Strategically Simple Mechanisms

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Mechanisms in which it is easy to determine one's optimal choice

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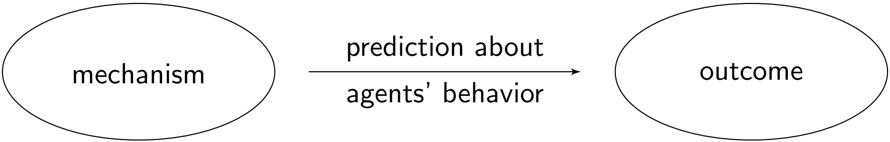
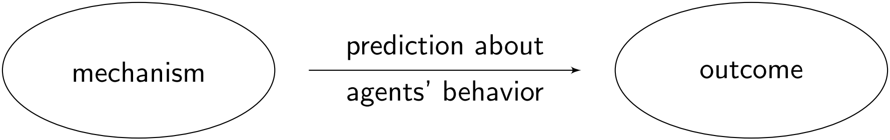


Mechanisms in which it is easy to determine one's optimal choice



- make it more likely that the designer's predictions are correct;

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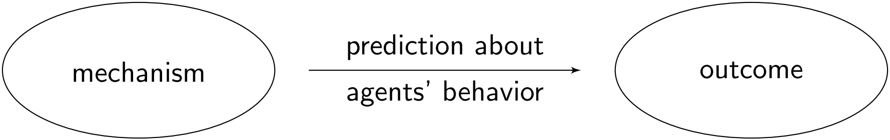

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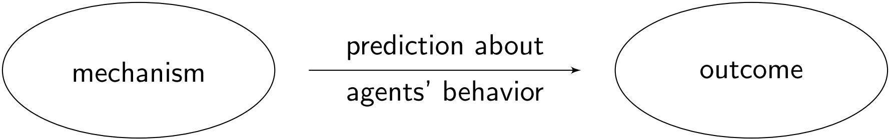

- make it more likely that the designer's predictions are correct;
- make it easier to persuade people to participate;
- don't discriminate on the basis of cognitive ability.

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Now imagine you are writing this paper...

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How would you model strategic simplicity?

In this paper, we

- propose a definition of strategic simplicity,
- and characterize all strategically simple mechanisms.

Possible Definition of Strategic Simplicity:

Strategic Simplicity = Dominant Strategy Mechanisms

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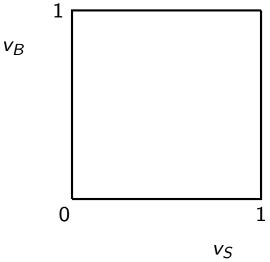
- Dominant strategy mechanisms - posted price mechanisms



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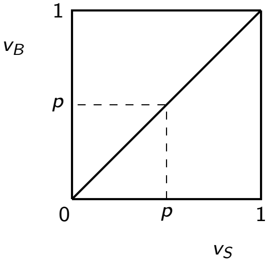

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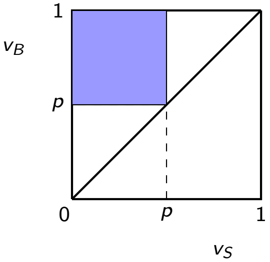

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- The seller may:
  - refuse trade;
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- If the seller has proposed trade at p', the buyer may:
  - reject trade;
  - accept trade at p'.

In this paper:

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Relevant:

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Example: Bilateral Trade (Myerson and Satterthwaite (1983)):

- Dominant strategy mechanisms:
  - posted price mechanisms.
- A strategically simple mechanism:
  - ultimatum bargaining (possibly with a price cap).
- Not strategically simple mechanism:
  - $\frac{1}{2}$-double auction.

Outline

- Definition
- Examples
- Characterization
- Related Literature
- Further Research

Definition

n agents: $i \in I = \{1, 2, \ldots, n\}$.

A finite set A of outcomes.

A mechanism:

- finite strategy sets S_i for each agent i,
- a function $g : S_1 \times S_2 \times \ldots \times S_n \to A$.

We are going to define the following:

- Utility function
- Utility belief
- Strategic belief
- Compatible strategic belief
- Best response
- Strategically simple mechanism

$u_i : A \to \mathbb{R}$: a utility function of agent i.

$\mathcal{U}$: set of all utility functions.

