{n \in P} a_n = +\infty \quad \text{and} \quad \sum_{n \in N} a_n = -\infty$$

where $P = \{n \in \omega : a_n > 0\}$ and $N = \{n \in \omega : a_n < 0\}$

Riemann's rearrangement theorem

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Suppose $\sum a_n$ *is conditionally convergent and* $r \in \mathbb{R} \cup \{+\infty, -\infty\}$*.* *Then there is a* rearrangement $\pi \in \mathrm{Sym}(\omega)$ *such that* $\sum a_{\pi(n)} = r$*.*

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Then there is a rearrangement $\pi \in \mathrm{Sym}(\omega)$ *such that* $\sum a_{\pi(n)} = r$*.*

Also there is $\pi \in \mathrm{Sym}(\omega)$ such that $\sum a_{\pi(n)}$ diverges *by oscillation*.
$(\liminf_k \sum_0^k a_{\pi(n)} < \limsup_k \sum_0^k a_{\pi(n)})$

The questions

How many permutations do we need such that for every conditionally convergent series $\sum a_n$ there is a permutation π in our family such that $\sum a_{\pi(n)}$ no longer converges to the same limit?

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What does it mean for real to compute a conditionally convergent series $\sum a_n$ of rationals such that for all computable permutations π, $\sum a_{\pi(n)} = \sum a_n$?

Rearrangement numbers

$\mathfrak{rr} := \min\{|\Pi| : \Pi \subseteq \mathrm{Sym}(\omega)$ and $\forall$ c.c. $\sum a_n \; \exists \pi \in \Pi$
$(\sum a_{\pi(n)} \neq \sum a_n)\}$ the *rearrangement number*

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Highness properties

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$\sum a_n$ is *weakly computably imperturbable* if for all computable permutations π,

$$\sum a_n = \sum a_{\pi(n)} \text{ or } \sum a_{\pi(n)} \text{ diverges by oscillation}$$

$\mathfrak{rr}$ versus $\mathfrak{rr}_o$

$\mathfrak{rr} := \min\{|\Pi| : \Pi \subseteq \mathrm{Sym}(\omega)$ and $\forall$ c.c. $\sum a_n \, \exists \pi \in \Pi$
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Fact: $\forall \pi \in \mathrm{Sym}(\omega) \, \exists \sigma_\pi \in \mathrm{Sym}(\omega)$ such that

- $\exists^\infty n \, (\sigma_\pi[\{0, ..., n-1\}] = \{0, ..., n-1\})$
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Π witness for $\mathfrak{rr} \implies \Pi \cup \{\sigma_\pi : \pi \in \Pi\}$ witness for $\mathfrak{rr}_o$

$\mathfrak{rr}$ versus non(meager)

non(meager) is the least size of a non-meager set of reals

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Theorem 2

$\mathfrak{rr} \leq \text{non}(\mathit{meager})$

Theorem 2': If X is computably imperturbable, then X is weakly meager engulfing ($=$ high or DNC)

$\mathfrak{rr}$ versus non(meager)

Theorem 2

$\mathfrak{rr} \leq \text{non}(\mathit{meager})$

<u>Proof:</u> $\sum a_n$ c.c. given. $K \in \omega$.

Theorem 2

$\mathfrak{rr} \leq$ non*(meager)*

Proof: $\sum a_n$ c.c. given. $K \in \omega$.

$\{\pi \in \mathrm{Sym}(\omega) : \exists n_0 \; (\sum_{n<n_0} a_{\pi(n)} > K)\}$ open dense.

$\mathfrak{rr}$ versus non(meager)

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Proof: $\sum a_n$ c.c. given. $K \in \omega$.

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Similarly with $< -K$ instead of $> K$.

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Similarly with $< -K$ instead of $> K$.

$\{\pi \in \mathrm{Sym}(\omega) : \sum a_{\pi(n)}$ diverges by oscillation$\}$ dense G_δ.

Done!

$\mathfrak{b} := \min\{|F| : F \subseteq \omega^\omega$ and $\forall g \in \omega^\omega \, \exists f \in F \, \exists^\infty n \, (g(n) < f(n))\}$

the *unbounding number*

$\mathfrak{rr}$ versus $\mathfrak{b}$

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Theorem 3

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Theorem 3

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Theorem 3': If X is high then it is computably imperturbable

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Proof: $A \subseteq \omega$, $\pi \in \mathrm{Sym}(\omega)$.

π *preserves* $A \iff \forall^\infty n < m \in A \, (\pi(n) < \pi(m))$

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Claim 1: $\mathfrak{j} \leq \mathfrak{rr}$

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$\exists A \in [\omega]^\omega$ s.t. all $\pi \in \Pi$ preserve A. $A = \{i_n : n \in \omega\}$.

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$$b_k = \begin{cases} a_n & \text{if } k = i_n \\ 0 & \text{otherwise} \end{cases}$$

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Then $\sum b_k = \sum a_n$ c.c. Also $\sum b_k = \sum b_{\pi(k)}$ for all $\pi \in \Pi$. Done!

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$\Pi \to \omega^\omega : \pi \mapsto f_\pi$ s.t. for all n

- $f_\pi(n) > n$
- $\forall m \leq n \quad \forall k \geq f_\pi(n) \quad (\pi(m) < \pi(k))$

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Let $A = \{a_n : n \in \omega\}$ s.t. $a_{n+1} \geq g(a_n)$.

