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Framing Game Theory

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1. Motivation and Examples

Real players fail "Hypothetical Thinking"

Shafir and Tversky (92), Evans (07), Charness and Levin (09), Esponda and Vespa (14,16), etc.

What is hypothetical thinking? Think in '**what-if**' manner.

Imperfect Information Games :

I cannot observe the opponent's choice, C or D.

"If the opponent selects C, I prefer A to B"
"If the opponent selects D, I prefer B to A"

Because of difficulty of hypothetical thinking, real players fail to play dominant strategies.

Example: Prisoners' Dilemmas

		player 2 C	player 2 D
Player 1	**C**	**1 - 3**	**- 1 0**
	D	**2 - 3**	**0 - 2**

D is a **dominant strategy** for player 1.

Hypothetical Thinking:

Hypothesis C: Player 2 selects C.
⇒ "I (player 1) prefer D to C, because 2 > 1."
Hypothesis D: Player 2 selects D.
⇒ "I prefer D to C, because 0 > - 1."

"I don't know which hypothesis is true, but D is always my best."

However, a bounded-rational player fails hypothetical thinking.

		player 2 C	player 2 D
Player 1	**C**	**1 - 3**	**- 1 0**
	D	**2 - 3**	**0 - 2**

Instead he or she incorrectly thinks in a **Strategy-Contingent** manner:

"I (player 1) select D and become pessimistic. I expect player 2 to select D."
$\Rightarrow$ "By selecting D, I expect to receive payoff 0."
"I select C and become optimistic. I expect player 2 to select C."
$\Rightarrow$ "By selecting C, I expect to receive payoff 1."

"**D is not "obviously" dominant, because 1 > 0. Hence, I prefer C to D**."

		player 2 C	player 2 D
Player 1	**C**	**1 - 3**	**- 1 0**
	D	**2 - 3**	**0 - 2**

D is a dominant strategy for player 2.
Player 2 selects D even if he or she incorrectly thinks in the strategy-contingent manner:

"I (player 2) select D and become pessimistic. I expect player 1 to select D."
$\Rightarrow$ "By selecting D, I expect to receive payoff 0."
"I select C and become optimistic. I expect player 1 to select C."
$\Rightarrow$ "By selecting C, I expect to receive payoff - 3."

"**D is obviously dominant, because 0 > - 3. Hence, I prefer D to C.**"

Obvious Dominance (instead of dominance)
Friedman and Shenker (1996), Friedman (2002), Li (2017)

Obviously dominated strategy: A player dislikes a strategy even if he is **optimistic**.
Obviously dominant strategy: A player prefers a strategy even if he is **pessimistic**.

Definition 2: A strategy $a_i \in A_i$ for player i is said to be ***obviously dominated*** in normal form (imperfect information) game $G = (N, A, u)$ if there exists $\hat{a}_i \in A_i$ such that he or she dislikes a_i even if he is optimistic, i.e.,

$$\max_{\hat{a}_{-i} \in A_{-i}} u_i(a_i, \hat{a}_{-i}) < \min_{\hat{a}_{-i} \in A_{-i}} u_i(\hat{a}_i, \hat{a}_{-i}).$$

A strategy $a_i \in A_i$ for player i is said to be ***obviously dominant*** in G if he likes a_i even if he is pessimistic, i.e.,

$$\min_{\hat{a}_{-i} \in A_{-i}} u_i(a_i, \hat{a}_{-i}) > \max_{\hat{a}_{-i} \in A_{-i}} u_i(\hat{a}_i, \hat{a}_{-i}).$$

We have various **anomalies** (probably) caused by failure of hypothetical thinking:

Winner's Curse	Crawford and Levin (2009)
Overbidding	Kagel, Harstad, and Levin (1987)
Non-pivotal Voting	Esponda and Vesta (2014)
Ellsberg Paradox	
Allais Paradox	
Sure-Thing Principle	Esponda and Vesta (2016)

Difficulty of hypothetical thinking is a growing concern in economics and psychology.

This study shows:

Frame design

motivates players to practice hypothetical thinking.

What is "frame" in this study ?

Cognitive procedure synchronized across players

defined as

Extensive (multi-stage) game form with imperfect information

		player 2 C	player 2 D
Player 1	**C**	**1 - 3**	**- 1 0**
	D	**2 - 3**	**0 - 2**

Prisoners' dilemma has three different frames:

Frame 0 (degenerate): Both players simultaneously select strategies.

Frame 1: Player 1 is first mover, but Imperfect Information

Frame 2: Player 2 is first mover, but Imperfect Information

Cf. Perfect Information (Physical rule (normal form game) is different)

By weakening obvious dominance, we introduce

Quasi-Obvious Dominance

Second mover correctly perceives
first mover has already selected a strategy
(even if he or she cannot observe which strategy selected)

Hence, we assume
second mover can practice hypothetical thinking, while
first mover remains a strategy-contingent thinker, failing hypothetical thinking.