$\mathbf{U}_i \subseteq \mathcal{U}$: set of all admissible utility functions of agent i.

$$\mathbf{U} \equiv \prod_{i \in I} \mathbf{U}_i \quad \mathbf{U}_{-i} \equiv \prod_{j \neq i} \mathbf{U}_i.$$

μ_i: a utility belief of agent i; a probability measure on $\mathbf{U}_{-i}$.

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$\hat{\mu}_i$: a strategic belief of agent i (a probability measure on S_{-i}).

$UD_i(u_i)$: set of strategies of i that are not weakly dominated given u_i.

Definition

A strategic belief $\hat{\mu}_i$ on S_{-i} is compatible with a utility belief μ_i if there is a probability measure ν_i on

$$\prod_{j \neq i} \{(u_j, s_j) \in \mathcal{U}_j \times S_j | s_j \in UD_j(u_j)\}$$

that has marginal μ_i on $\mathcal{U}_{-i}$ and marginal $\hat{\mu}_i$ on S_{-i}.

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$$UD_j(u_j) = \{L, M\}.$$

$$UD_j(u'_j) = \{M, R\}.$$

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$\hat{\mu}_i^3$	1	0	0	not compatible

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Notation: $\mathcal{M}_i(\mu_i)$: set of strategic beliefs of i compatible with utility belief μ_i.

$BR_i(u_i, \hat{\mu}_i)$: set of strategies of i that maximize expected utility if i has utility function u_i and strategic belief $\hat{\mu}_i$.

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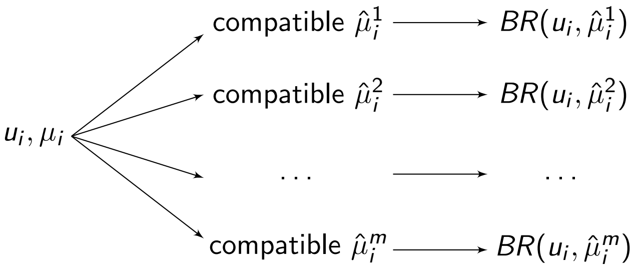

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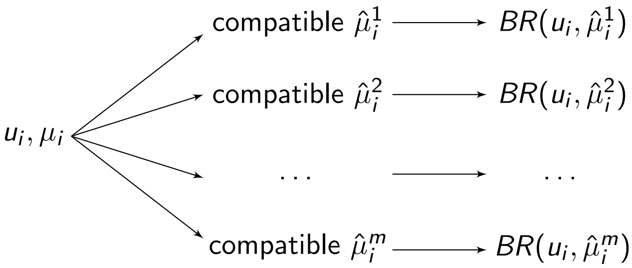

Definition

A mechanism is strategically simple if for all agents $i \in I$, utility functions $u_i \in \mathbf{U}_i$, and utility beliefs $\mu_i \in \mathbf{M}_i$:

$$\bigcap_{\hat{\mu}_i \in \mathcal{M}_i(\mu_i)} BR_i(u_i, \hat{\mu}_i) \neq \emptyset.$$

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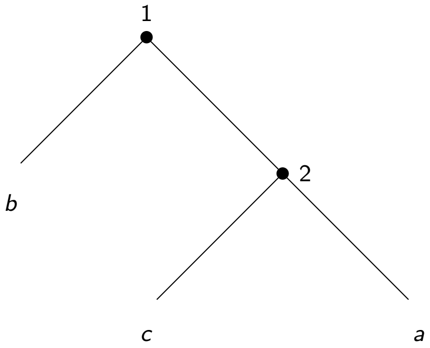
- Built In Robustness: higher order beliefs don't matter.
- Simplicity with Complete Robustness: large sets $\mathbf{U}_i$ and $\mathbf{M}_i$.
- Simplicity without Complete Robustness: small sets $\mathbf{U}_i$ and $\mathbf{M}_i$.

