Multi-period Matching with Commitment

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Outline

1. **Introduction**
   - Commitment in Multi-period Matching Market
   - Related Literature: Dynamic Matching Market
   - Preview of the Results
2. **Modeling Different Types of Commitment**
   - Multi-period Matching with Full Commitment
   - Multi-period Matching with Two-sided Commitment
3. **Conclusion**

Matching theory has been widely applied to two-sided markets without money:

- **One-to-one marriage market** (Gale and Shapley, 1962):
- Kidney Exchange, Housing Market, College admissions...

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Differences Between Static and Dynamic Matching Market

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1. Preferences are defined over complete partnership plans, instead of individuals;
2. Agents may enter or leave the market as time passes by;
3. Agents' ability to terminate an existing relationship is somehow restricted by commitment, an exogenous nature of the market.

Existence of Commitment in Multi-period Matching

Many multi-period markets involve some level of commitment

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"I (___), take you, (___), to be my lawfully wedded (husband /wife), to have and to hold, from this day forward, for better, for worse, for richer, for poorer, in sickness and in health, until death do us part."

— A Catholic Wedding Vow

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Related Literature: Modeling Dynamic Matching Market

1. Full Commitment & Arrivals and (or) Departures

- Tradeoff: Waiting for a thicker market v.s Avoiding waiting cost;
- Ünver (2010), Baccara, Lee and Yariv (2015), Thakral (2015), Akbarpour, Li and Oveis Gharan (2014), Doval (2016).

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   - Pereyra (2013): Teachers are entitled with the right to continue in the school to which they were assigned in the previous period;
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4. No Commitment & Fixed Set of Agents
   - Kadam and Kotowski (2015a, b, KK2015a, b for short).

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What we do

This paper:

- Models different dynamic matching markets with a given type of commitment:
  - Full, Two-sided, One-sided, and no commitment;
- Studies how commitment functions in different dynamic matching markets and defines dynamic stability with commitment;
- Provides sufficient conditions under which a dynamically stable matching exists in matching markets with commitment.

Summary of Main Results

Table: Summary of Main Results

Commitment Type	Full	Two-sided	One-sided	No (KK2015a)
Feature	Once-for-all	Mutual Consent	Asymmetric	No Constraint
Stability	DSFC	DSTC	DSOC	DSNC
Algorithm (Preferences)	P-DAFC (General)	T-DA (Rankability+SI)	P-DAA (Rankability+SI)	P-DAA (SIC+SA/RDS)
Efficiency	$\surd$	Rankability (IR matchings)	Rankability (IR matchings)	Inertia (consistent μ)
Strategy-Proofness	$\times$	$\times$	$\times$	$\times$
SP for one side	$\surd$	Rankability (Restricted)	SIC,WREP (Restricted)	SIC (Restricted)
Entries and Exits	$\surd$	Special case	Special case	$\times$

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Model Setup:

- Two-period marriage market $(M_1 \cup M_2, W_1 \cup W_2, R)$;
- Each agent has a strict preference over all possible partnership plans or agents, based on his /her appearance on the market;
- $\mu = (\mu_1, \mu_2)$ is a **multi-period matching** if $\mu_t : M_t \cup W_t \to M_t \cup W_t$ is a spot matching on the static market for period t;

Full commitment: When the matching is reasonably once-for-long or once-for-all...

- Consistent spot preferences over time (e.g. kidney exchange (Ünver, 2010));
- Relatively short economic scope of interest (e.g. child-adoption (Baccara, Lee and Yariv, 2015) and public housing (Thakral, 2015)).

Dynamic Stability with Commitment

In a 2-period matching market with a commitment requirement, a feasible matching is dynamically stable if for both periods it does not have any blocking agent or pair that respects the commitment requirement.

Def.1: Dynamic Stability with Full Commitment (DSFC)

*A matching μ in (M_1, M_2, W_1, W_2, R) is **dynamically stable with full commitment** if*

1. *μ satisfies full commitment, that is, if $\mu_1(x) \neq x$, then $\mu_1(x) = \mu_2(x)$, $\forall x \in M \cup W$;*
2. *μ is not blocked by any individual, that is, $\nexists x \in M \cup W$ such that $xx \succ_x \mu(x)$;*
3. *μ is not blocked by any pair of agents, that is, $\nexists(m, w)$ such that (i). $ww \succ_m \mu(m)$ and $mm \succ_w \mu(w)$ or*
   (ii). If $m \in M_1 \cap M_2, w \in W_1 \cap W_2$, $mw \succ_m \mu(m)$ and $wm \succ_w \mu(w)$;
   If $m \in M_2 \backslash M_1, w \in W_1 \cap W_2$, $w \succ_m \mu_2(m)$ and $wm \succ_w \mu(w)$;
   If $m \in M_1 \cap M_2, w \in W_2 \backslash W_1$, $mw \succ_m \mu(m)$ and $m \succ_w \mu_2(w)$;
   If $m \in M_2 \backslash M_1, w \in W_2 \backslash W_1$, $w \succ_m \mu_2(m)$ and $m \succ_w \mu_2(w)$;

- Natural extension of static stability under the restriction of once-for-all matching;

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- Natural extension of static stability under the restriction of once-for-all matching;
- Identical blocking rule no matter when agents enter the market.