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Show all $\pi \in \Pi$ preserve A:

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Let $A = \{a_n : n \in \omega\}$ s.t. $a_{n+1} \geq g(a_n)$.
Show all $\pi \in \Pi$ preserve A:
If $g(a_n) \geq f_\pi(a_n)$, then $\pi(a_n) < \pi(a_{n+1})$. Done!

$\mathfrak{rr}_i, \mathfrak{rr}_f$ versus $\mathfrak{d}$

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$$\mathfrak{d} := \min\{|F| : F \subseteq \omega^\omega \text{ and } \forall g \in \omega^\omega \, \exists f \in F \, \forall^\infty n \, (g(n) < f(n))\}$$

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Theorem 4

$\mathfrak{d} \leq \mathfrak{rr}_i, \mathfrak{rr}_f$

Theorem 4': If X is hyperimmune then it is weakly computably imperturbable

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$\mathfrak{d} := \min\{|F| : F \subseteq \omega^\omega$ and $\forall g \in \omega^\omega \; \exists f \in F \; \forall^\infty n \; (g(n) < f(n))\}$
the *dominating number*

Theorem 4

$\mathfrak{d} \leq \mathfrak{rr}_i, \mathfrak{rr}_f$

Theorem 4': If X is hyperimmune then it is weakly computably imperturbable

Proof similar to Theorem 3.

$\mathfrak{rr}$ versus cov(null)

cov(null) is the least size of a family of null sets covering the reals

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Note: Since a high degree computes a Schnorr random, this strengthens Theorem 3'. However, in set theory, Theorems 3 and 5 are independent.

$\mathfrak{rr}$ versus cov(null)

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$\text{cov}(\mathit{null}) \leq \mathfrak{rr}$

Rademacher's Lemma

Let $(c_n : n \in \omega)$ *be a sequence of reals. Set*

$$A = \{f \in 2^\omega : \sum_n (-1)^{f(n)} c_n \textit{ converges}\}$$

Then

$$\mu(A) = \begin{cases} 1 & \textit{if } \sum_n c_n^2 \textit{ converges} \\ 0 & \textit{otherwise} \end{cases}$$

$\mathfrak{rr}$ versus cov(null), 2

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cov$(null) \leq \mathfrak{rr}$

Proof: $\pi \in \mathrm{Sym}(\omega)$. $\sum \frac{1}{\pi(n)^2}$ convergent.

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Hence $\sum \frac{(-1)^{f(n)}}{n}$ c.c. and $\sum_n \frac{(-1)^{f(\pi(n))}}{\pi(n)}$ converges for all $\pi \in \Pi$.

$\mathfrak{rr}$ versus cov(null), 2

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Thus $\mathfrak{rr}_o \geq$ cov(null). By Theorem 1, $\mathfrak{rr} \geq$ cov(null). Done!

Consequences

Corollary 6

$CON\ (\mathfrak{d} < \mathfrak{rr})$

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CON $(\mathfrak{d} < \mathfrak{rr})$

Proof: In the random model, $\text{cov}(\text{null}) > \mathfrak{d}$. So follows from Theorem 5.

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Corollary 7

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Proof: In the Laver / Hechler model, $\text{cov}(\text{null}) < \mathfrak{b}$. So follows from Theorem 3.

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$CON\ (\mathfrak{d} < \mathfrak{rr})$

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Corollary 7

$CON\ (\mathrm{cov}(null) < \mathfrak{rr})$

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Question 1

CON ($\mathfrak{rr} <$ non(meager))?

Upper bounds for $\mathfrak{rr}_i, \mathfrak{rr}_f$?

No known upper bounds for $\mathfrak{rr}_i, \mathfrak{rr}_f$

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Proof Idea: Start with a model of $\mathfrak{c} > \omega_1$.

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(This needs some preliminary forcing.)
Thus the ω_1 permutations adjoined along the iteration witness $\mathfrak{rr}_i = \omega_1$.

Upper bounds for $\mathfrak{rr}_i, \mathfrak{rr}_f$, 2?

$\mathfrak{rr}_f := \min\{|\Pi| : \Pi \subseteq \mathrm{Sym}(\omega)$ and $\forall$ c.c. $\sum a_n \; \exists \pi \in \Pi$
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Theorem 9

CON $(\mathfrak{rr}_f < \mathfrak{c})$

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Theorem 9

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Theorem 9

CON $(\mathfrak{rr}_f < \mathfrak{c})$

Proof Idea: Start with a model of $\mathfrak{c} > \omega_1$.
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Use the Lévy-Steinitz Theorem (finite-dimensional version of Riemann's Theorem).

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Conjecture 1

$\mathfrak{rr}_i \leq \mathfrak{rr}_f$

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Conjecture 2

CON $(\mathfrak{rr}_i < \mathfrak{rr}_f)$

The subseries number

If $\sum a_n$ is c.c. then it has a divergent subseries $\sum_{n \in X} a_n$, $X \in [\omega]^\omega$.

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<u>Known:</u> (1) cov(null) $\leq$ ß $\leq$ non(meager)

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Theorem 10

CON$(ß < \mathfrak{b})$; *so also CON*$(ß < \mathfrak{rr})$

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Proof idea: this holds in the Laver model.

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Question 2

CON$(\mathfrak{rr} < ß)$?