Particle rolling MCMC

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Outline

1. Introduction

Motivation

We consider modelling economic time series during the long sample period where we

- estimate model parameters,
- forecast on the time series
- for the risk management and policy simulation ...

but, in the economic time series, it is well-known that

- there have been major structural changes (due to such as the financial crisis in 2008)
- parameters are not necessary constant and subject to changes in the long sample period

Motivation

To deal with the structural change in economic times series, for example,

- divide the sample periods into several subsample periods
  $\rightarrow$
  1. How many structural changes?
  2. When does the structural changes occur?
- fix the length of the sample period and implement the rolling estimation
  $\rightarrow$
  1. How to decide the length of the sample period?
  2. It takes time to estimate parameters and forecast using MCMC.

State space model

This paper considers the state space model

$$
\begin{aligned}
p(y_t \mid y_{1:t-1}, \alpha_{1:t}, \theta) &= p(y_t \mid \alpha_t, \theta) \equiv g_\theta(y_t \mid \alpha_t), \\
p(\alpha_t \mid \alpha_{1:t-1}, \theta) &= p(\alpha_t \mid \alpha_{t-1}, \theta) \equiv f_\theta(\alpha_t \mid \alpha_{t-1}), \\
& \quad p(\alpha_1 \mid \theta) \equiv \mu_\theta(\alpha_1),
\end{aligned}
$$

where

y_t: an observation vector at time t,

α_t: an unobserved state vector at time t,

θ: a static parameter vector,

$y_{s:t} = (y_s, \ldots, y_t)$ and $\alpha_{s:t} = (\alpha_s, \ldots, \alpha_t)$. The correlation between y_t and α_{t+1} is also taken into account (e.g. leverage in stochastic volatility models).

Rolling parameter and state estimation

We estimate the posterior distribution of θ and $\alpha_{s:t}$ given the observations $y_{s:t}$ with $t = s + L$ for $s = 1, 2 \ldots$:

$$\begin{aligned}
&\pi(\theta, \alpha_{s:t} \mid y_{s:t}) \\
&\quad \propto \quad p(\theta)\mu_\theta(\alpha_s)g_\theta(y_s \mid \alpha_s)\left\{\prod_{j=s+1}^{t} f_\theta(\alpha_j \mid \alpha_{j-1}, y_{j-1})g_\theta(y_j \mid \alpha_j)\right\} \\
&\quad \propto \quad p(\theta)\mu_\theta(\alpha_s)\left\{\prod_{j=s+1}^{t} f_\theta(\alpha_j \mid \alpha_{j-1})g_\theta(y_{j-1} \mid \alpha_j, \alpha_{j-1})\right\}g_\theta(y_t \mid \alpha_t),
\end{aligned}$$

where $p(\theta)$: the prior probability density function of θ.

Contribution of the paper

This paper considers the state space model for economic times series and

- proposes the efficient rolling estimation method for both parameter θ and states $\alpha_{s:t}$.
- gives the theoretical justification of the algorithm by proving the marginal density of parameters and states.
- the algorithm is based on Particle Gibbs
- proposes, as a special case, the sequential Bayesian estimation method (which is alternative to SMC^2 based on Particle MH).

2. Particle rolling MCMC (PRMCMC)

Simple particle rolling MCMC

Suppose, at time $t-1$, we have

- $(\theta^n, \alpha^n_{s-1:t-1})$: a collection of particles for $n = 1, \ldots, N$
- $W^n_{s-1:t-1}$: the importance weight which is a discrete approximation of $\pi(\theta, \alpha_{s-1:t-1} \mid y_{s-1:t-1})$.

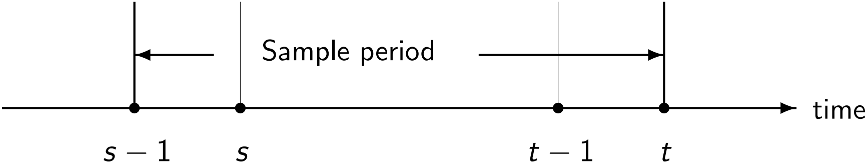
We update these particles in two steps:

1. Step 1. Add a new observation y_t.
   $(\theta^n, \alpha^n_{s-1:t-1}) \to (\theta^n, \alpha^n_{s-1:t})$, $W^n_{s-1:t-1} \to W^n_{s-1:t}$.
2. Step 2. Discard the old observation y_{s-1}.
   $(\theta^n, \alpha^n_{s-1:t}) \to (\theta^n, \alpha^n_{s:t})$, $W^n_{s-1:t} \to W^n_{s:t}$.

