Strategic "Mistakes": Implications for Market Design Research

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Received knowledge: mistakes

- One of the desiderata of market design: strategy-proofness.
  - Allows participants to submit their true preferences to the mechanism, without attempting to second-guess the strategies of others
- Participants "strategize" and do not submit their true preferences.
  - Lab experiments
  - Hassidim, Romm & Shorrer (2018): 19% of applicants to Israeli postgraduate programs in psychology made "clear" mistakes.
  - Similar observations in Hungarian university admission (Shorrer and Sóvágó, 2017), National Resident Matching Program (Rees-Jones, 2017)

This paper: mistakes

Look deeper into (one particular type of) mistakes:

- Do they change the outcome?
  - Most mistakes are not payoff-relevant (80% to 98%)
- Payoff-irrelevant mistakes: data consistent with applicants skipping infeasible (out-of-reach) programs
- Which individual characteristics correlated with mistakes?
  - P-irrelevant: negatively correlated with ability, private school
  - P-relevant mistakes: not correlated with any of the above
  - Change in opposite directions "over time" (p-irrelevant increase, p-relevant decrease)
- Applicants do not appear to strategize in a misguided attempt to get a better assignment.

Received knowledge: Truth-telling vs. stability

Why strategy-proofness?

- Easier to avoid mistakes ($\approx$ mistakes are payoff-irrelevant?)
- No disadvantage to unsophisticated players ($\approx$ lower ability not correlated with payoff-relevant mistakes?)
- Observed behavior is easy to interpret as true preferences are submitted to the mechanism
  - Predict welfare changes theoretically
  - Estimate applicants' preferences assuming truth-telling (TT):
  - Draw counterfactual welfare comparisons
    - Abdulkadiroglu, Agarwal & Pathak (2015), Abdulkadiroglu, Pathak & Roth (2009), Che & Tercieux (2016a,b) . . .

Received knowledge: Truth-telling vs. stability

If truth-telling is not used as identifying assumption, what is an alternative?

- Use stability ($\approx$ ROL where "it matters") for estimates/conterfactuals
  - In centralized school choice and college admissions: Fack, Grenet & He (2015).
  - In other settings: Agarwal (2015) – centralized; Chiappori, Galichon, Fox, Menzel, Salanié – decentralized matching.

This paper: implications for market research, truth-telling vs. stability

- Theoretical analysis: account for mistakes and explore implications to identifying assumptions of empirical studies (truth-telling vs. stability)
  - Participants are allowed to make ϵ-costly mistakes
  - Market grows large
  - Stability is robust to mistakes
  - but truth-telling is not
- Monte Carlo Simulations: Confirm the theory and quantify the effects of mistakes on estimates based on alternative assumptions and on the counterfactuals.
  - Bias vs. variance

Data Analysis: Institutional Background

- Centeralized mechanism via a platform **VTAC** (Victorian Tertiary Admissions Centre)
- An applicant submits a rank-ordered list (ROL) of university courses (university-major combination) up to 12 choices.
- Courses rank students (almost exlusively) by ENTER (Equivalent National Tertiary Entrance Rank) — we shall call it Score (the higher the better).
- The algorithm is similar to **Serial Dictatorship**
  - The highest ranked student "choose" a course, then the second ...
  - Everyone "chooses" among the remaining courses.
  - How one "chooses" is determined by the submitted ROL.
- Note: not strategyproof due to length restriction;

Fee/no fee, "Skip" and its payoff relevance: definitions

- Many courses (about 881 out of 1899) are available in two versions, and treated as "separate courses" with separate cutoffs applying.
  - Full Fee (FF): Full tuition; median AUD17,000 (USD13,000) per year.
  - Reduced Fee (RF): About half of tuition; can be borrowed on a subsidized loan.
  - They are otherwise identical.
- Skip: Not listing RF version of a course but listing FF version is clearly a weakly dominated strategy when listing < 12 choices.
- A skip is declared to be payoff relevant if an applicant would have been assigned to the skipped course if that course had been listed.