Def.2: Extended Marriage Market

*Denote $M = M_1 \cup M_2$, $W = W_1 \cup W_2$, then $(M, W, \overline{R})$ is an **extended marriage market** of $(M_1 \cup M_2, W_1 \cup W_2, R)$ if*

- For $m \in M$, $\succ_m$ is defined over $(W^m)^2$;
- For $m \in M_1 \cap M_2$, $\succ_m = \succ_m^0$ over plans in $W_1^m \times W_2^m$;
- For $m \in M_1 \backslash M_2$, $mm \succ_m x_1 x_2$ if $x_2 \neq m$ and $y_1 m \succ_m y_2 m \iff y_1 \succ_m^0 y_2$ for $y_1, y_2 \in W_1^m$;
- For $m \in M_2 \backslash M_1$, $mm \succ_m x_1 x_2$ if $x_1 \neq m$ and $my_1 \succ_m my_2 \iff y_1 \succ_m^0 y_2$ for $y_1, y_2 \in W_2^m$;
- Symmetric for $w \in W$

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- For $m \in M_2 \backslash M_1$, $mm \succ_m x_1 x_2$ if $x_1 \neq m$ and $my_1 \succ_m my_2 \Longleftrightarrow y_1 \succ_m^0 y_2$ for $y_1, y_2 \in W_2^m$;
- Symmetric for $w \in W$

- **Claim:** The extended marriage market coincides with the original market in terms of individually rational matchings and DSFC matchings.

Theorem 1

The set of dynamically stable matchings with full commitment is nonempty for any extended marriage market $(M, W, \overline{R})$.

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Algorithm 1 (P-DAFC)

The man-proposing plan deferred acceptance algorithm with full commitment identifies a matching μ^* as follows:

1). For each $m \in M$, let $X_m^0 \equiv \{ww, mw, mm : w \in W\}$. Initially, no plans in X_m^0 have been rejected; 2). In round $\tau \geqslant 1$,

(a). Let $X_m^\tau \subset X_m^{\tau-1}$ be the set of plans that have not been rejected in the previous rounds and m propose the most preferred plan in X_m^τ. Proposing *mm* implies that agent *m* has been rejected by any acceptable plans involving a women.

(b). Let X_w^τ denote the set of plans proposed to w in round τ. If $ww \succ_w x_1x_2$ for all $x_1x_2 \in X_w^\tau$, then *w* rejects all the proposals. Otherwise, *w* keeps her most preferred plan in X_w^τ tentatively and rejects all others.

3). Repeat procedure 2) until no rejections occur. Define μ^* correspondingly.

Theorem 2 : Mutual Interests on the Same Side

Let $(M, W, \overline{R})$ be a two-period marriage market in which all preferences are strict, then the M-optimal (W-optimal) dynamically stable matching with full commitment exists, and coincides with the outcome of corresponding P-DAFC μ^M (μ^W).

- Intuition: no agent on the proposing side has any achievable plan rejected.

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Theorem 3 : Conflicting Interests on Opposite Sides

Let $(M, W, \overline{R})$ be a two-period marriage market in which all preferences are strict, and $\mu^1, \mu^2 \in S$ as two stable matchings with full commitment.
Then $\mu^1 \succ_M \mu^2 \Longleftrightarrow \mu^2 \succ_W \mu^1$.

- By $\succ_M$, all men are weakly better off and at least one man is strictly better off;

Theorem 4 : Strategic Issues

*(1). No stable matching mechanism with full commitment exists for which stating the true preferences is a (weakly) dominant strategy **for every agent**;*
*(2). Let $(M, W, \overline{R})$ be a two-period marriage market in which all preferences are strict, then the M-proposing P-DAFC makes it a (weakly) dominant strategy for each **man** to state his true preference. Symmetric for the W-proposing P-DAFC.*

- (1). The argument from Roth (1982) remains valid since static market is a special case of the multi-period market, where everyone enters in the last period;
- (2). One-sided strategy-proofness is enough on some markets where the preference (priority) of the other side is common knowledge or determined by a rational social planner.

Properties of Dynamic Stability with Full Commitment (3)

Theorem 5 : Efficiency

(1). Let $(M, W, \overline{R})$ be a two-period marriage market in which all preferences are strict, then every dynamically stable matching with full commitment is Pareto efficient within matchings with full commitment.

