Rearrangements

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ABSTRACT

Let $\sum_n a_n$ be a conditionally convergent series of real numbers. The Riemann rearrangement theorem says that by choosing a permutation p of the natural numbers $\mathbb{N}$ appropriately, the rearranged series $\sum_n a_{p(n)}$ can be made to diverge or to converge to any prescribed real number. The rearrangement number $\mathfrak{rr}$ is the least size of a family $\mathcal{P}$ of permutations such that for every conditionally convergent $\sum_n a_n$ there is $p \in \mathcal{P}$ such that $\sum_n a_{p(n)}$ no longer converges to the same limit. We compare $\mathfrak{rr}$ to other cardinal invariants of the continuum and also discuss some of its relatives. This is joint work with A. Blass, W. Brian, J. Hamkins, M. Hardy, and P. Larson [1].

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Weakly Homogeneous Structures

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ABSTRACT

We continue the investigation from [1] on the notion of weakly ultrahomogeneous structures and their effective categoricity. It was shown that any computable ultrahomogeneous structure is Δ^0_2 categorical. A structure $\mathcal{A}$ is said to be *weakly ultrahomogeneous* if there is a finite (*exceptional*) set of elements $a_1, \ldots, a_n$ such that $\mathcal{A}$ becomes ultrahomogeneous when constants representing these elements are added to the language. Characterizations were obtained for weakly ultrahomogeneous linear orderings, equivalence structures, injection structures and trees, and these were compared with characterizations of the computably categorical and Δ^0_2 categorical structures. Index sets can be used to determine the complexity of the notions of ultrahomegenous and weakly ultrahomogeneous for various families of structures. We report on recent work on weakly ultrahomogeneous Boolean algebras and Abelian p-groups.

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Gödel's Program and Ultimate L

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ABSTRACT

In this talk we discuss Gödel's program for solving Continuum Problem by finding new axioms beyond ZFC and Professor Hugh Woodin's concept of Ultimate L. We will try to argue that the axiom of V=Ultimale L, if it is well justified, would be a realization of Gödel's program. Moreover, this great progress in set theory could be treated as an achievement of Platonism in philosophy of mathematics and foundation studies.

The Reverse Mathematics of Model Theory and First-Order Principles

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ABSTRACT

I will discuss some results in the computability theory and reverse mathematics of basic model-theoretic notions, and their connections with first-order principles, drawing in particular from my recent paper *Induction, bounding, weak combinatorial principles, and the Homogeneous Model Theorem* with Lange and Shore.

Computability and model-theoretic aspects of families of sets and its generalizations

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ABSTRACT

In the talk I will discuss different approaches to the study of computability of families of sets. The most of these aproaches are based on a presentation of a family as a special alebraic structure. This allows to carry model-theoretic notions and properties to the familes and their generalizations.

Roots of polynomials in fields of generalized power series

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ABSTRACT

We consider roots of polynomials in fields of formal power series. Newton [1] and Puiseux [2], [3] showed that if K is algebraically closed of characteristic 0, then the field of Puiseux series with coefficients in K is also algebraically closed. Maclane [6] extended the Newton-Puiseux Theorem to Hahn fields. Our goal is to measure the recursion-theoretic complexity of the root-taking process in these fields. Puiseux series have length at most ω. In this setting, we already have results bounding the complexity. Hahn series have varying ordinal length, and complexity seems to go up with length. The first two authors have results bounding the lengths of roots of polynomials, in terms of the lengths of the coefficients [4], [5]. The work on complexity is ongoing.

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Redeveloping Takeuti-Yasumoto forcing

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ABSTRACT

Forcing method turned out to be a powerful tool in bounded arithmetic when Ajtai [1] proved that $I\Delta_0(R)$ cannot prove that R is not a bijection from $n+1$ to n. Krajíček [2], [3] also used forcing construction to give specific models of theories such as PV or V^1. Inspired by these results, Takeuti and Yasumoto [4], [5] gave Boolean valued model constructions for Buss' hierarchy and presented their connections with the separation of complexity classes.

In this talk, we will reformulate Takeuti-Yasumoto's forcing in two-sort bounded arithmetic and reprove some of their results, In particular, we show that if $P = NP$ holds in the standard model then Takeuti-Yasumoto generic extension is a model of Σ_1^B induction .

We will also show that the forcing construction of Krajíček and Takeuti-Yasumoto are identical in the sense that constructions by Krajíček can be obtained by Takeuti-Yasumoto construction. As a result, we conclude that there is a Takeuti-Yasumoto generic extension which is a model of Σ_1^B induction.