Simple particle rolling MCMC

Step 1. Generate α_t^n given $(\theta^n, \alpha_{s-1:t-1}^n)$ and $y_{s-1:t}$

Update $W_{s-1:t-1}^n \to W_{s-1:t}^n$



where

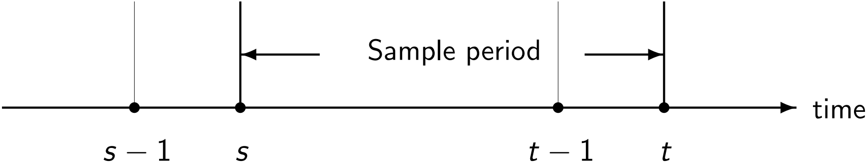
$$W_{s-1:t}^n \quad \propto \quad \frac{f_{\theta^n}(\alpha_t^n \mid \alpha_{t-1}^n, y_{t-1}) g_{\theta^n}(y_t \mid \alpha_t^n)}{q_{t,\theta^n}(\alpha_t^n \mid \alpha_{t-1}^n, y_t)} W_{s-1:t-1}^n,$$

$q_{t,\theta^n}(\cdot \mid \alpha_{t-1}^n, y_t)$: proposal density to generate α_t^n.

Simple particle rolling MCMC

Step 2. Discard α_{s-1}^n and construct $(\theta^n, \alpha_{s:t}^n)$ given $y_{s:t}$

Update $W_{s-1:t}^n \to W_{s:t}^n$ for $n = 1, \dots, N$



$$W_{s:t} \quad \propto \quad g_{\theta^n}(y_{s-1} \mid \alpha_{s-1}^n, \alpha_s^n)^{-1} W_{s-1:t},$$

$$g_\theta(y_{s-1} \mid \alpha_{s-1}, \alpha_s) \quad = \quad \frac{\mu_\theta(\alpha_{s-1}) f_\theta(\alpha_s \mid \alpha_{s-1}) g_\theta(y_{s-1} \mid \alpha_{s-1}, \alpha_s)}{p(\alpha_{s-1} \mid \alpha_s, \theta) \mu_\theta(\alpha_s)}$$

Simple particle rolling MCMC

Remarks.

- If some degeneracy criterion is fulfilled (such as ESS $< 0.5N$), resample all the particles by implementing MCMC algorithm.
- We often use a prior density $f_\theta(\alpha_t|\alpha_{t-1}, y_{t-1})$ as a proposal density $q_{t,\theta}$ to generate α_t in Step 1. It is known to cause the weight degeneracy problem since the prior density does not take account of the new observation y_t.
- Even if we are able to use a fully adapted proposal density, $q_{t,\theta}(\alpha_t \mid \alpha_{t-1}, y_{s-1:t}) = p(\alpha_t \mid \alpha_{t-1}, y_t, \theta)$, we still have the weight degeneracy phenomenon.

Simple particle rolling MCMC

Example 1.

$$
\begin{aligned}
y_t &= \alpha_t + \epsilon_t,\ \ \epsilon_t \sim \mathcal{N}(0,\sigma^2),\ \ t = 1,\ldots,2000 \\
\alpha_{t+1} &= \mu + 0.25(\alpha_t - \mu) + \eta_t,\ \ \eta_t \sim \mathcal{N}(0,2\sigma^2),\ \ t = 1,\ldots,2000, \\
\alpha_1 &= \mu + \frac{\eta_0}{\sqrt{1-0.25^2}},\ \ \eta_0 \sim \mathcal{N}(0,2\sigma^2),
\end{aligned}
$$

where $\theta = (\mu,\sigma^2)'$, $\mu \mid \sigma^2 \sim \mathcal{N}(0,10\sigma^2)$, $\sigma^2 \sim \mathcal{IG}(5/2, 0.05/2)$. The window size $= 1000$. In order to evaluate the weight degeneracy in Steps 1a and 2a, we define two ratios:

$$
R_{1t} = \frac{ESS_{s-1:t}}{ESS_{s-1:t-1}}, \quad R_{2t} = \frac{ESS_{s:t}}{ESS_{s-1:t}}, \quad ESS_{s:t} = \frac{1}{\sum_{n=1}^{N}(W_{s:t}^{n})^2}.
$$

Simple particle rolling MCMC

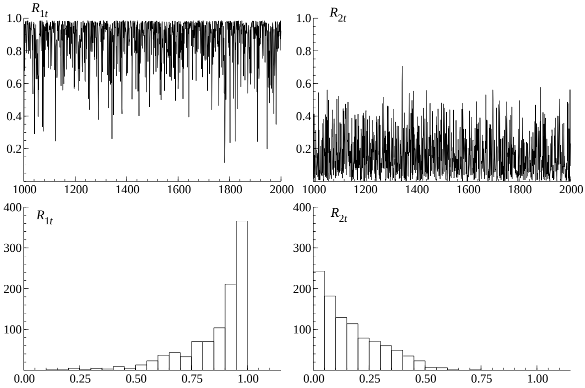


Figure: Rolling window estimation of $p(\theta, \alpha_{t-999:t} \mid y_{t-999:t})$, $t = 1001, \ldots, 2000$ in the linear Gaussian state space model. The fully adapted proposal density is used for $q_{t,\theta}$.