(2). Furthermore, there does not exist any individually rational matching μ s.t. $\mu \succ_m \mu^M$, $\forall m \in M$. Similarly, there does not exist any individually rational matching μ s.t. $\mu \succ_w \mu^W$, $\forall w \in W$.

Concrete Example

Example 1:

$M = \{m_1, m_2\}$, $W = \{w_1, w_2\}$ and the preferences:

$m_1 : w_2 w_1 \succ_{m_1} w_1 w_2 \succ_{m_1} w_2 w_2 \succ_{m_1} w_1 w_1$

$m_2 : w_1 w_2 \succ_{m_2} w_2 w_1 \succ_{m_2} w_1 w_1 \succ_{m_2} w_2 w_2$

$w_1 : m_2 m_1 \succ_{w_1} m_1 m_2 \succ_{w_1} m_1 m_1 \succ_{w_1} m_2 m_2$

$w_2 : m_1 m_2 \succ_{w_2} m_2 m_1 \succ_{w_2} m_2 m_2 \succ_{w_2} m_1 m_1$

- DSFC: $\mu^M(m_1) = w_2 w_2$, $\mu^M(m_2) = w_1 w_1$, and $\mu^W(m_1) = w_1 w_1$, $\mu^W(m_2) = w_2 w_2$;
- μ^M, μ^W are efficient among matchings with full commitment, but can be dominated by matchings with separations.

Outline

Model Setup:

- The same as the case with full commitment

Two-sided commitment: When only mutual-consent (no-fault) separations are permitted ...

- Hindu Marriage act since 1976;
- Mutual consent is a new no fault ground for absolute divorce in Maryland since 2015;[1]
- A large enough fixed amount of liquidated damages for violation of certain commercial contract;
- Coasian bargaining and implicit interior transfers make sure that only mutually efficient divorces can happen.

[1] https://www.peoples-law.org/no-fault-divorce

Notion of Stability: DSTC

Def.3: Dynamic Stability with Two-sided Commitment (DSTC)

A matching μ in $(M, W, \overline{R})$ is ***dynamically stable with two-sided commitment*** *if*

1. *μ is individually rational, that is, μ does not have a period-1 or period-2 blocking agent that respects two-sided commitment;*
2. *μ does not have a period-1 blocking pair that respects two-sided commitment;*
3. *μ does not have a period-2 blocking pair that respects two-sided commitment.*

- It is common knowledge that agents can unilaterally prevent divorces;
- Again, we can use extended marriage market for illustration.

Blocking Agent

A matching μ has a period-1 blocking agent i that respects two-sided/no commitment if $(i, i) \succ_i \mu(i)$.

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A matching μ has a period-2 blocking agent i that respects **two-sided commitment** if one of the followings holds: (i) $(\mu_1(i), \mu_1(i)) \succ_i \mu(i)$; (ii) $j \equiv \mu_1(i)$, $(j, i) \succ_i \mu(i)$ and $(i, j) \succeq_j \mu(j)$.

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A matching μ has a period-2 blocking agent i that respects **no commitment** if one of the followings holds: (i) $j \equiv \mu_1(i)$, $(j, j) \succ_i \mu(i)$ and $(i, i) \succ_j \mu(j)$; (ii) $(\mu_1(i), i) \succ_i \mu(i)$.

Period-1 Blocking Pair

A matching μ has a period-1 blocking pair (m, w) that respects **two-sided commitment** if one of the followings holds:

- $ww \succ_m \mu(m)$ and $mm \succ_w \mu(w)$;
- $mw \succ_m \mu(m)$ and $wm \succ_w \mu(w)$;
- $wm \succ_m \mu(m)$, $mw \succ_w \mu(w)$, $wm \succ_m ww$ and $mw \succ_w mm$.

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- $mw \succ_m \mu(m)$ and $wm \succ_w \mu(w)$;
- $wm \succ_m \mu(m)$, $mw \succ_w \mu(w)$, $wm \succ_m ww$ and $mw \succ_w mm$.

A matching μ has a period-1 blocking pair (m, w) that respects **no commitment** if one of the followings holds:

- $ww \succ_m \mu(m)$, $mm \succ_w \mu(w)$, $ww \succ_m wm$ and $mm \succ_w mw$;
- $mw \succ_m \mu(m)$ and $wm \succ_w \mu(w)$;
- $wm \succ_m \mu(m)$ and $mw \succ_w \mu(w)$.

