Gradual Bargaining in Decentralized Asset Markets

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Background

Models of decentralized asset markets

- to explain asset/market liquidity

Two approaches

- New Monetarist approach: Assets as media of exchange
- Finance approach: Illiquid assets traded over the counter

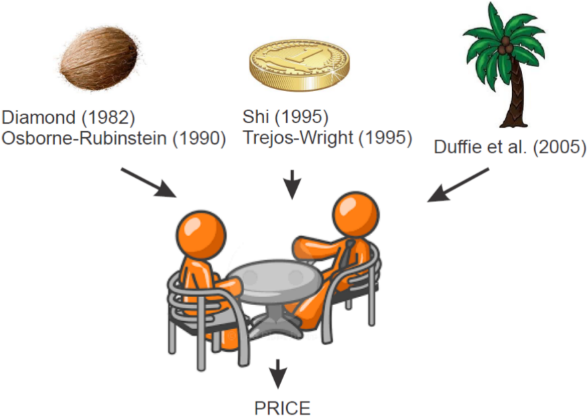
Based on search paradigm with two core components:

1. search frictions and pairwise meetings
2. bargaining

This paper is about **bargaining**

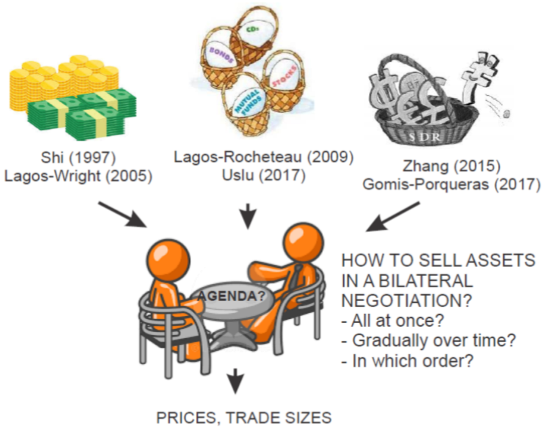
Background: 2nd generation of models

Restricted asset holdings: $a \in \{0, 1\}$



Background: 3rd generation of models

Portfolio of divisible assets: $\mathbf{a} \in \mathbf{R}_+^J$



Background: How is bargaining handled?

Bargaining with $\mathbf{a} \in \mathbf{R}_+^J$ like with $a \in \{0, 1\}$

- Generalized Nash or Kalai solution
- Agents negotiate their portfolio all at once

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Questions

- Is this agenda (all-at-once bargaining) restrictive?
- Is it the agenda that agents/society would choose?
- Does the agenda matter for allocations and prices?

Insights

1. **Bargaining theory**
   Extensive-form bargaining games, endogenous agenda
2. **Asset prices**
   Negotiability premia, distributions of asset returns and velocities
3. **Monetary theory**
   rate-of-return dominance, exchange rate determination, OMOs

Time, goods, agents

Time: $t = 0, 1, 2..., \infty$

- Each period has two stages:
  1. Decentralized market (DM) for goods and assets, with pairwise meetings and bargaining
  2. Centralized market (CM) for goods and assets

- DM good is perishable, and CM good taken as numeraire

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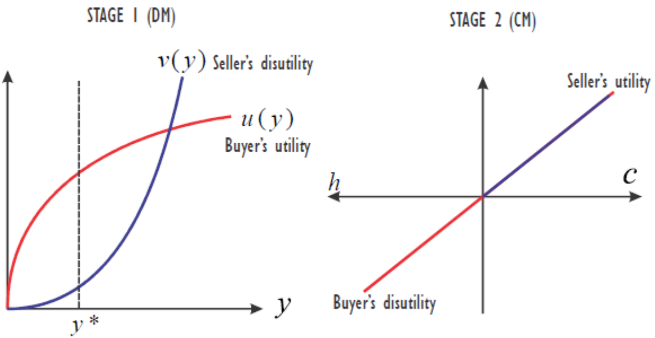
Agents: divided into two types, unit measure of each