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Dickson's lemma and weak Ramsey theory

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ABSTRACT

The weak Paris–Harrington principle is a weak version of the Paris–Harrington principle, which was originally used as a convenient intermediate version in showing lower bounds for the Paris–Harrington principle for pairs [1]. We compare it with Dickson's lemma, which is a combinatorial theorem originally used in algebra, in particular for showing Hilbert's basis theorem [2]. We give a construction which shows the direct, level by level, equivalence between the weak Paris–Harrington principle for pairs and the Friedman-style miniaturization of Dickson's lemma.

Our studies result in a cascade of consequences:

- An explicit expression for weak Ramsey numbers for pairs.
- A sharp classification of the complexity classes of weak Paris–Harrington–Ramsey numbers.
- Bounds for weak Ramsey numbers in higher dimensions.
- A phase transition for the weak Paris–Harrington principle which is different from that for the Paris–Harrington principle [3].
- Level by level equivalence of Dickson's lemma and a relativized version of the weak Paris–Harrington principle.

All of these are established in RCA_0^*.

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Can we fish with Mathias forcing?

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ABSTRACT

Ramsey's theorem and its consequences received a great attention in reverse mathematics and computable analysis, due to their chaotic computability-theoretic features. Many simply stated questions became long-standing open questions and the topic quickly became a major subject of research in reverse mathematics.

Many tools were invented to answer these questions. Over the years, the technics were simplified, uniformized, and many questions were answered about Ramsey's theorem and its variants. Besides the nature of these answers, this systematic study led to the important observation that the minimalistic framework of *Mathias forcing* combined with a *CJS argument* was sufficient to give very precise and even often optimal answers to most questions.

This observation can be considered as a sign of maturity of the domain. From a personal point of view, the ultimate goal of a field is not to have all the questions answered, but to find a general technic which would enable one to answer easily any further question that one might have, following the famous proverb "give a man a fish and you feed him for a day; teach a man to fish and you feed him for a lifetime". However, some open questions seem to resist this framework, and in particular the question of the relationship between stability and cohesiveness. Can these questions be answered by a careful use of the existing framework, or are they requesting new tools? Have we found with Mathias forcing the right framework enabling us to fish in the sea of Ramsey's theory?[a]

[a]The metaphor is dubious, and may explain why the author ended-up doing mathematics.

Clique homological simplification problem is NP-hard

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ABSTRACT

We consider the problem that input as a pair (G, H) of graphs and asks whether there exists a simplicial complex X which realizes the persistent homology of $Cl(G)$ into $Cl(H)$. We show that this problem is NP-hard.

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Cototal enumeration degrees and the skip operator

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ABSTRACT

This is a joint work with U. Andrews, H. Ganchev, R. Kuyper, S. Lempp, J. Miller and M. Soskova. The talk will be an overview on the properties of the cototal enumeration degrees which form a proper substructure of the degree structure of the enumeration degrees closed under least upper bound and the enumeration jump operator. The cototal enumeration degrees properly extend the substructure of the total enumeration degrees. A set A is cototal if it is enumeration reducible to its complement. An enumeration degree is cototal if it contains a cototal set. The skip is a monotone operator such that the skip of the set A is the uniform upper bound of the complements of all sets enumeration reducible to A. These are closely connected: A has cototal degree if and only if it is enumeration reducible to its skip. We study cototality, using the skip operator and give some examples of classes of enumeration degrees that either guarantee or prohibit cototality. The skip has many of the nice properties of the Turing jump, even though the skip of A is not always above A. In fact, there is a set that is its own double skip.

Andrews et al. [1] provide the first paper substantially focused on the cototal enumeration degrees. The skip inversion theorem proved there shows that the range of the skip operator is the upper cone with base $\mathbf{0}'$. So, not every e-degree is reducible to its skip. The e-degrees reducible to its skip are exactly the cototal degrees. The cototal enumeration degrees have many further characterizations: they are the enumeration degrees of complements of maximal independent sets for infinite computable graphs on ω [1]; McCarthy [4] gave three more characterizations — the enumeration degrees of complements of maximal antichains in $\omega^{<\omega}$, the enumeration degrees of uniformly enumeration pointed trees, the enumeration degrees of languages of minimal subshifts. Another characterization comes from computable analysis. Miller [5] introduced a reducibility between points in computable separable metric spaces. This reducibility gave rise to the structure of the continuous degrees and Miller showed that this structure also properly embeds in the structure of the enumeration degrees, and forms a proper extension of the Turing degrees. Andrews et al. [1] showed that the image of the continuous degrees is contained in the cototal enumeration degrees. Kihara and Pauly [2] extended Miller's reducibility to capture points in any represented metric space.

Recently Miller and M. Soskova [6] proved that the cototal enumeration degrees form a dense substructure of the structure of the enumeration degrees. Moreover they proved that these are the only enumeration degrees which have a good approximations in the sense of Lachlan and Shore [3].