Particle rolling MCMC with double block sampling

We update these particles in two steps:

Step 1. Add a new obs y_t (forward block sampling)

1. Generate $\alpha^n_{t-K:t}$ given $(\theta^n, \alpha^n_{s-1:t-K-1})$ and $y_{s-1:t}$
2. Construct a collection of particles $(\theta^n, \alpha^n_{s-1:t})$ with the importance weight $W^n_{s-1:t}$
3. Implement the particle simulation smoother to improve the mixing property.

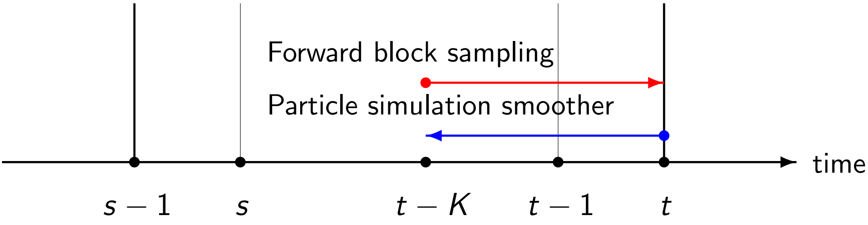
Step 2. Remove the old obs y_{s-1} (backward block sampling)

1. Generate $\alpha^n_{s-1:s+K-1}$ given $(\theta^n, \alpha^n_{s+K:t})$ and $y_{s:t}$.
2. Construct a collection of particles $(\theta^n, \alpha^n_{s:t})$ with the importance weight $W^n_{s:t}$
3. Discard α^n_{s-1} and implement the particle simulation smoother.

Particle rolling MCMC with double block sampling

Step 1. Generate $\alpha^n_{t-K:t}$ **given** $(\theta^n, \alpha^n_{s-1:t-K-1})$ **and** $y_{s-1:t}$

Update $W^n_{s-1:t-1} \to W^n_{s-1:t}$



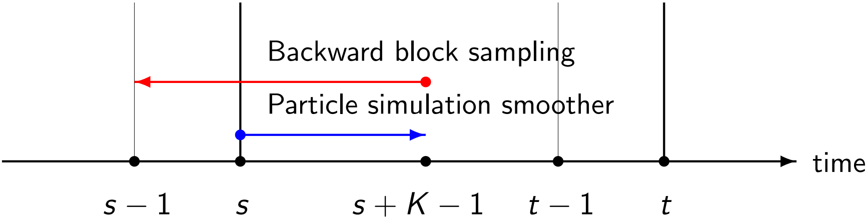
where

$$W^n_{s-1:t} \quad \propto \quad \hat{p}(y_t \mid y_{s-1:t-1}, \alpha^n_{t-K-1}, \theta^n) \times W^n_{s-1:t-1}.$$

Particle rolling MCMC with double block sampling

Step 2. Generate $\alpha^n_{s-1:s+K-1}$ given $(\theta^n, \alpha^n_{s+K:t})$ and $y_{s:t}$

Discard α^n_{s-1} and update $W^n_{s-1:t} \to W^n_{s:t}$



where

$$W^n_{s:t} \propto \frac{1}{\hat{p}(y_{s-1} \mid y_{s:t}, \alpha^n_{s+K}, \theta^n)} W^n_{s-1:t}$$

Particle rolling MCMC with double block sampling

Forward block sampling

1. 'Move' the particles $\alpha^n_{t-K:t-1}$ using the conditional SMC update for each $n = 1, \ldots, N$ based on particle Gibbs (Andrieu et al. (2010)).
   (1) Generate a number of candidates $\alpha^{n,m}_{t-K:t-1}$ $(m = 1, \ldots, M)$ where the current 'lineage' $\alpha^n_{t-K:t-1}$ is fixed as one of candidates.
   (2) The $\alpha^{n,m}_t$ is also generated conditional on $\alpha^{n,m}_{t-K:t-1}$.
2. Determine which lineage is appropriate as $\alpha^n_{t-k:t}$ and compute its importance weight for each $n = 1, \ldots, N$.