1. Consumers: consume DM good and produce numeraire
2. Producers: produce DM good and consume numeraire

In DM, $\alpha \in (0, 1]$ pairwise meetings b/w consumers and producers



- Discount factor $\beta = 1/(1+\rho)$
- Efficient DM output: $u'(y^*) = v'(y^*)$

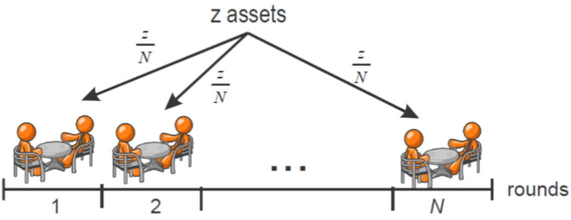
Assets

- Lucas trees: pay off $d \geq 0$ in the CM
  - Fiat money: $d = 0$
- Exogenous supply: $A_{t+1} = (1 + \pi)A_t$
  - if $d > 0$, $\pi = 0$
- Asset price in terms of the numeraire: ϕ_t
- No private IOUs: no record-keeping and no commitment

Bargaining game

Game has N rounds

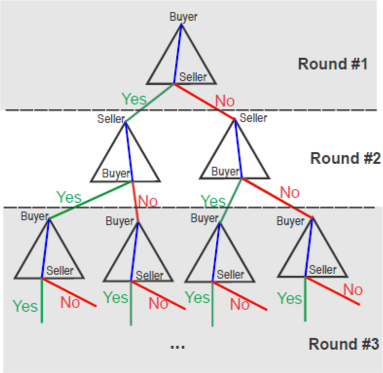
- Asset owner has z units of assets (in terms of numeraire)
- Divided into N equal sizes: z/N
- In each round, agents negotiate sale of z/N assets for some output y



Alternative ultimatum offer game

N two-stage rounds, identity of the proposer alternates

- Stage 1: One player makes an offer
- Stage 2: Other player accepts/rejects



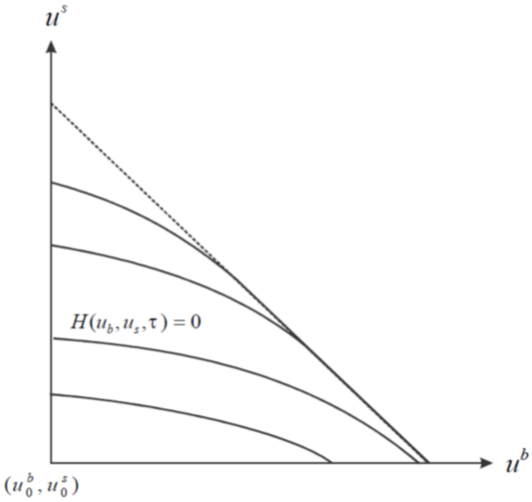
Intermediate Pareto frontier

- Denote $\tau \equiv nz/N$ where $n = 1, ..., N$
- For each τ, feasibility constraint on asset sales: $p(\tau) \leq \tau$
- For each τ, a Pareto frontier:

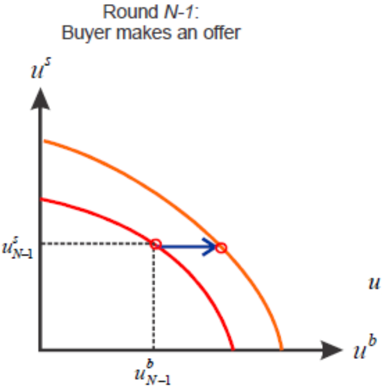
$$\max u_b(\tau) \quad \text{s.t.} \quad u_s(\tau) \geq u_s \text{ and } p(\tau) \leq \tau$$

$$\Downarrow$$

$$H(\overset{-}{u^b}, \overset{-}{u^s}, \overset{+}{\tau}) = 0.$$


u^s
$H(u_b, u_s, \tau) = 0$
u^b
(u_0^b, u_0^s)