Skipping and Payoff-Relevant Skipping

Skips and Mistakes among V16 Applicants Listing Fewer than 12 Courses

	All	FF listed	Skips	Payoff-relevant mistakes Upper bound	Payoff-relevant mistakes Lower bound
% All	100.00	10.61	3.61	0.72	0.05
% FF listed		100.00	34.05	6.78	0.47
% Skips			100.00	19.92	1.39
Total	27,922	2,963	1,009	201	14

- Potentially non-negligible number of applicants skip,
- but the majority of the skips are payoff irrelevant.

Regressions: Who Skips? Who Makes Mistakes?

$$Skip_i = \alpha + \beta Score_i + \gamma \mathbf{X}_i^T + \epsilon_i, \quad (1)$$

where

- Skip or not: 100% or 0
- $\mathbf{X}_i^T$:
  - General Achievement Test (**GAT**): distinct from Score, taken as a proxy for cognitive ability (separate from Score)
  - School fee (dummy): the applicant attends a school that charges more than AUD11,000 (approx. USD8,000) in fees.
  - gender, income (postal-code average), region of birth, citizenship status, language spoken at home
  - high school fixed effects
  - number of FF courses ranked (dummies)

$$\text{Payoff-relevant mistake}_i = \alpha + \beta Score_i + \gamma \mathbf{X}_i^T + \epsilon_i, \quad (2)$$

More able / better advised make fewer skips, but no fewer payoff-relevant mistakes

Probability of Skips and Payoff-relevant mistakes

	All mistakes (Skips)		Payoff-relevant mistakes		
	Full sample	FF s/sample	Full sample	FF s/sample	Skip s/sample
	(1)	(2)	(3)	(4)	(5)
Score	-0.04***	-0.56***	0.01***	0.19***	0.61***
	(0.01)	(0.07)	(0.00)	(0.04)	(0.15)
GAT	-0.04***	-0.33***	0.00	0.01	0.28
	(0.01)	(0.10)	(0.01)	(0.06)	(0.19)
(Private school)	-0.03***	-0.05**	0.01	-0.11*	0.22
$\times$ *Score*	(0.01)	(0.02)	(0.01)	(0.06)	(0.27)
Other controls	Yes	Yes	Yes	Yes	Yes
# of Applicants	26,325	2,766	26,325	2.766	947
R^2	0.36	0.17	0.14	0.25	0.48

- Negative correlation of skips with GAT/Private school: more able and better advised make fewer skips;
- No correlation of p-rel mistakes with GAT/Private school: less able and worse advised make as good decisions as more able better advised "where it matters";

Correlation of Score with Skips and Payoff-relevant Mistakes

Probability of Skips and Payoff-relevant mistakes

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- Negative correlation of skips with Score: Consistent with omitting out-of-reach courses (lower score → more courses are out-of-reach and dropped);
- Positive correlation of p-rel mistakes with Score: Consistent with “random noise” (applicant omits any course with equal probability → more likely to be payoff-relevant for a higher-ability applicant).

Skips increase, Payoff-Rel Mistakes decrease

- Unique feature: Applicants revise ROLs over time: before & after Scores are revealed.
- Regressions:

$$\Delta(\textit{Skips}_i) = \tau^s + \upsilon^s(\mathbf{X}_i^T - \overline{\mathbf{X}}_i^T) + \epsilon_i$$

$$\Delta(\textit{Payoff-rel Mistakes}_i) = \tau^m + \upsilon^m(\mathbf{X}_i^T - \overline{\mathbf{X}}_i^T) + \epsilon_i,$$

	All mistakes (Skips)		Payoff-relevant mistakes	
	(1)	(2)	(3)	(4)
Constant	1.05***	0.72***	-0.12**	-0.19***
	(0.14)	(0.11)	(0.05)	(0.05)
Change in # FF courses		43.24***		8.69***
		(2.75)		(1.26)
Other Controls	Yes	Yes	Yes	Yes
# of Applicants	26,325	26,325	26,325	26,325
R^2	0.02	0.13	0.04	0.42

With more information (about one's score), applicants

- Make fewer payoff-relevant mistakes, despite the fact that they
- Skip more

Summary of previous tables

- Payoff-irrelevant mistakes have some systematic patterns, but
- Payoff-relevant mistakes are rare and look like non-systematic noise.

