What Do Mediators Do?

Information and Bargaining Design

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Mediation

"Mediation is a structured negotiation process in which an independent person, known as a mediator, assists the parties to identify and assess options and negotiate an agreement to resolve their dispute."

Federal Court of Australia

- Common process of dispute resolution
  - the most favored process in both federal and US state courts
  - used in different contexts: courts, border disputes, commercial transactions
- Voluntary – the parties sign a contract to mediate
  - but courts increasingly require mediation as part of a litigation plan

Mediation

- Mediator's role according to the legal literature:
  - facilitative – helping the parties to agree, or
  - evaluative – providing parties with a "reality check" about the merits of the case
- The mediator often engages in shuttle diplomacy
  - meets with each side in a private caucus
  - leaks some information to the other side
- The mediator can propose a settlement price or a settlement range
- Parties must agree to the resolution proposed by the mediator
  - unlike an arbitration panel or a judge, the mediator is not empowered to render a judgment

The Economic Questions

- What does the mediator do to "help" the parties?
- Why do the parties need help?

Law and Economics

- Models in law and economics (Brown and Ayres, 1994; Doornik, 2014):
  - rely on asymmetric information to explain the role of mediation
  - assume parties have full information about their values
  - cannot explain the "reality check" function of the mediator
- If the parties know their own private information, why do they need a "reality check" from the mediator?
- Reality checks are pretty common; we ask someone else's opinion
- This paper:
  - a mechanism design model in which agents do not have full information about their values for a transaction
  - the mediator controls the information flow to parties
  - the disclosure of such information provides "reality checks"

Example

- Agent 1 inherits a business in which she has little expertise
- Agent 2, is interested in acquiring the business, conditional on the positive outcome of a profitability investigation
- A reputable mediator proposes a procedure that selects the information Agent 1 and 2 obtain and the price at which a sale might take place
- Mediator maximizes the gains from trade
- Should the two agents use the mediator?
- Should agents demand to be fully informed?
- What kind of mediation procedure should the mediator design?

Information and Bargaining Design

- When buyer and seller have full, private information about their value and cost no Bayesian mechanism achieves efficiency (Myerson and Satterthwaite, 1983)
- We focus on mechanisms in which the mediator sets a single price, but add the information design problem to the classic mechanism design problem
  - robustness of our mechanism; we require ex post individual rationality and – except for the case of full-scale shuttle diplomacy – ex post equilibrium; then the bargaining mechanism must be a posted price (Hagerty and Rogerson, 1987; Čopič and Ponsati, 2016, Čopič 2017)

Information Design Literature

- Bayesian persuasion - Kamenica and Gentzkov (2011), ...approach as summarized by Bergemann and Morris (2017): the "information designer ... can commit to providing information about the state..., but has no ability to change the mechanism"

- Surplus extraction literature - single item seller chooses disclosure policy
  - Bergemann and Pesendorfer (2007): static disclosure, but a Bayesian approach. The optimal mechanism is complex - finite, interval, asymmetric partitions even if the buyers priors are the same
  - Esö and Szentes (2007): buyers have some private information and the seller may disclose additional, "orthogonal" information - optimal surplus extraction requires full disclosure of the orthogonal information
  - Bergemann and Wambach (2015): participation constraint can be strengthened if the seller uses a dynamic mechanism
  - Li and Shi (2017): with a more general class of "direct disclosure" policies, discriminatory, as opposed to full, disclosure may be optimal

The Plan

Mediator maximizes the gains from trade

- Static Information Disclosure and Trading Mechanisms
- Single-Ride Shuttle Diplomacy: Sequential Information Disclosure and Trading Mechanisms
- Full-Scale Shuttle Diplomacy: Dynamic Information Disclosure and Trading Mechanisms

Values and Costs

- The buyer's value is $v \in [0, 1]$
- The seller's cost is $c \in [0, 1]$
- $F_0(v) = \Pr_0(\text{value} \leq v)$ and $G_0(c) = \Pr_0(\text{cost} \leq c)$ are the true distributions from which value and cost are drawn. Assume they have no atoms.