Subgame Perfect Equilibrium



Solution to alternating ultimatum offer bargaining game

Take the limit as N approaches ∞

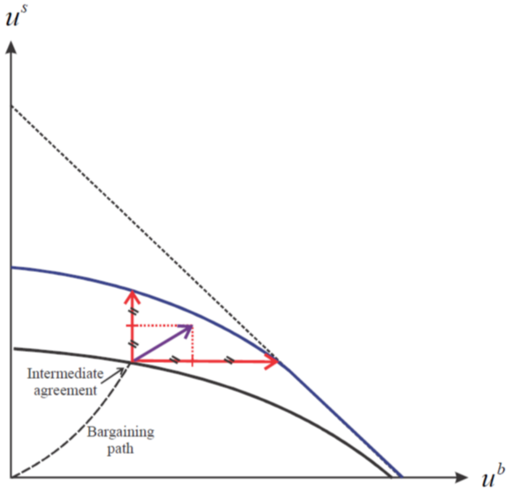
- SPE exists with $\left\{u^b(\tau), u^s(\tau)\right\}$ converging to solution to:

$$u^{\chi\prime}(\tau) = -\frac{1}{2}\frac{\overbrace{\partial H(u^b, u^s, \tau)/\partial\tau}^{\text{shift of Pareto frontier over time}}}{\underbrace{\partial H(u^b, u^s, \tau)/\partial u^\chi}_{\text{expressed in utils of player }\chi}}, \quad \chi \in \{b, s\}$$

Robustness: coincides with axiomatic gradual bargaining solution by O'Neill et al. (2004)

- Pareto optimality, scale invariance, symmetry, directional continuity, time consistency

Gradual bargaining path



Solution in terms of allocations

Asset price (in terms of DM goods) solves:

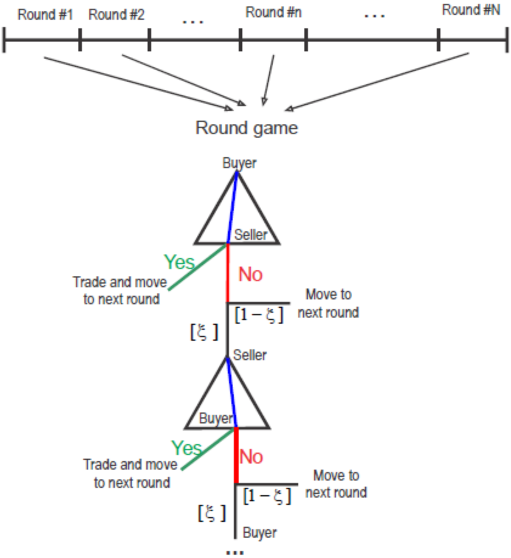
$$y'(\tau) = \frac{1}{2}\left(\overbrace{\frac{1}{\upsilon'(y)}}^{\text{ask price}} + \overbrace{\frac{1}{u'(y)}}^{\text{bid price}}\right) \quad \text{for all } y < y^*$$

Suppose $\upsilon'(y) = 1$. Asset price is:

$$\frac{1}{2}\left(1 + \frac{1}{u'(y)}\right).$$

- Price increases with the size of the trade

Alternative Extensive Game



Bundled vs gradual sales

- Intermediate output levels, $\{y_n\}_{n=1}^{N}$, solve:

$$\int_{y_{n-1}}^{y_n} \overbrace{\frac{\upsilon'(y_n)}{u'(y_n)+\upsilon'(y_n)}}^{\text{producer's share}} u'(x) + \overbrace{\frac{u'(y_n)}{u'(y_n)+\upsilon'(y_n)}}^{\text{consumer's share}} \upsilon'(x)dx = \frac{z}{N}$$

Proposition: *Consumers (asset owners) prefer $N = +\infty$ to any $N < +\infty$.*

Asset negotiability

Agenda indexed by time, τ

- An implicit mapping between τ and z

New asset characteristic: Negotiability

- $\delta > 0$ units of assets can be sold per unit of time
- What is negotiability in practice:
  - time to authenticate assets
  - time to value complex assets
  - time to execute trade and transfer ownership (e.g., blockchain technologies)