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Non-depth-first search of an AND-OR tree

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We investigate an algorithm probing truth values of leaves of an AND-OR tree. The goal of an algorithm is to find the truth value of the root. The cost of an algorithm is measured by the number of leaves probed during the search. In the case where a probability distribution on the truth values of leaves is given, the cost means the expected value of the cost. An algorithm A is *depth-first* if at any internal node x, once A probes a leaf that is descendant of x, A does not probe leaves that are not descendant of x until A finds the value of x. Otherwise, A is *non-depth-first*.

A tree is called *balanced* if (1) all the leaves have the same distance from the root and (2) any two internal nodes that have the same distance from the root have the same number of child nodes.

An *independent distribution* (ID for short) on a tree T denotes a probability distribution on the truth assignments on the leaves of T such that each leaf has a certain probability p of having value 0, where p depends on a leaf, and the value of of each leaf is given independently. If, in addition, p is the same for all the leaves then the distribution is called an *independent and identical distribution* (IID for short).

Tarsi (1983) shows that if T is a balanced AND-OR tree and d is an IID on T such that

(*) "the probability of the root having value 0 is neither 0 nor 1"

then there exists a depth-first algorithm that minimizes the cost among all the algorithms. The method of Tarsi heavily depends on the hypothesis that a given distribution is an IID.

Saks and Wigderson (1986) investigate the case where non-depth-fist algorithms and correlated distributions are taken into consideration. However, they do not investigate non-depth-first search against an independent distribution.

Liu and Tanaka (2007) shed light on Saks-Wigderson result again. They assert, without a proof, that among IDs on a uniform binary AND-OR tree, the minimum cost (achieved by an algorithm) is maximized only by an IID.

In this decade, the above assertion is justified under a hypothesis that only depth-first algorithms are taken into consideration. The uniform binary tree case is shown by Suzuki and Niida (2015). The result is extended to the uniform balanced trees and IDs satisfying (*) by Peng et al. (2017).

We extend the results of Suzuki-Niida (2015) and Peng et al. (2017) to the case where non-depth-first algorithms are taken into consideration. We also show some examples.

Main theorem Given a uniform binary AND-OR tree, we take non-depth-first algorithms into consideration. Then (1) the minimum cost is maximized only by an IID (say, d_0), and (2) the best algorithm for d_0 is chosen from depth-first ones. The same assertions hold for a uniform balanced AND-OR tree and IDs satisfying (*).

Example 1 If T is a uniform binary AND-OR tree of height 2 and d is an ID then there exists a depth-first algorithm that achieves the minimum cost. If, in addition, the probability of having value 0 is not 0 at every leaf, then the minimum cost is achieved only by a depth-first algorithm.

Example 2 There exists an ID d on a uniform binary OR-AND tree of height 3 such that the following hold. At any leaf, the probability of having value 0 is neither 0 nor 1, and the minimum cost is achieved only by a **non-**depth-first algorithm.

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Set theoretic geologies

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ABSTRACT

The study of the structure of the ground models of the universe is called the set theoretic geology. In this talk we present some progresses on this field. We also consider variations of set-theoretic geology, the set-theoretic geology without the axiom of choice, and the set-theoretic geology of pseudo-grounds. I discuss some connections between these geologies and Woodin's HOD-conjecture.

On the first-order part of Ramsey's theorem for pairs

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ABSTRACT

Deciding the first-order part of Ramsey's theorem for pairs is one of the important problems in reverse mathematics. I will overview the recent developments of this study. To decide the first-order part, a standard approach is proving Π^1_1-conservation over some induction or bounding axiom by showing, omega-extension property. Cholak/Jockusch/Slaman showed $WKL_0 + RT^2_2$ is Π^1_1-conservative over RCA_0 plus Σ^0_2-induction and $WKL_0 + RT^2$ is a Π^1_1-conservative over RCA_0 plus Σ^0_3-induction, and they posed whether they are Π^1_1-conservative over Σ^0_2-bounding and Σ^0_3-bounding respectively. In this talk, we will approach this question by using the omega-extension property and a hybrid method of indicator argument and forcing.

Active Learning of Classes of Recursive Functions by Ultrametric Algorithms

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ABSTRACT

We study active learning of classes of recursive functions by asking value queries about the target function f, where f is from the target class. That is, the query is a natural number x, and the answer to the query is $f(x)$. The complexity measure in this paper is the worst-case number of queries asked. We prove that for some classes of recursive functions *ultrametric active learning* algorithms can achieve the learning goal by asking *significantly fewer* queries than deterministic, probabilistic, and even nondeterministic active learning algorithms. This is the first ever example of a problem, where ultrametric algorithms have advantages over nondeterministic algorithms.