Particle rolling MCMC with double block sampling

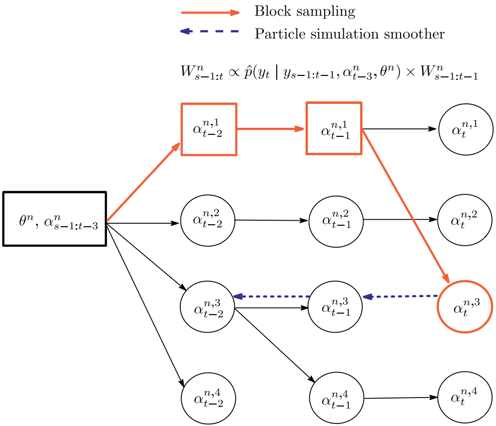


Figure: $(\theta_n, \alpha^n_{s-1:t-1})$ is fixed and shown in the rectangle. Other state variables in the circle are generated.

Particle rolling MCMC with double block sampling

Backward block sampling

1. Generate a cloud of particles of $\alpha^{n,m}_{s+K-1}, \alpha^{n,m}_{s+K-2}, \ldots, \alpha^{n,m}_{s-1}$ $(m = 1, \ldots, M)$ given $(\alpha^{n}_{s+K:t}, \theta^{n})$ and $y_{s:t}$ targeting $\pi(\theta, \alpha_{s-1:t} \mid y_{s:t})$
2. Stochastically determine the new values of $\alpha^{n}_{s:t}$ (discarding α^{n}_{s-1}) with the updated importance weight.

Particle rolling MCMC with double block sampling

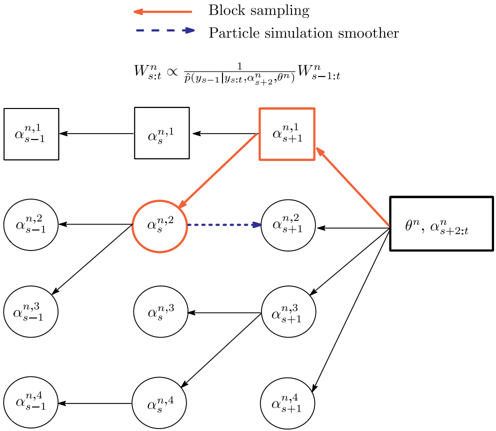


Figure: $(\theta_n, \alpha^n_{s-1:t-1})$ is fixed and shown in the rectangle. Other state variables in the circle are generated.

Initialization of the rolling MCMC

1. In order to sample from the initial posterior distribution, it is straightforward to use MCMC methods as in the warm-up period for the practical filtering in Polson et al. (2008).
2. SMC-based methods are preferred when we need to compute the marginal likelihood $p(y_{1:s-1})$.
3. Thus we implement only forward block sampling (with particle simulation smoother) to sample $\alpha_{1:L+1}$ and θ where $L = t - s$.

If some degeneracy criterion is fulfilled (such as ESS $< 0.5N$), resample all the particles by implementing MCMC algorithm.

Estimation of the marginal likelihood

We compute

$$
\begin{aligned}
p(y_{s:t}) &= \int p(y_{s:t} \mid \alpha_{s:t}, \theta) p(\alpha_{s:t} \mid \theta) p(\theta) d\alpha_{s:t} d\theta \\
&= \frac{p(y_t \mid y_{s-1:t-1})}{p(y_{s-1} \mid y_{s:t})} p(y_{s-1:t-1}),
\end{aligned}
$$

using the estimates

$$
\begin{aligned}
\hat{p}(y_t \mid y_{s-1:t-1}) &= \sum_{n=1}^{N} W^n_{s-1:t-1} \hat{p}(y_t \mid y_{s-1:t-1}, \alpha^n_{t-K-1}, \theta^n), \\
\hat{p}(y_{s-1} \mid y_{s:t}) &= \sum_{n=1}^{N} W^n_{s-1:t} \hat{p}(y_{s-1} \mid y_{s:t}, \alpha^n_{s+K}, \theta^n).
\end{aligned}
$$

Estimation of the marginal likelihood

The initial estimate is given by

$$\hat{p}(y_{1:L+1}) = \hat{p}(y_1) \prod_{j=2}^{L+1} \hat{p}(y_j \mid y_{1:j-1}),$$

where

$$\begin{aligned}\hat{p}(y_1) &= \sum_{n=1}^{N} \hat{p}(y_1 \mid \theta^n), \\ \hat{p}(y_j \mid y_{1:j-1}) &= \sum_{n=1}^{N} W_{j-1}^n \hat{p}(y_j \mid y_{1:j-1}, \alpha_{j-K-1}^n, \theta^n),\end{aligned}$$

3. Theoretical justification of PRMCMC

Forward block sampling

Proposition 4.1

The marginal density of $(\theta, \alpha_{s-1:t-K-1}, \alpha_{t-K}^{k_{t-K}^*}, \ldots, \alpha_t^{k_t^*})$ for the artificial target density

$$\hat{\pi}(\theta, \alpha_{s-1:t-K-1}, \alpha_{t-K:t}^{1:M}, a_{t-K:t-1}^{1:M}, k_t^* \mid y_{s-1:t})$$

is $\pi(\theta, \alpha_{s-1:t-K-1}, \alpha_{t-K}^{k_{t-K}^*}, \ldots, \alpha_t^{k_t^*} \mid y_{s-1:t})$ for the forward block sampling where a_j: 'parent' index variable and k_j^*: (random) index $(j = t-K, \ldots, t)$.