Not specific to Australia

- For all mistakes (skips), results are similar to those for Israeli medical match (Hassidim, Romm & Shorrer, 2018) and Hungarian college admission (Shorrer and Sóvágó, 2017).
- Shengwu Li "Obviously Strategy-Proof Mechanisms" (2017): data from his Serial Dictatorship experiment.

	All mistakes	Payoff-relevant mistakes
Score	-0.36***	0.60***
	(0.06)	(0.13)
Subject FE	Yes	Yes
Period FE	Yes	Yes
N	720	209
R^2	0.53	0.37

Theoretical Analysis

Solution Concept Permissive of Mistakes: Environment

- Azevedo-Leshno Continuum Model $E = [\eta, S]$: finite number of colleges, mass of seats $S = (S_1, ..., S_C)$, and mass of applicants given by
  - distribution η of applicant types (preferences and scores); assume atomless, full support.
  - $\Rightarrow$ Unique stable matching (or DA outcome) characterized by "cutoffs" in scores.
- A sequence of finite (random) economies $\{F^k\}_k = \{[\eta^k, S^k]\}_k$, generated by k iid sampling of applicants according to η and proportionate scaling of supply S^k. F^k converges to E in a strong sense ($\eta^k \to \eta$ uniformly, a.s.).
- We consider a sequence of DA games that applicants play in $\{F^k\}_k$.

Solution Concept Permissive of Mistakes: Environment

Definition (Robust equilibrium)

A sequence of strategy profiles $\{(\sigma_i^k)_{1\leq i\leq k}\}_k$ by students (wrt ROLs) with the property that, for any $\epsilon > 0$, $(\sigma_i^k)_{1\leq i\leq k}$ must form an interim ϵ-BNE for all k large enough.

- Consider economy F^k
- Equilibria: all strategy profiles where applicants cannot gain more than ϵ by deviating;
  - We allow applicants to make small mistakes
- How small is ϵ allowed to be? For sufficiently large economy, ϵ can be made arbitrarily small.

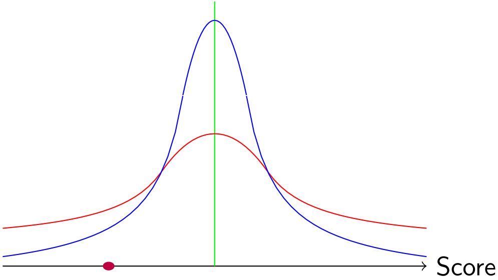
Truth-telling is not the only robust equilibrium strategy in a large market

Theorem

There exists a robust equilibrium in which a positive fraction of applicants submit untruthful ROLs. The fraction can be arbitrarily close to one as $k \to \infty$.

- Implication: Truth-telling assumption is not robust to "mistakes."

Intuition:
Market grows large
$\to$ admission cutoffs become tight
$\to$ ϵ loss if an applicant skips a program she is not assigned to in E.



- Intuition: Market grows large $\to$ admission cutoffs become tight $\to$ ϵ loss if an applicant skips a program she is not assigned to in E.

- Details: cutoffs are an outcome of the game the applicants play. Need to

Robust equilibria result in a stable outcome

Theorem

In any "regular" robust equilibrium, the outcome becomes virtually stable as $k \to \infty$: the fraction of applicants getting their stable assignment (given the prevailing cutoffs) converges to one in probability as $k \to \infty$.

- "Stable": every applicant gets best ex-post feasible college (score above realized cutoff)
- "Regular": each student plays TT with some arbitrarily small probability.
- Implication: Stability is robust to "mistakes" in a large enough market.

Robust equilibria result in a stable outcome, intuition for proof

Theorem

In any "regular" robust equilibrium, the outcome becomes virtually stable as $k \to \infty$: the fraction of applicants getting their stable assignment (given the prevailing cutoffs) converges to one in probability as $k \to \infty$.