The expectations according to these distributions are:

$$v_0 = \int_0^1 v dF_0(v)$$

and

$$c_0 = \int_0^1 c dG_0(c)$$

Feasible Distributions...

- Buyer and seller receive a signal, which can be interpreted as an unbiased estimate of value or cost
- No restrictions on signals. The feasible signal distributions are all the distributions of which the priors are mean preserving spreads:

$$\mathcal{F} = \left\{ F : \int_0^1 v dF(v) = v_0 \ \& \int_0^x F(v) dv \leq \int_0^x F_0(v) dv \ \ \forall x \in [0,1] \right\}$$

$$\mathcal{G} = \left\{ G : \int_0^1 c dG(c) = c_0 \ \& \int_0^x G(c) dc \leq \int_0^x G_0(c) dc \ \ \forall x \in [0,1] \right\}$$

...Distributions

- For the buyer:
  - Acquiring no information corresponds to the signal distributions that puts an atom of mass one on v_0
  - Full information acquisition (i.e., discovering the item's value) corresponds to the signal distribution $F_0(v)$

- For the seller:
  - Acquiring no information corresponds to the signal distributions that puts an atom of mass one on c_0
  - Full information acquisition (i.e., discovering the item's cost) corresponds to the signal distribution $G_0(c)$

Static Information Disclosure and Trading Mechanisms

- The mediator simultaneously chooses price p and signal distributions F and G for buyer and sellers so as to maximize the gains from trade
- Interpretation: buyer and seller use a mediator; they ask the mediator to release information and select the trading price. The mediator's problem is:

$$\max_{p\in[0,1],\, F\in\mathcal{F},\, G\in\mathcal{G}} \int_p^1 \int_0^p (v-c)dG(c)dF(v) \tag{1}$$

Two-Point Discrete Distributions

Lemma

Given any solution to the maximization problem (1), there is a payoff equivalent solution in which the mediator chooses two-point discrete distributions.

Feasible Two-Point Signal Distributions for the Buyer

- Let $\{v_L, v_H\}$ be the buyer's signals with probabilities $f_L, 1 - f_L$.
- Using $v_L f_L = v_0 - v_H(1 - f_L)$, the following constraint must hold for F_0 to be a mean preserving spread of the signal distribution:

$$x f_L - v_0 + v_H(1 - f_L) - \int_0^x F_0(v)dv \leq 0 \quad \text{for } x \in [v_L, v_H) \tag{2}$$

- The lhs of (2) is concave in x, maximized at x such that $f_L = F_0(x)$.
- Thus (2) holds if it holds for such x and we can write it as:

$$v_H \leq \int_x^1 v \frac{dF_0(v)}{1 - F_0(x)}$$

Feasible Two-Point Signal Distributions for the Seller

- Let $\{c_L, c_H\}$ be the seller's signals with probabilities $g_L, 1 - g_L$.
- Repeating the same argument for the seller, using y instead of x, with $G_0(y) = g_L$ leads to the following constraint on the two-point distribution for the seller:

$$c_L \geq \int_0^y c \frac{dG_0(c)}{G_0(y)}$$

The Intermediary's Problem

Thus, we can write the mediator's problem as follows:

$$\max_{v_H, c_L, x, y} (v_H - c_L)\, G_0(y)\, [1 - F_0(x)] \quad \text{s.t.}$$

$$v_H \leq \int_x^1 v \frac{dF_0(v)}{1 - F_0(x)}$$

$$c_L \geq \int_0^y c \frac{dG_0(c)}{G_0(y)}$$

It is immediate to see that both constraints must bind, otherwise the mediator would profit from raising v_H or lowering c_L.