Examples

Strategically Simple



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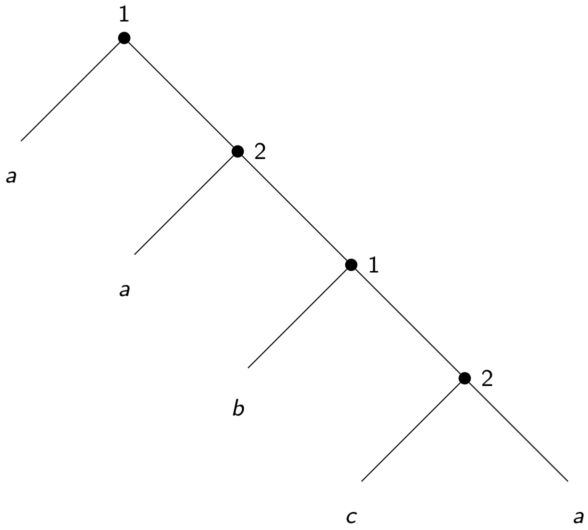

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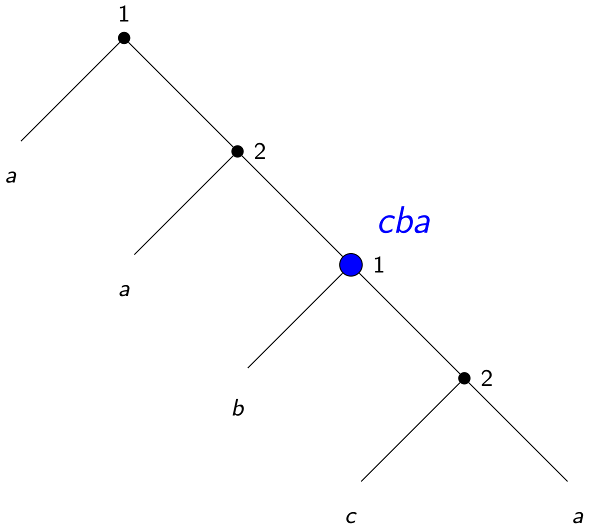

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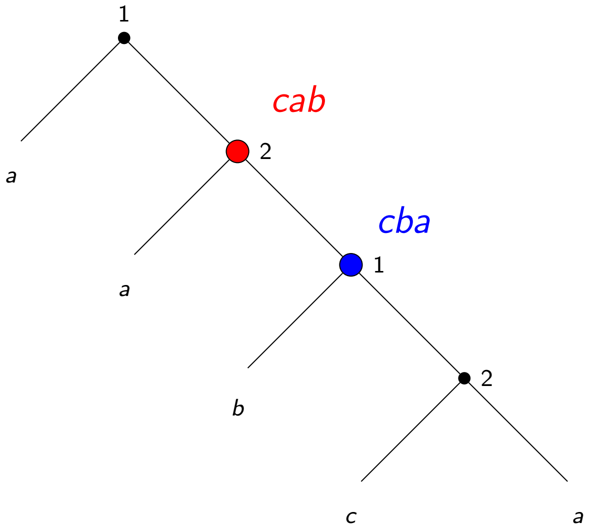

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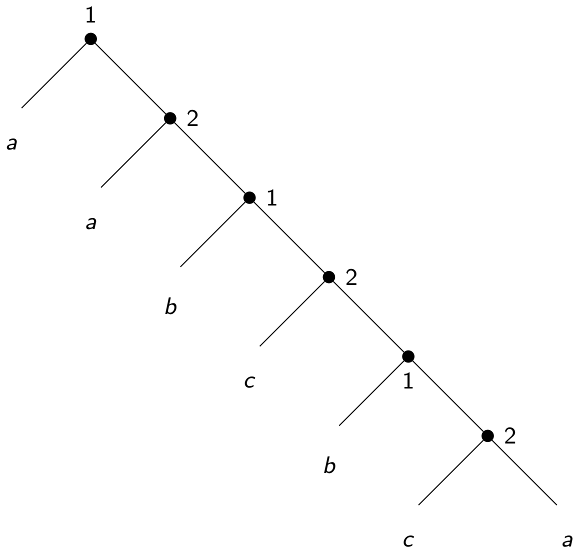

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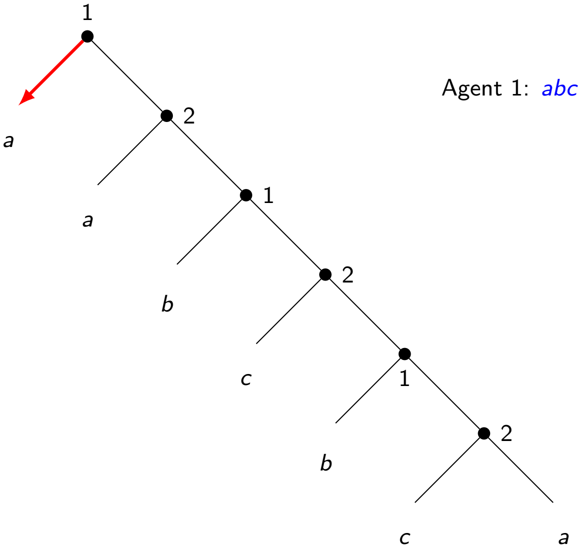