Forward block sampling

Proposition 4.2

$$
\begin{aligned}
& E[\hat{p}(y_t \mid y_{s-1:t-1}, \alpha_{t-K-1}, \theta)|y_{s-1:t}] \\
&= E[p(y_t \mid y_{s-1:t-1}, \alpha_{t-K-1}, \theta)|y_{s-1:t}, \alpha_{t-K-1}, \theta] \\
&= p(y_t \mid y_{s-1:t-1}).
\end{aligned}
$$

Particle simulation smoother

Proposition 4.3 The joint conditional density of $(k^*_{t-K}, \ldots, k^*_t)$ is given by

$$
\begin{aligned}
&\hat{\pi}(k^*_{t-K}, \ldots, k^*_t | \theta, \alpha_{s-1:t-K-1}, \alpha^{1:M}_{t-K:t}, a^{1:M}_{t-K:t-1}, y_{s-1:t}) \\
&\quad = \hat{\pi}(k^*_t | \theta, \alpha_{s-1:t-K-1}, \alpha^{1:M}_{t-K:t}, a^{1:M}_{t-K:t-1}, y_{s-1:t}) \\
&\quad \times \prod_{t_0=t-1}^{t-K} \hat{\pi}(k^*_{t_0} | \theta, \alpha_{s-1:t-K-1}, \alpha^{1:M}_{t-K:t_0}, a^{1:M}_{t-K:t_0-1}, \alpha^{k^*_{t_0+1}}_{t_0+1}, \ldots, \\
&\qquad\qquad \alpha^{k^*_t}_t, k^*_{t_0+1:t}, y_{s-1:t}),
\end{aligned}
$$

as in Whitely et al. (2010) and the discussion of Whitely following Andrieu et al. (2010).

Backward block sampling

Proposition 4.4

The marginal density of $(\theta, \alpha_s^{k_s^*}, \ldots, \alpha_{s+K-1}^{k_{s+K-1}^*}, \alpha_{s+K:t})$ for the artificial target density

$$\breve{\pi}(\theta, \alpha_{s-1:s+K-1}^{1:M}, \alpha_{s+K:t}, a_{s:s+K-1}^{1:M}, k_{s-1}, k_s^* \mid y_{s:t})$$

is $\pi(\theta, \alpha_s^{k_s^*}, \ldots, \alpha_{s+K-1}^{k_{s+K-1}^*}, \alpha_{s+K:t} \mid y_{s-1:t})$ for the backward block sampling where a_j: 'parent' index variable and k_j^*: (random) index $(j = t-K, \ldots, t)$.

Backward block sampling

Proposition 4.5

$$
\begin{aligned}
& E[\hat{p}(y_{s-1} \mid y_{s:t}, \alpha_{s+K}, \theta)^{-1}] \\
& \quad = \quad E[\hat{p}(y_{s-1} \mid y_{s:t}, \alpha_{s+K}, \theta)^{-1} \mid y_{s:t}, \alpha_{s+K}, \theta] \\
& \quad = \quad p(y_{s-1} \mid y_{s:t}, \theta)^{-1}.
\end{aligned}
$$

4. Numerical examples

Example 1 (Continued)

$$\begin{aligned} y_t &= \alpha_t + \epsilon_t,\ \ \epsilon_t \sim \mathcal{N}(0,\sigma^2),\ \ t=1,\ldots,2000 \\ \alpha_{t+1} &= \mu + 0.25(\alpha_t-\mu) + \eta_t,\ \ \eta_t \sim \mathcal{N}(0,2\sigma^2),\ \ t=1,\ldots,2000, \\ \alpha_1 &= \mu + \frac{\eta_0}{\sqrt{1-0.25^2}},\ \ \eta_0 \sim \mathcal{N}(0,2\sigma^2), \end{aligned}$$

where $\theta=(\mu,\sigma^2)'$, $\mu \mid \sigma^2 \sim \mathcal{N}(0,10\sigma^2)$, $\sigma^2 \sim \mathcal{IG}(5/2, 0.05/2)$.

- The window size $=1000$
- The number of particles: $N=1000$.
- The size of block $K=1,2,3,5$
- For each $n=1,\ldots,N$, we generate M particle paths in the forward and backward block sampling.