- Strategy does not deliver a stable outcome $\Rightarrow$ deviate to TT $\Rightarrow$ get a better college $\Rightarrow$ discrete gain $\Rightarrow$ not equilibrium (not robust eq?).
- Given any sequence of economies and strategies, there exists a subsequence of economies $\{F^k\}$ which induce a sequence of (random) cutoffs $\{P_j^k\}$ that converges to cutoffs $\bar{P}$ in the limit continuum economy.
- In such an economy, an agent who gets a non-stable outcome can deiate to TT to get a better match (determined by $\{P_j^k\}$)
- This entails a gain in payoff that does not converge to zero (as $\{P_j^k\}$ converge to $\bar{P}$)
- Then such a strategy is not a robust equilibrium.

Summary of Theoretical Results

Theorem

There exists a robust equilibrium in which a positive fraction of applicants submit untruthful ROLs. The fraction can be arbitrarily close to one as $k \to \infty$.

Theorem

In any "regular" robust equilibrium, the outcome becomes virtually stable as $k \to \infty$*: the fraction of applicants getting their stable assignment (given the prevailing cutoffs) converges to one in probability as* $k \to \infty$.

Corollary

The limit outcome of a regular robust equilibrium would be the same as if all applicants reported their preferences truthfully.

If participants play robust equilibrium, truth-telling is

- *not* a good predictor of behavior;
- a good predictor of the outcome.

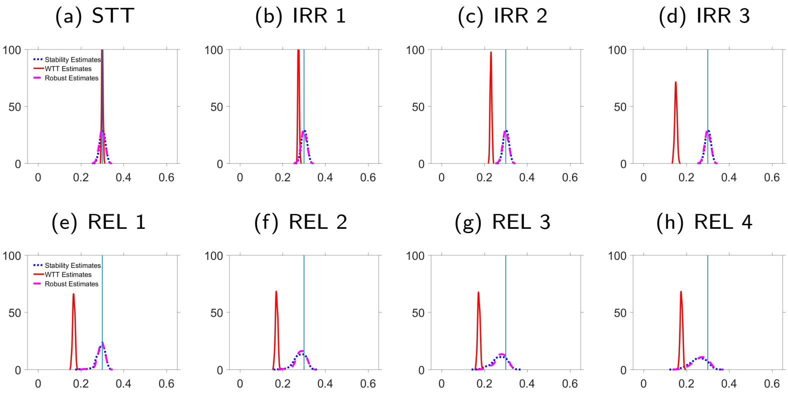
Monte Carlo Simulation

Simulations: estimating parameters

- Monte Carlo Simulations: Confirm the theory and quantify the effects of mistakes on estimates based on alternative assumptions and on the counterfactuals.
  - Obtain the distribution of cutoffs ("historical data"): serial dictatorship on 1800 samples
  - Simulate 200 "real-life" samples
    - "real-life": make mistakes (variety of % of payoff-irrelevant and payoff-relevant mistakes)
  - Estimate parameters of random utility model using (i) truth-telling; (ii) stability and (iib) robust stability.

How do mistakes affect estimation?

Distribution of estimates based on TT (red), Stability (blue), and Robust (purple) [True Value: $\beta_1 = 0.3$]

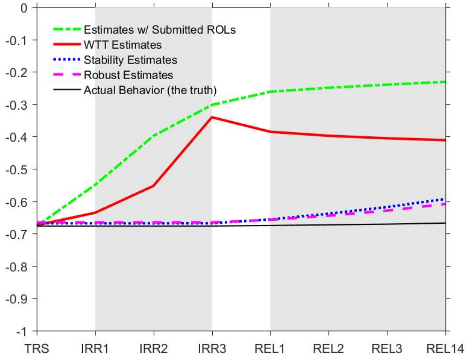


- Bias vs. variance: Estimates based on stability and the robust approach is more robust than those based on TT to mistakes.