Agent 1: *abc*, *acb*

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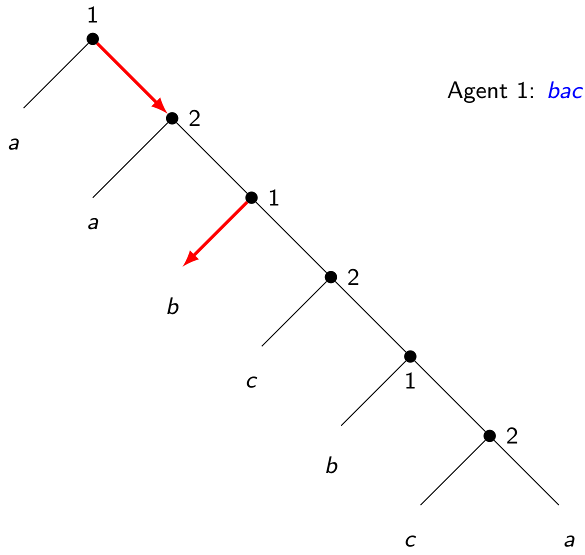

Agent 1: *bac*, *bca*

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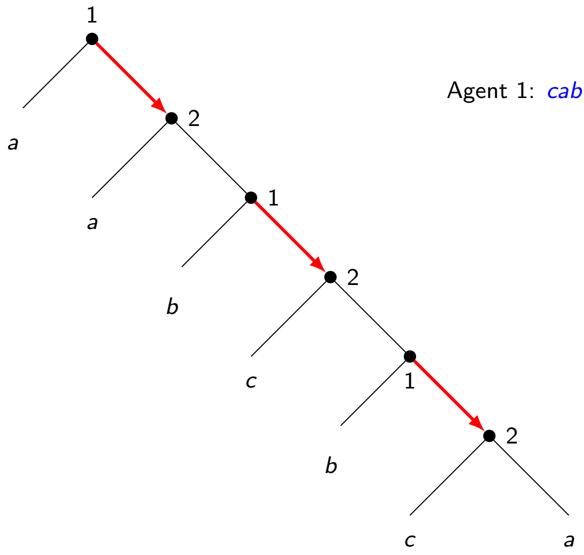

Agent 1: *cab*

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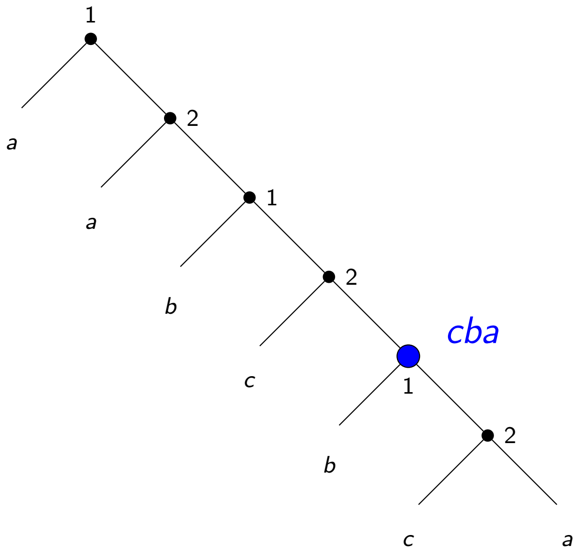

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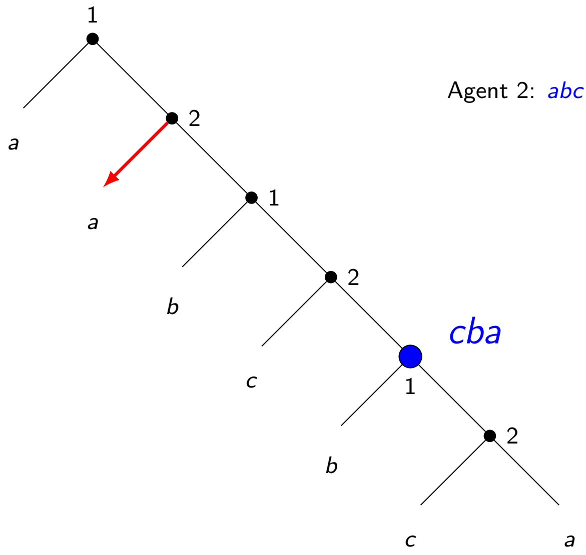