Example 1 (Continued)

Table: # resampling steps in PRMCMC with block sampling ((2))

Estimation period	K	(1) Fully adapted	(2) M 100	300	500	(3) Simple sampling
	1	30	54	37	32	184
Initial estimation	2	10	39	21	15	
($t = 1, \ldots, 1000$)	3	8	38	19	15	
	5	6	37	18	15	
	10	8	38	18	14	
	1	48	104	71	61	1027
Rolling estimation	2	8	74	33	23	
($t = 1001, \ldots, 2000$)	3	7	74	31	22	
	5	6	72	32	22	
	10	5	69	31	23	

Example 1 (Continued)

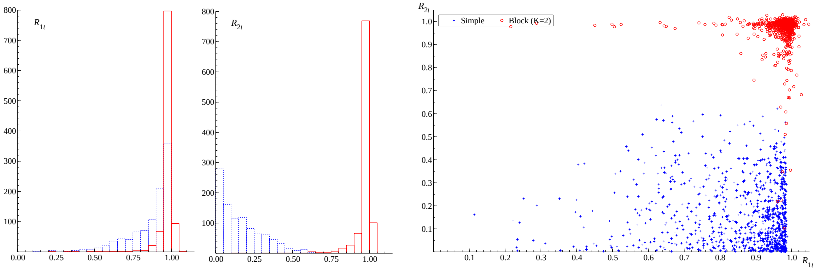


Figure: The histograms (left) and scatter plots (right) of R_{1t} and R_{2t} (left) ($t = 1001, \ldots, 2000$) for the simple sampler (dotted blue) and the sampler with block sampling with $K = 2$ and $M = 100$ (solid red).

Example 1 (Continued)

Table: Summary statistics of R_{1t} and R_{2t} for the simple sampling and the block sampling ($M = 100, K = 2$) for $t = 1001, \ldots, 2000$

	Method	Mean	Median	Std. dev.
R_{1t}	Simple	0.862	0.924	0.145
	Block	0.975	0.988	0.057
R_{2t}	Simple	**0.161**	**0.127**	**0.139**
	Block	**0.970**	**0.988**	**0.068**

Example 1 (Continued)

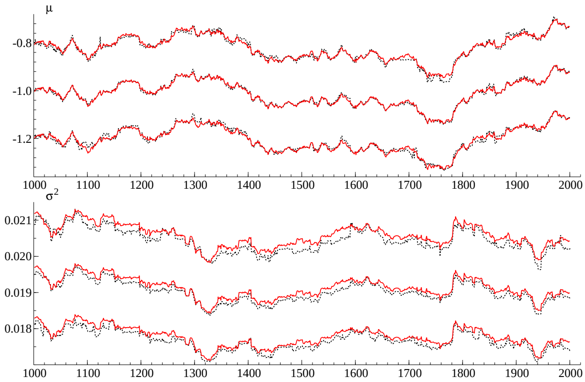


Figure: True posterior means and 95% credible intervals (dotted black) with their estimates (solid red) for μ and σ^2.

Example 1 (Continued)

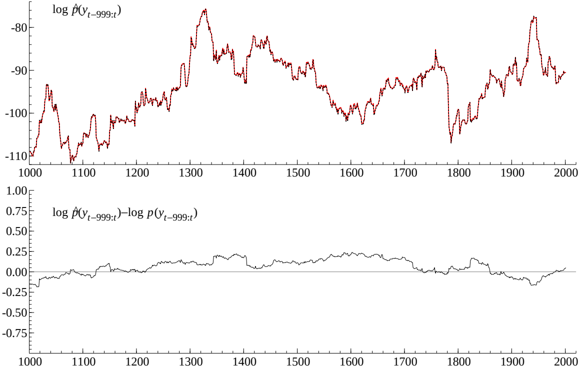


Figure: Top: true log marginal likelihoods $\log p(y_{t-999:t})$ (dotted black) and their estimates (solid red). Bottom: estimation errors $\log \hat{p}(y_{t-999:t}) - \log p(y_{t-999:t})$ for $t = 1001, \dots, 2000$.

Example 2 (Realized stochastic volatility model)

Let $y_{1,t}$: daily stock log return and $y_{2,t}$: log realized volatility.

$$
\begin{aligned}
y_{1,t} &= \exp(\alpha_t/2)\epsilon_t,\ \ \epsilon_t \sim \mathcal{N}(0,1),\ \ t=1,\ldots,T \\
y_{2,t} &= \alpha_t + \xi + u_t,\ \ u_t \sim \mathcal{N}(0,\sigma_u^2),\ \ t=1,\ldots,T \\
\alpha_{t+1} &= \mu + \phi(\alpha_t - \mu) + \eta_t,\ \ \eta_t \sim \mathcal{N}(0,\sigma_\eta^2),\ \ t=1,\ldots,T, \\
\alpha_1 &= \mu + \frac{1}{\sqrt{1-\phi^2}}\eta_0,\ \ \eta_0 \sim \mathcal{N}(0,\sigma_\eta^2),
\end{aligned}
$$

where

$$
\begin{pmatrix} \epsilon_t \\ u_t \\ \eta_t \end{pmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & \rho\sigma_\eta \\ 0 & \sigma_u^2 & 0 \\ \rho\sigma_\eta & 0 & \sigma_\eta^2 \end{bmatrix} \right).
$$