Simulations: Welfare comparisons

- Use three versions of estimates above (+ naive assumption that the same rank-ordered list would be submitted) to evaluate welfare implications of a new policy (affirmative action: a group of applicants have a higher priority than anyone else) on non-target group

Comparison of the Three Approaches: Mis-Predicted Welfare Changes for non-targeted group



Conclusion

- Most mistakes are payoff-irrelevant
- Payoff-relevant mistakes appear to be "random noise"

⇒ Strategy-proofness performs reasonably well in real life.

- Using strategies predicted by an "exact" solution concept (that does not permit mistakes) may not be robust as a prediction or as an identification condition for empirical studies.
- Instead outcome predicted by the proposed robust solution concept appears to work more reliably both as a prediction and as an empirical method.

Outcomes speak louder than strategies!

Thank you!

Comments? Suggestions?

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Institutional Background

- Centralized assignment of applicants to programs (university-major)
  - 1899 programs in total
  - 881 offer full-fee (FF) and reduced-fee (RF) place.
- An applicant submits a rank-ordered list (ROL) of up to 12 choices.
  - Not strategy-proof; focus on these listing less than 12 programs
- Programs rank applicants (almost only) by Score (the higher the better).
- The algorithm, a variant of DA, is similar to **Serial Dictatorship**
  - The highest ranked applicant "choose" a program, then the second ...
  - Everyone "chooses" among the remaining programs.
  - How one "chooses" is determined by the submitted ROL.

“Skip”: mistakes and payoff relevant mistakes

- Everyone strictly prefer RF to FF program
- **Skip**: Not listing RF but listing FF – a dominated strategy
  - Example: C_1 = RF of course C; C_2 = FF of course C.
    - (A_1, A_2, C_2) – skipping C_1
- Skip is **payoff-relevant** if, given others' actions, adding C_1 back into ROL results in the applicant getting C_1.
  - Depends on where in ROL we put back C_1:
  - Upper bound: adding C_1 on top;
    - e.g., $(A_1, A_2, C_2) \rightarrow (C_1, A_1, A_2, C_2)$
  - Lower bound: adding C_1 right above C_2.
    - e.g., $(A_1, A_2, C_2) \rightarrow (A_1, A_2, C_1, C_2)$

Data

- Sample period: 2007 application (for enrollment in 2008), with 75k applicants, about 1899 programs (881 offering both tuition types, RF/FF)
- Our focus: "V16 <12" — 12th grader applicants who filled out fewer than 12 (making "skip" a dominated strategy)
- 27,992; of whom 2,963 listed at least one FF program.

Back

Not a misguided attempt to improve an allocation

- Do applicants skip courses hoping to get a better allocation (e.g. mistake the mechanism for Boston/Immediate Acceptance algorithm)?
- H_2: skip courses from the top, but not from the bottom $\Rightarrow$ skip positively correlated with compressed ROL. Negative coefficient on δ – no support.

$$\begin{aligned} &\textit{Cutoffs top-ranked courses}_i - \textit{Cutoffs bottom-ranked courses}_i \\ &\qquad = \gamma + \delta \textit{Skip}_i + \zeta \textit{Score}_i + \eta \mathbf{X}_i^T + \epsilon_i . \end{aligned}$$

	Applicants Listing at Least One Full-Fee Course					
	(1)	(2)	(3)	(4)	(5)	(6)
Skip	-1.38	-0.41	0.09	0.06	0.32	0.69
	(1.11)	(0.93)	(1.02)	(1.18)	(1.00)	(1.06)
Score	0.08**	0.08***	0.07**	0.06*	0.07**	0.07**
	(0.04)	(0.03)	(0.03)	(0.04)	(0.03)	(0.03)
ROL Length				Yes	Yes	Yes
Other Controls	No	No	No	Yes	Yes	Yes
N	2825	2598	2080	2797	2570	2055
R^2	0.18	0.18	0.25	0.21	0.21	0.28

Back

Real effects of Payoff-Relevant Mistakes

- Applicants making payoff-relevant mistake suffer financial loss
- Payoff-relevant mistake also significantly decrease the probability of enrolment.