Agent 2: *abc*, *acb*

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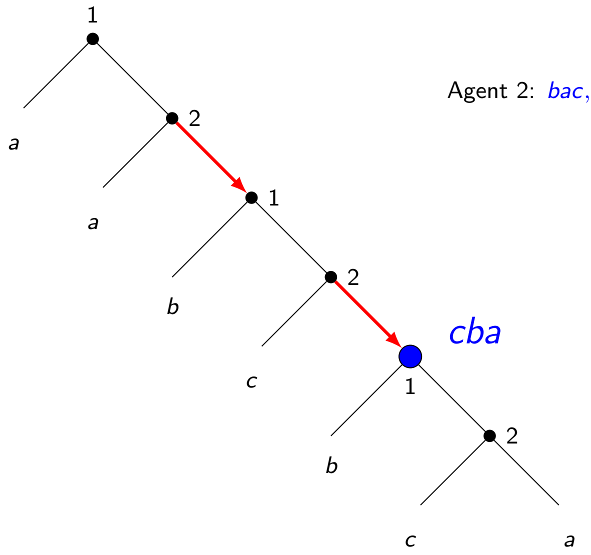

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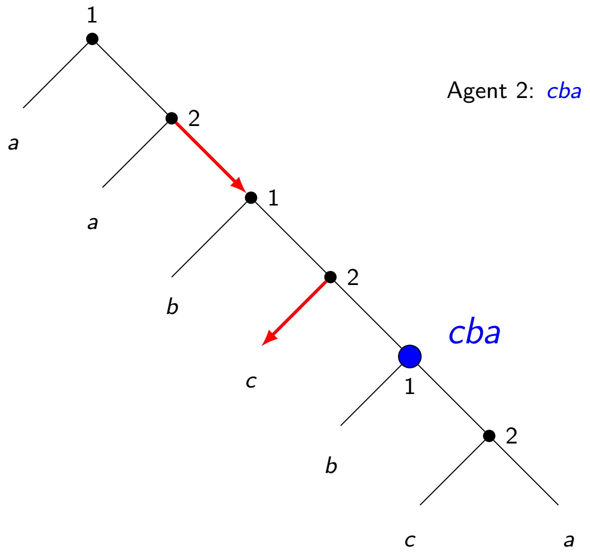

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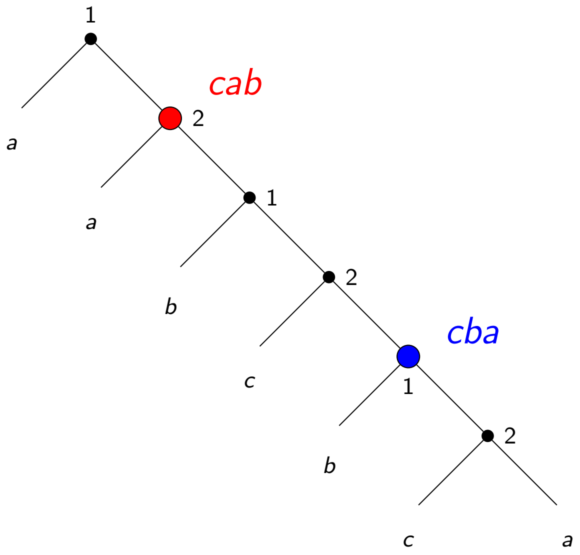

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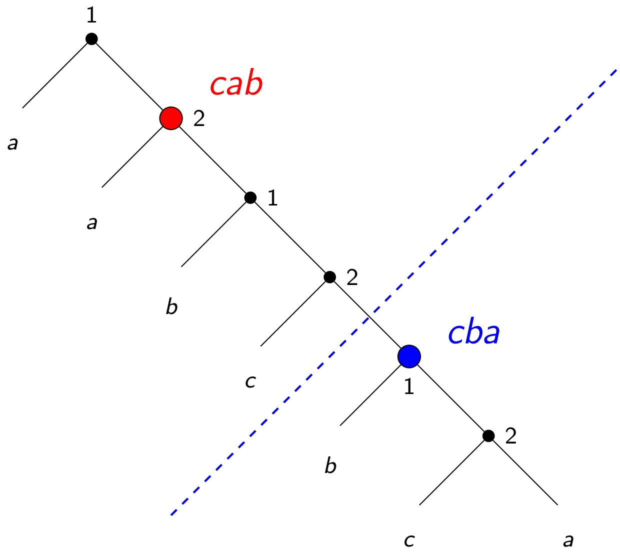

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$\mathcal{R}$: the set of all linear orders on A.