(The correlation between ϵ_t and η_t is introduced to express the leverage effect)

Example 2 (Realized stochastic volatility model)

Data

- y_{1t} and y_{2t}: the daily log returns and log realized volatilities of Standard and Poor's (S&P) 500 index (obtained from Oxford-Man Institute Realized Library)
- The initial estimation period: from January 1, 2000 ($t = 1$) to December 31, 2007 ($t = 1988$) with $L + 1 = 1988$.
- The rolling estimation started from sampling from the posterior distribution using this initial sample period and moved the window until December 30, 2016 ($T = 4248$).
- $K = 10$, $M = 300$ and $N = 1000$.
- We implement 10 MCMC iterations for the comparison below.

Example 2 (RSV)

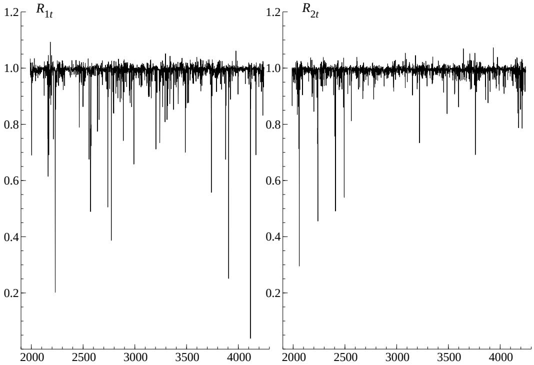


Figure: Traceplot of R_{1t} (left) and R_{2t} (right), $(t = 1988, \ldots, 4248)$ in RSV model.

Example 2 (RSV)

	K	Mean	Median	Std. dev.
R_{1t}	5	0.981	0.995	0.058
	10	0.985	0.996	0.053
	15	0.986	0.997	0.055
R_{2t}	5	0.983	0.993	0.044
	10	0.988	0.994	0.036
	15	0.988	0.994	0.035

Table: Summary statistics for R_{1t} and R_{2t} ($t = 1988, \ldots, 4248$) in RSV model with leverage using $K = 5, 10$ and 15.

Example 2 (RSV)

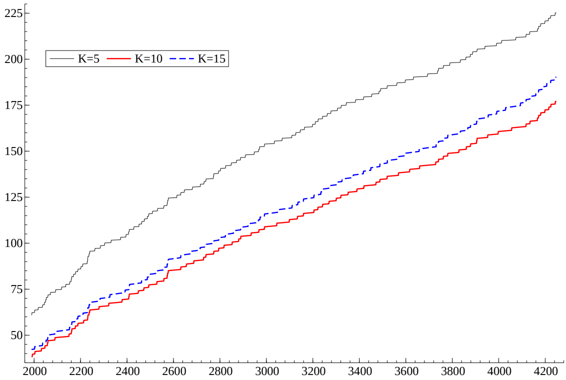


Figure: Cumulative computation times (wall time, unit time $=1000$ seconds) using $K = 5, 10$ and 15 ($t = 1988, \ldots, 4248$) in RSV model.

Example 2 (RSV)

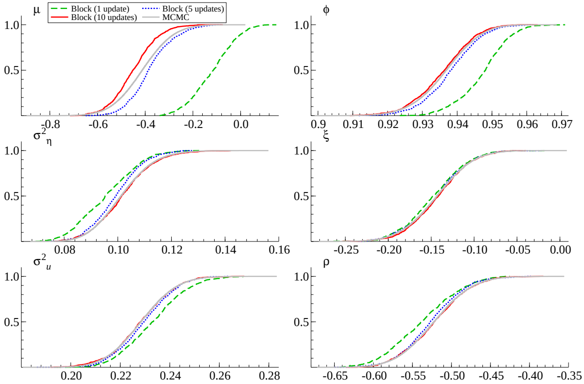


Figure: The estimated posterior distribution functions of θ for $t = 2261, \ldots, 4248$. MCMC and Particle rolling MCMC: 1, 5 and 10 iterations.