The Optimal Static Mechanism

Proposition

Under the static information disclosure and trading mechanism that maximizes the gains from trade, the buyer observes whether the value is strictly below or at least as high as x and the seller observes whether the cost is strictly above or at least as low as y, where

$$\mathbb{E}_{G_0}[c|c \leq y] = x \quad \textit{and}$$
$$\mathbb{E}_{F_0}[v|v \geq x] = y$$

The trading price is any $p \in \left[\mathbb{E}_{G_0}[c|c \leq y], \mathbb{E}_{F_0}[v|v \geq x]\right] = [x, y]$.

The Uniform Example

- True distributions of values and costs F_0 and G_0 are uniform
- The solution is: $x = 1/3$ and $y = 2/3$.
  - buyer observes whether the value is above or below $1/3$;
  - seller observes whether the cost is above or below $2/3$
  - any price $p \in [1/3, 2/3]$ is a solution
- Expected welfare is $\left(\frac{2}{3} - \frac{1}{3}\right) \frac{2}{3} \frac{2}{3} = \frac{4}{27}$ or 89% of the first best level $\frac{1}{6}$
  - higher than $\frac{9}{64}$ or 84% of the first best level, the welfare in the optimal Bayesian mechanism when traders are fully informed, in which trade occurs iff $v \geq c + 1/4$

Static Information Disclosure and Trading Mechanisms: Insight

- Full information is not optimal; it does not generate enough trade
  - trade if $v \geq p \geq c$ (all trades are efficient)
  - efficient trades lost: (i) $p > v > c$, (ii) $v > c > p$
  - most valuable of lost trades:

    (i) $v = p - \varepsilon_v$ and $c = \varepsilon_c$; (ii) $v = 1 - \varepsilon_v$ and $c = p + \varepsilon_c$
- First effect of the optimal static information disclosure and trading mechanism: complete the most valuable efficient trades lost under full information
- Second effect: complete some inefficient trades with relatively small welfare losses: (i) $c = y - \varepsilon_c > v = x + \varepsilon_v$

Single-Ride Shuttle Diplomacy

A *sequential disclosure and trading mechanism* is a triple $\langle G, \Phi, P \rangle$.

- The mediator lets one trader (possibly randomly chosen) obtain information, say the seller: $G \in \mathcal{G}$ is the chosen posterior signal distribution of the seller
- the seller reports what signal she observed.
- The function $\Phi : [0, 1] \to \mathcal{F}$ assign a posterior signal distributions of the buyer's value $F_c \in \mathcal{F}$ to each cost report c of the seller.
- The function $P : [0, 1] \to \mathbb{R}_+$ assigns a trading price $P(c) \in \mathbb{R}_+$ to each cost report c of the seller.

Mediator's Problem

- After the buyer observes her signal, buyer and seller decide whether they want to trade at the specified price $P(c)$
- Mediator maximizes the gains from trade; her problem is:

$$\max_{\langle G,\Phi,P\rangle} \int_0^1 \int_{P(c)}^1 [(v-c)dF_c(v)]\, dG(c)$$

Two-Point Buyer Signal Distribution

- As in static case, it is optimal to select a buyer disclosure policy that reveals to the buyer whether her values is above or below a threshold x.
- An upper bound on what the mediator could achieve with shuttle diplomacy is given by the ex post efficient outcome.

Ex Post Efficiency and Full Information to Seller

Lemma

Any sequential disclosure and trading mechanism $\langle G, \Phi, P \rangle$ *that realizes the first best, or efficient, gains from trade informs the buyer whether her value is above or below the seller's reported cost and must fully inform the seller about her costs. That is, the buyer signal distribution puts mass* $F_0(c)$ *on* $v_L^0(c) = \mathbb{E}_{F_0}[v|v < c]$ *and mass* $R_{F_0}(c)$ *on* $v_H^0(c) = \mathbb{E}_{F_0}[v|v \geq c]$*, while the seller's signal distribution must be* $G(c) = G_0(c)$ *for all except a zero measure set of costs* c*.*