Frame 2 (player 1 is second) is a good design

		player 2 C	player 2 D
Player 1	C	1 - 3	- 1 0
	D	2 - 3	0 - 2

Second mover (player 1) can practice hypothetical thinking, selecting dominant strategy D. First mover (player 2) remains a strategy-contingent thinker, but he selects D, because D is obviously dominant.

⇒ D is **quasi-obviously dominant** for both players.

Frame 1 (player 1 is first) is a bad design

		player 2 C	player 2 D
Player 1	C	1 - 3	- 1 0
	D	2 - 3	0 - 2

First mover (player 1) remains a strategy-contingent thinker. He does not select D, because D is not obviously dominant.

Player 2 selects D, because D is obviously dominant.

The recipe for good frame design is
"put problematic players (player 1, in this example) on later steps"

Example: Auction

Ascending Auction (AA): Open-bid. popular since long ago
Second-Price Auction (SPA): Sealed-bid. not popular historically.

Experimental subjects play sincere bidding in AA, while they **overbid** in SPA.

Li (2017): SPA→AA implies: **"Change physical rule from imperfect information to perfect information".**

In AA, a bidder can observe whether others quitted before. Hence, he doesn’t need hypothetical thinking, making AA easier to play than SPA.

This study considers:

Ascending Proxy Auction (APA): **Sealed-bid.**
Same normal form game as SPA.
But, now popular in net auction sites.

SPA→APA implies "Frame Design"

APA frame regards proxy bid p_i as action sequence $(0,....,1,1,....)$, where
0 implies "stay", while 1 implies "quit"

$t-th$ **component (auctioneer's price t) is** **0 (stay) if $t < p_i$**
1 (quit) if $t \geq p_i$

Quasi-obvious dominance assumes:

When a bidder determines 0 or 1 for t-th component, he perceives that the others have determined for every earlier component.

Hence, he can correctly understand that stay 0 is better than quit 1, if and only if auctioneer's price t is lower than his valuation.

In SPA (degenerate frame), a bidder overbids:
when overbidding, he optimistically expects the others to make low bids.

The more general recipe for good frame design is
"put problematic strategies (high bids, in this example) on later steps"

2. Quasi-Obvious Dominance (General)

A Frame is defined as extensive (T-step) game form with imperfect information

$$\Gamma = (T, (A_{i,t}, \tilde{A}_{i,t}(\cdot))_{t \in T}, \delta_i)_{i \in N})$$

At each step $t \in \{1, ..., T\}$, each player i selects action $a_{i,t} \in \tilde{A}_{i,t}(a_i^{t-1}) \subset A_{i,t}$.

A complete action sequence $a_i^T = (a_{i,1}, ..., a_{i,T})$ shapes a single strategy $a_i = \delta_i(a_i^T) \in A_i$.

Quasi-obviously dominated strategy: A player dislikes a strategy even if he is **optimistic about later-step determinations**.
Quasi-obviously dominant strategy: A player prefers a strategy even if he is **pessimistic about later-step determinations**.

Definition 3: A strategy $a_i \in A_i$ for player i is said to be ***quasi-obviously dominated*** *in a game with frame* (G,Γ) if there exist $t \in \{1,...,T\}$ and $\hat{a}_i \in A_i$ such that

$$\hat{a}_i \in A_i(a_i^{t-1}),\ \hat{a}_{i,t} \neq a_{i,t},$$

and

$$\max_{\hat{a}_{-i} \in \underset{j\neq i}{\times} A_j(a_j^{t-1})} u_i(a_i, \hat{a}_{-i}) < \min_{\hat{a}_{-i} \in \underset{j\neq i}{\times} A_j(a_j^{t-1})} u_i(\hat{a}_i, \hat{a}_{-i}) \text{ for all } a_{-i}^{t-1} \in A_{-i}^{t-1},$$

where we denote $A_i(a_i^t) \equiv \{\hat{a}_i \in A_i \mid \hat{a}_i^t = a_i^t\}$. It is said to be ***quasi-obviously dominant*** *in* (G,Γ) if for every $t \in \{1,...,T\}$ and $\hat{a}_i \in A_i$, whenever

$$\hat{a}_i \in A_i(a_i^{t-1}) \text{ and } \hat{a}_{i,t} \neq a_{i,t},$$

then

(2) $$\min_{\hat{a}_{-i} \in \underset{j\neq i}{\times} A_j(a_j^{t-1})} u_i(a_i, \hat{a}_{-i}) > \max_{\hat{a}_{-i} \in \underset{j\neq i}{\times} A_j(a_j^{t-1})} u_i(\hat{a}_i, \hat{a}_{-i}) \text{ for all } a_{-i}^{t-1} \in A_{-i}^{t-1}.$$

Specification: Strategy-Order Frame $\Gamma^{\rho} = (T, \bar{a}, \rho)$

Fix $\bar{a} = (\bar{a}_1, \ldots, \bar{a}_n) \in A$ as default profile. A **strategy order** is defined as

$$\rho : \bigcup_{i \in N} A_i \setminus \{\bar{a}_i\} \to \{1, \ldots, \sum_{i \in N} |A_i| - n\}.$$

At each step $t = \rho(a_i)$, the corresponding (single) player i decides whether to select $a_i = \rho^{-1}(t)$ or not.