$$\begin{matrix} Enroll_i \\ Defer_i \\ Reject_i \end{matrix} = \omega^m \textit{Payoff-relevant Mistake}_i + \omega^s Skip_i + \aleph \mathbf{X}_i^T + \epsilon_i$$

	Enroll (1)	Defer (2)	Reject (3)
Payoff-relevant mistake	-15.75*** (3.81)	13.06*** (3.34)	2.69 (2.94)
Skip	-4.07** (1.99)	4.76*** (1.57)	-0.69 (1.73)
Score	0.38*** (0.02)	0.12*** (0.01)	-0.51*** (0.02)
Other controls	Yes	Yes	Yes
N	23774	23774	23774
R^2	0.17	0.13	0.18

Monte Carlo Simulation: Model Specification

- $I = 1800$ students; $S = 12$ schools with quotas $\{S_c\}_{c=1}^{12} = \{150, 75, 150, 150, 75, 150, 150, 75, 150, 150, 75, 150\}$.
- Mechanism: Serial Dictatorship where higher index student has a higher (superior) priority.
- random utility model:

$$u_{i,s} = \beta_1 s - d_{i,s} + \beta_2 T_i A_s + \sigma \epsilon_{i,s}, \forall i \text{ and } s;$$

  where $(\beta_1, \beta_2, \sigma) = (0.3, 2, 1)$ true quality/preference parameters; the goal is to recover them by estimation)

  - $d_{i,s}$: distance from i to s (uniform random within a circle with radius 1, schools on a circle with radius 1/2);
  - Student type: $T_i = 1$ ("disadvantaged") or 0 ("advantaged"); T_i is 1 wp 2/3 for $i \leq 900$; $T_i = 0$ for all $i > 900$.
  - School type: $A_s = 1$ (odd numbered schools; good for disadvantaged) or 0 (even numbered schools)
  - Idiosyncratic shock: $\epsilon_{i,s} \sim$ a type-I extreme value.

Data Generating Processes (DGP)

- First run 1800 Serial Dictatorship under truthful reporting with random draws of $\{d_{i,s}, \epsilon_{i,s}\}_s$ and T_i to obtain **distribution of cutoffs**.
- Next simulate behavior/outcome with another 200 samples with new independent draws of $\{d_{i,s}, \epsilon_{i,s}\}_s$ and T_i under 8 different behavioral models:

1. **STT (Strict Truth-Telling)**: Every student submits a rank-ordered list of 12 schools according to her true preferences.
2. **IRR (Payoff Irrelevant Drops)**: Varying fractions of students skip schools with which they would never match given cutoff distributions, at the top ("out of reach" schools) and at the bottom ("dominated"). Fractions: 1/3 (IRR1), 2/3 (IRR2) and 1 (IRR3)
3. **REL (Payoff Relevant Mistakes)**: In addition to IRR3, students drop schools with small match probabilities (given the simulated cutoff distribution): 7.5% (REL1), 15% (REL2), 22.5% (REL3), and 30% (REL4).

Data Generating Processes

Skips and Mistakes in Monte Carlo Simulations (Percentage Points)

	Scenarios (Data Generating Processes)							
	Strict Truth-telling	Payoff Irrelevant Skips			Payoff Relevant Mistakes			
	STT	IRR 1	IRR 2	IRR 3	REL 1	REL 2	REL 3	REL 4
WTT: *Weak Truth-Telling*	100	90	76	62	61	61	61	61
Matched w/ favorite feasible sch.	100	100	100	100	96	93	89	85
Skippers	0	24	53	82	82	82	82	82
By number of skips:								
11	0	14	31	46	57	61	65	68
10	0	4	9	14	18	18	16	14
9	0	3	6	10	7	4	2	1
8	0	3	5	8	1	0	0	0
7	0	1	2	3	0	0	0	0
6	0	0	0	1	0	0	0	0
5	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0
STT: *Strict Truth-telling*	100	76	47	18	18	18	18	18
Reject TT: Hausman Test	5	8	57	100	97	93	90	85