Making time relevant

Random time to negotiate asset sales: $\bar{\tau} \sim \text{Exp}(\lambda)$

- negotiation breakdown, proxy for discounting

Formally:

$$\overbrace{p(y)}^{\text{Asset sales}} \leq \overbrace{\delta}^{\text{Negotiability}} \times \overbrace{\bar{\tau}}^{\text{Time to negotiate}}$$

Pricing of Lucas trees

Interest rate spread (liquid vs non-liquid):

$$\overbrace{s}^{\text{spread}} = \overbrace{\alpha}^{\text{search}} \times \overbrace{\theta}^{\text{bargaining}} \times \overbrace{e^{-\frac{\lambda}{\delta}p(y)}}^{\text{negotiability}} \times \overbrace{\ell(y)}^{\text{liquidity needs}} .$$

where $\ell(y) \equiv u'(y)/\upsilon'(y) - 1$

- $e^{-\frac{\lambda}{\delta}p(y)}$ akin to a pledgeability coefficient
  - endogenous with $\neq$ comparative statics
- s decreases with Ad but increases with δ and $1/\lambda$

Endogenous negotiability

Consumers choose δ when a match is formed but before $\bar{\tau}$ is realized

- Cost to enhance negotiability: $\psi(\delta)$

Proposition

1. *If A is not too large, an increase in A reduces s, but raises δ.*

2. *If A is not too large, asset negotiability is too low for all bargaining powers.*
    - *a pecuniary externality*

Multiple assets

J one-period lived trees, one unit of each pays off one unit of numeraire

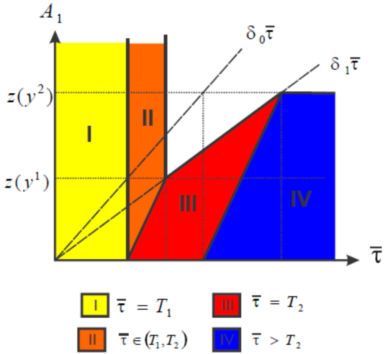
- Fiat money: $j = 0$; asset j has fixed supply A_j, $j = 1, ..., J$
- Negotiability of asset j is δ_j with $\delta_0 \geq \delta_1 \geq ... \geq \delta_J$
  - Pecking order: sell assets with high negotiability first

Asset prices:

$$\overbrace{s_j}^{\text{spread}} = \overbrace{\alpha\theta}^{\text{search\&bargaining}} \overbrace{\lambda \sum_{k=j+1}^{J} \int_{T_k}^{T_{k+1}} \frac{(\delta_j - \delta_k)}{\delta_j} e^{-\lambda\tau} \ell[y(\tau)] d\tau}^{\text{negotiability premium}}$$

$$+\alpha\theta \overbrace{e^{-\lambda T_{J+1}} \ell[y(T_{J+1})]}^{\text{liquidity premium}}$$

OMOs: negotiability vs liquidity



Regime 1: CIA **Regime 2**: Marginally illiquid bonds
Regime 3: Effective OMOS **Regime 4**: Rate of return equality

- In Regime 3, increase in A_1 (bond supply) leads to reduction in output

Multiple fiat monies

Multiple cryptocurrencies: Bitcoins, Litecoin, Ethereum

- Confirmation times vary across currencies, modeled as different δ
- 2 currencies: 0 and 1, with inflation rates $\pi_0 > \pi_1$ but with $\delta_0 > \delta_1$

Dual currency equilibrium

- For intermediate $\bar{\tau}$'s a unique eq. exists with both currencies valued
- $\partial y/\partial \pi_0 < 0$ and $\partial y/\partial \pi_1 > 0$
- Currency 0 appreciates vis-a-vis currency 1 as α or θ increases or as $\bar{\tau}$ decreases
    - because agents put more weight on negotiability

Conclusion

New approach to bargaining over portfolios in decentralized asset markets

- Axiomatic and strategic foundations
- Tractable
- Encompasses Nash and Kalai solutions for specific agendas

Insights

- Normative: gradualism desirable individually and socially
- Positive: negotiability premia, distribution of asset returns, determinacy of exchange rate, OMOs