Theorem 1: *There exists a frame Γ such that a strategy profile a^* is quasi-obviously dominant in (G, Γ) if and only if there exists a strategy order ρ such that a^* is quasi-obviously dominant in (G, Γ^{ρ}).*

To make a^* quasi-obviously dominant, we design a strategy order ρ that **puts problematic (not obviously dominant) strategies on later steps.**

By replacing strict inequalities with weak inequalities, we define

Weak Quasi-Obvious Dominance

Theorem 2 (Parallel to Theorem 1): *There exists a frame such that* a^* *is weakly quasi-obviously dominant if and only if there exists a strategy order* ρ *such that it is weakly quasi-obviously dominant in* (G, Γ^{ρ}).

Ex. Ascending Proxy Auction

3. Iterative Quasi-Obvious Dominance

Bounded rationality has various aspects:

Hypothetical Thinking
Higher-Order Reasoning
Computational Complexity
Social Preferences

This section assumes:

a player is bounded-rational in hypothetical thinking, but he or she is rational in higher-order reasoning.

We define
Iterative Quasi-Obvious Dominance (IQOD)
by replacing "dominance" in Iterative Dominance (ID) with "quasi-obvious dominance"

Example: Prisoners' Dilemma (Symmetric Case)

		player 2: C	player 2: D
Player 1	C	1 1	-1 2
	D	2 -1	0 0

D is a dominant strategy for both players.
D is not obviously dominant for both players.
D is not quasi-obviously dominant for first mover, while it is for second mover.

However, **irrespective of who is first mover,** (D, D) is the unique iteratively quasi-obviously undominated strategy profile.

Example:

		player 2 C	D
Player 1	A	1 1	1 2
	B	2 -1	0 0

D is a dominant strategy for player 2 but not obviously dominant.
D is not a dominant strategy for player 1 but is the unique iteratively undominated strategy.

With the frame that lets **player 1 first mover**, (D, D) is the unique iteratively quasi-obviously undominated strategy profile.

However, with the frame that lets player 2 first mover, (D, D) is **not** the unique iteratively quasi-obviously undominated strategy profile.

[Solvable in ID] $\Leftrightarrow$ [Solvable in IQOD]

Theorem 3: *There exists a strategy order* ρ *such that a strategy profile* a^* *is the unique iteratively quasi-obviously undominated strategy profile in* (G, Γ^ρ) *if and only if it is the unique iteratively undominated strategy profile in* G.

In contrast to QOD,
the recipe for good frame design w.r.t. IQOD is
"put strategies eliminated earlier in ID on later steps"

In other words,
"put problematic strategies on earlier steps"
(cf. Theorems 1 and 2)

4. Further Results (Omitted)

4.1. Detail-Free Frame Design:

Fix an arbitrary frame, and check the range of games that are solvable in IQOD.
A single frame solves the difficulty of hypothetical thinking in wider range of games.

Application: Implementation Theory (Abreu-Matsushima Mechanisms)
Possibility Theorem in ID $\Rightarrow$ **Possibility Theorem in IQOD**
Detail-Free Frame Design: fine only last deviants

4.2. Incomplete Information:

Regarding Bayesian game as agent-normal form game, we directly apply this study to Bayesian environment.
Computational Complexity: the set of all players $\boldsymbol{N} = \{\boldsymbol{1,...,n}\}$ is replaced with the set of all type-dependent agents $\underset{i \in N}{\times} \Omega_i$.
We investigate "detail-free" frame design defined, not on $\underset{i \in N}{\times} \Omega_i$, but on N.

5. Experiments (Work in Process)

This study was a theory with introspective routes.
We need experimental evidences.

Frame Design ≈ Instruction (Education) Design

Prisoners' Dilemma: How is the impact of good frame design ?
Compare it with the impact of perfect information.

Ascending Proxy Auction Compare APA, SPA, and AA.
Order of Experiments matters:
SPA → AA → APA → SPA

We have various aspects of bounded rationality:

Hypothetical Thinking
Computational Complexity,
Higher-Order Reasoning
Social Preference
Which actually matters ?