Proof

- Suppose $\langle G, \Phi, P\rangle$ is an efficient mechanism.
- Buyer observes whether the value is strictly below or above x. Suppose $x \neq c$. Then:

$$\int_0^1 \int_x^1 (v-c)dF_0(v)dG(c) < \int_0^1 \int_c^1 (v-c)dF_0(v)dG(c)$$

  contradicting the efficiency of $\langle G, \Phi, P\rangle$. Hence $x = c$.
- Suppose $G(c) \neq G_0(c)$ for a positive set measure of costs c:

$$\int_0^1 \left[\int_c^1 (v-c)dF_0(v)\right] dG(c) < \int_0^1 \int_c^1 (v-c)dF_0(v)dG_0(c)$$

  since term in square brackets is a convex function of c and G_0 is a mean preserving spread of G.
  - This contradicts efficiency of $\langle G, \Phi, P\rangle$. $\square$

Sequential Information Disclosure and Trading Mechanisms

- To obtain ex post efficiency the seller must discover her true cost and the buyer observe whether her value is above or below the cost reported by the seller.
- But can the mediator incentivize the seller to report her true cost?
- Let $\widehat{c}$ be the seller's cost report
- Let $p\left(\widehat{c}\right)$ be the price chosen by the intermediary
- The buyer ex post individual rationality constraint:

$$p\left(\widehat{c}\right) \leq \mathbb{E}_{F_0}\left[v \middle| v \geq \widehat{c}\right]$$

- The seller to accept ex post individual rationality constraint:

$$p\left(\widehat{c}\right) \geq \widehat{c}$$

The Seller's Incentive Constraint

- The payoff of a seller who reports $\widehat{c}$ while her true cost is c is:

$$u_S(\widehat{c}; c) = [p(\widehat{c}) - c][1 - F_0(\widehat{c})]$$

- Let $u_S(c) = u_S(c; c)$; by the envelope theorem:

$$u_S'(c) = -[1 - F_0(c)]$$

  or, integrating and using the boundary condition $p(1) = 1$:

$$[p(c) - c][1 - F_0(c)] = u_S(c) = \int_c^1 [1 - F_0(\widetilde{c})]\, d\widetilde{c}$$

  integrating by parts and rearranging we obtain

$$p(c) = \int_c^1 \frac{\widetilde{c}}{1 - F_0(c)} dF_0(\widetilde{c}) = \mathbb{E}_{F_0}[v | v \geq c]$$

- Price function satisfies posterior IR constraints of buyer and seller

The Optimal Sequential Mechanism

Proposition

Under the sequential information disclosure and trading mechanism that maximizes the gains from trade, one trader, say the seller, obtains full information about her type and reports it to the mediator. The mediator chooses an information policy that lets the buyer (the other trader) observe whether her value is above or below the reported type (cost) of the seller. As a function of the report $\widehat{c}$*, the mediator chooses a price function* $p(\widehat{c})$ *that gives all the gains from trade to the seller (the fully informed trader):*

$$p\left(\widehat{c}\right) = \mathbb{E}_{F_0}[v | v \geq \widehat{c}]$$

Efficient trading results as an ex post perfect equilibrium of the mechanism.

Single-Ride Shuttle Diplomacy Insight

- Informing first one trader and asking her to report her information allows the mediator to condition the information received by the other trader and the price on the first trader's type
- The second trader can be given the minimum information needed to implement ex post efficient trading, conditional on the report
- Choosing a price that gives the fully informed trader all the gains from trade induces her to report truthfully, as it aligns her private incentives with the social goal
- It is less obvious, but perhaps not surprising, that no other price function would work

Full-Scale Shuttle Diplomacy

- Is it possible to design information disclosure and trading mechanisms that are ex post efficient but in which the gains from trade are shared (ex post)?
- The answer is yes, by using dynamic information disclosure and trading mechanisms, i.e., full-scale shuttle diplomacy

Full-Scale Shuttle Diplomacy Disclosure Policy

- The mediator selects a "fair" price p^F, a natural candidate is a solution to

$$\mathbb{E}_{F_0}[v|v \geq p^F] - p^F = p^F - \mathbb{E}_{G_0}[c|c \leq p^F]$$

- p^F is the price at which trade takes place if the buyer's value is above it and the seller's cost is below it
- the mediator discloses information slowly over time, alternating between disclosures to the buyer and to the seller;
    - for concreteness say the seller is the first to receive some information.