Example 2 (RSV)

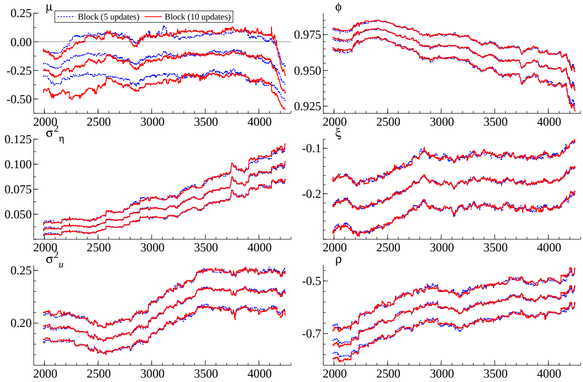


Figure: Traceplot of estimated posterior means and 95% credible intervals for parameters using S&P500 return in RSV model (from December 31, 2007 to December 30, 2016).

Example 2 (RSV)

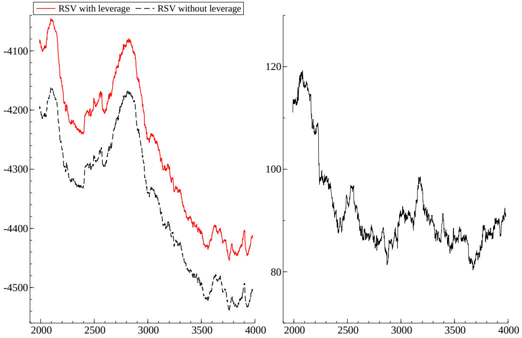


Figure: Left: Estimates of $\log p(y_{t-1987:t})$ $(t = 1988, \ldots, 4248)$ for S&P 500 index return in RSV model with leverage (solid red) and in RSV model without leverage (dotted black). Right: Difference between two log marginal likelihoods.

Comparison with SMC^2 (without rolling window)

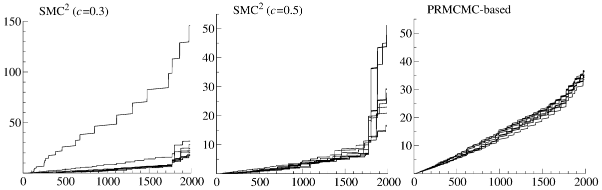


Figure: Cumulative computation times (wall time, unit time =1000 seconds) over 10 runs with SMC^2 with $c = 0.3$ (left), 0.5 (middle) and with the PRMCMC-based algorithm (right).

In the particle MH update steps of SMC^2, the parameter θ is generated using the random walk MH algorithm using the normal proposal $\mathcal{N}(\theta^*, c\hat{\Sigma})$ where θ^* is a current value of the parameter θ, c is a tuning parameter (0.3 or 0.5), and $\hat{\Sigma}$ is the estimated covariance matrix.

Comparison with SMC^2 (without rolling window)

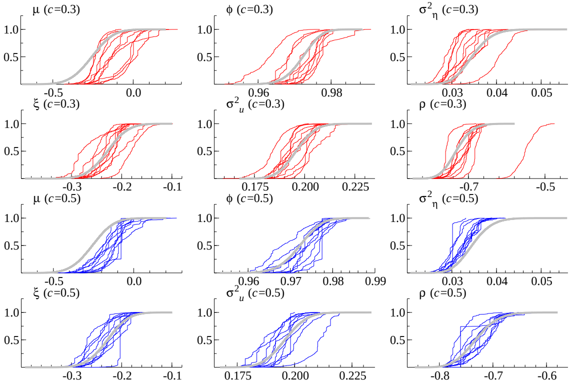


Figure: Estimated posterior cumulative distribution functions of θ given $y_{1:1988}$ over 10 runs with SMC^2 with $c = 0.3, 0.5$ (red and blue, respectively) compared to the result of MCMC (gray).

Comparison with SMC^2 (without rolling window)

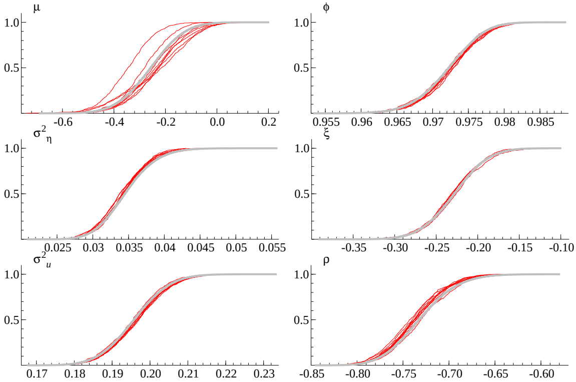


Figure: Estimated posterior cumulative distribution functions of θ given $y_{1:1988}$ over 10 runs with the PRMCMC-based method (red) compared to the result of MCMC (gray).

Conclusion

This paper considers the state space model for economic times series and

- proposes the efficient rolling estimation method for both parameter θ and states $\alpha_{s:t}$.
- gives the theoretical justification of the algorithm by proving the marginal density of parameters and states.
- the algorithm is based on Particle Gibbs
- proposes, as a special case, the sequential Bayesian estimation method (which is alternative to SMC^2 based on Particle MH).

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