Definition

Let R_i be a linear order on A. A strategy $s_i \in S_i$ of agent i is weakly dominated given R_i if there is another strategy $\hat{s}_i \in S_i$ such that for all $s_{-i} \in S_{-i}$

$$g(\hat{s}_i, s_{-i}) R_i g(s_i, s_{-i}) \text{ or } g(\hat{s}_i, s_{-i}) = g(s_i, s_{-i}).$$

and, for some $s_{-i} \in S_{-i}$

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$UD_i(R_i) \subseteq S_i$: set of all strategies of agent i that are not weakly dominated given R_i.

Theorem

Suppose for every agent i,

- *there is a set $\mathcal{R}_i \subseteq \mathcal{R}$ such that $\mathbf{U}_i = \bigcup_{R_i \in \mathcal{R}_i} \mathcal{U}(R_i)$,*

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Then a mechanism is strategically simple if and only if:

for every $R \in \times_{i \in I} \mathcal{R}_i$ there is a local dictator $i^* \in I$,

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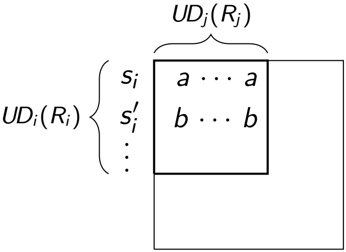
i.e. for every strategy $s_{i^*} \in UD_{i^*}(R_{i^*})$ there is an alternative $a \in A$ such that:

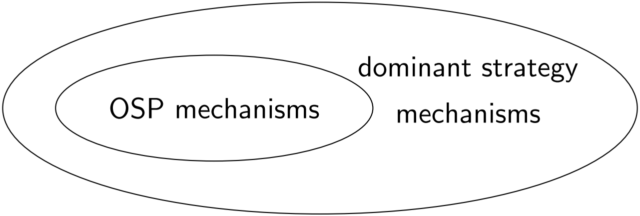
$$g(s_{i^*}, s_{-i^*}) = a \text{ for all } s_{-i^*} \in UD_{-i^*}(R_{-i^*}).$$



Related Literature

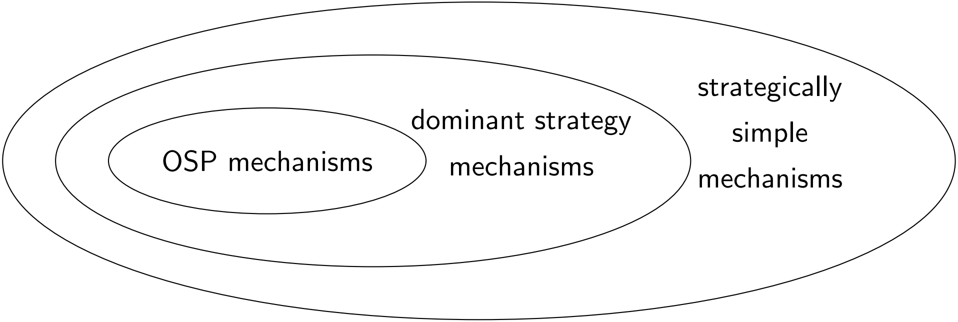
Li (2017) studies obviously strategy-proof (OSP) mechanisms.

- A subset of the set of all dominant strategy mechanisms.
- What is obvious?
    - Agents immediately recognize optimal strategies.



We study strategically simple mechanisms.

- A superset of the set of dominant strategy mechanisms.
- What is strategically simple?
  - Agents can be offered a convincing explanation of optimal strategy choices.



For environments with quasilinear preferences.

Robust mechanism design:

- The design has no information about agents' beliefs.
- Chen and Li (2017)
- Yamashita and Zhu (2017)

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If attention is restricted to a narrow subset of beliefs:

- Cremer and Riordan (1985)

Further Research

- Further characterizations of strategically simple mechanisms.

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- Second order belief? Finite order belief?
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- Testing strategic simplicity; Borgers, Calford, and Li (WIP).