Disclosure Policy: The Shuttle Diplomacy Phase

In Period t:

1. seller discovers whether her cost is in $[1 - t\Delta^S, 1 - (t-1)\Delta^S]$
   - seller reports if she has discovered her cost
     - if report is *Yes*, then go to Final Stage
     - if report is *No*, then:
2. buyer discovers whether her value is in $[(t-1)\Delta^B, t\Delta^B]$
   - buyer reports if she has discovered her value
     - if report is *Yes*, then go to Final Stage
     - if report is *No*, then:
3. if $t < T$ go to period $t + 1$;
4. if $t = T$ go to Final Stage

- Selects Δ^S and Δ^B so that at at the end of period T buyer and seller know whether their value and cost are above or below p^F.

Disclosure and Pricing Policy: The Final Stage

1. if $t < T$ and the seller has reported to have discovered her cost c_t:
   - the buyer observes whether her value is above c_t
   - the posted price $p^S(c_t)$ is set so that the seller with costs c_t and the buyer who has observed that her value is above c_t are willing to trade
2. if $t < T$ and the buyer has reported to have discovered her value v_t:
   - the seller observes whether her cost is below v_t
   - the posted price $p^B(v_t)$ is set so that the buyer with value v_t and the seller who has obseved that her cost is below v_t are willing to trade
3. if $t = T$, then the posted price is p^F and buyer and seller will be willing to trade if they have sincerely reported not to have yet discovered their value and cost

Full-Scale Shuttle Diplomacy

- under sincere reporting the described procedure is approximately ex post efficient and becomes exactly ex post efficient as the number of periods, i.e., T, converges to infinity.

Proposition

Under "mild technical conditions", there are continuous price functions $p^B(v)$ and $p^S(c)$ under which it is a perfect Bayesian equilibrium for traders to sincerely report their value and cost immediately after discovering it. The equilibrium allocation of the dynamic information disclosure and trading mechanism is ex post efficient.

The Uniform Case

- A price function pair and fair price (not unique) that induce sincere reporting and ex post efficient trade are $p^B(v) = v$, $p^S(c) = c$, $p^F = \frac{1}{2}$
- after discovering that her value is v, or her cost is c, it is a weakly dominant strategy for a trader to report sincerely
- suppose the buyer only knows that her value is above v and the seller only knows that her cost is below $c(v) = 1 - v$
  - by reporting that her value is v the buyer obtains:

$$\left(\frac{1+v}{2} - v\right) \frac{v}{c(v)}$$

  - under sincere reporting by the seller, by waiting for a small time δ and then reporting that her value is $v + \delta$ the buyer expects to obtain:

$$\left(\frac{1+v+\delta}{2} - (v+\delta)\right) \frac{1-(v+\delta)}{1-v} \frac{v+\delta}{c(v)} + \left(\frac{1+c(v+\delta)}{2} - c(v+\delta)\right) \frac{1-c(v+\delta)}{1-v} \frac{\delta}{c(v)}$$

  - since $c(v+\delta) = 1 - v - \delta$, waiting is optimal for all $v < \frac{1}{2}$

Conclusions

- Information Disclosure and Mechanism Design: Both the allocation and the information given to agents are chosen by the designer
- General insights:
  - Some obfuscation is optimal in order to induce increased trade: do not give full information to all agents
  - with static procedures optimal bfuscation generates additional efficient and inefficient trades, but the former are more valuable than the latter
  - There are efficiency gains in using sequential or dynamic information disclosure procedures, as information disclosure and trading price can be conditioned on past disclosures
- In the future: apply the information disclosure and mechanism design approach to other important problems (e.g., public good decisions)