Period-2 Blocking Pair

A matching μ has a period-2 blocking pair (m, w) that respects **two-sided commitment** if one of the followings holds:

- If $\mu_1(m) = m$, $\mu_1(w) = w$, then
  $mw \succ_m \mu(m)$ and $wm \succ_w \mu(w)$;
- If $\mu_1(m) = w'(\neq w)$, $\mu_1(w) = w$, then
  $mw' \succeq_{w'} \mu(w')$, $w'w \succ_m \mu(m)$ and $wm \succ_w \mu(w)$;
- If $\mu_1(w) = m'(\neq m)$, $\mu_1(m) = m$, then
  $wm' \succeq_{m'} \mu(m')$, $m'm \succ_w \mu(w)$ and $mw \succ_m \mu(m)$;
- $\mu_1(m) = w'(\neq w)$, $\mu_1(w) = m'(\neq m)$, then
  $mw' \succeq_{w'} \mu(w')$, $wm' \succeq_{m'} \mu(m')$, $w'w \succ_m \mu(m)$ and $m'm \succ_w \mu(w)$; or
  $mm' \succeq_{w'} \mu(w')$, $ww' \succeq_{m'} \mu(m')$, $w'w \succ_m \mu(m)$ and $m'm \succ_w \mu(w)$.

Period-2 Blocking Pair

A matching μ has a period-2 blocking pair (m, w) that respects **no commitment** if the following holds:

- $(\mu_1(m), w) \succ_m \mu(m)$ and $(\mu_1(w), m) \succ_w \mu(w)$.

Our period-1 blocking pair definition:

A matching μ has a period-1 blocking pair (m, w) that respects **no commitment** if one of the followings holds:

- $ww \succ_m \mu(m)$, $mm \succ_w \mu(w)$, $ww \succ_m wm$ and $mm \succ_w mw$;
- $mw \succ_m \mu(m)$ and $wm \succ_w \mu(w)$;
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Comparison with KK2015a

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Period-1 blocking pair definition in KK2015a:

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- $ww \succ_m \mu(m)$, $mm \succ_w \mu(w)$;
- $mw \succ_m \mu(m)$ and $wm \succ_w \mu(w)$;
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Existence of Stable Matching

- The set of dynamically stable matchings with two-sided commitment may be empty for some marriage market;
- Three approaches to deal with it:
  1. Limited Blocking Power of Entries: Not allowing Period-1 Blocking Pairs of Agents Entering the Market at Different Periods;
  2. Restriction on Period-2 Blocking: Binding Engagement;
  3. Restriction on Preference: Preferences for Early Filling (Strong Impatience Assumption).

Theorem 7

If either one of the three restrictive assumption holds and the preferences are rankable for agents who exist in both periods, the set of dynamically stable matchings with two-sided commitment is nonempty for any extended marriage market $(M, W, \overline{R})$.

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Insights of the **man-proposing two-stage deferred acceptance algorithm (T-DA)**:

- Stage 1: Run an adjusted version of M-proposing P-DAFC to get the interim matching μ^I, which is not period-1 blocked by any individual or pair of agents. $\mu_1^* = \mu_1^I$;
- Stage 2: Exercise M-proposing Adjustments for Plans with Separation in period two to carry out mutual-consent separations and rematches to get μ_2^F. $\mu_2^* = \mu_2^F$,
  - μ_2^I serves as the outside option for Stage 2 algorithm.

Properties of Dynamic Stability with Two-sided Commitment: Negative Results

Fact 1:

(i). *In* $(M, W, \overline{R})$*, a dynamically stable matching with two-sided commitment can be Pareto-dominated by another dynamically stable matching.*

(ii). *T-DA algorithm is not strategy-proof for the proposing side.*

Fact 1:

(i). *In $(M, W, \overline{R})$, a dynamically stable matching with two-sided commitment can be Pareto-dominated by another dynamically stable matching.*

(ii). *T-DA algorithm is not strategy-proof for the proposing side.*

Example 1:

$M = \{m_1, m_2\}$, $W = \{w_1, w_2\}$ and the preferences:

$m_1 : w_2 w_1 \succ_{m_1} w_1 w_2 \succ_{m_1} w_2 w_2 \succ_{m_1} w_1 w_1$

$m_2 : w_1 w_2 \succ_{m_2} w_2 w_1 \succ_{m_2} w_1 w_1 \succ_{m_2} w_2 w_2$

$w_1 : m_2 m_1 \succ_{w_1} m_1 m_2 \succ_{w_1} m_1 m_1 \succ_{w_1} m_2 m_2$

$w_2 : m_1 m_2 \succ_{w_2} m_2 m_1 \succ_{w_2} m_2 m_2 \succ_{w_2} m_1 m_1$

- DSTC: $\mu^A(m_1) = w_2 w_1$, $\mu^A(m_2) = w_1 w_2$, and $\mu^B(m_1) = w_1 w_2$, $\mu^B(m_2) = w_2 w_1$;
- DSFC: $\mu^M(m_1) = w_2 w_2$, $\mu^M(m_2) = w_1 w_1$, and $\mu^W(m_1) = w_1 w_1$, $\mu^W(m_2) = w_2 w_2$;
- μ^A Pareto dominates μ^B and w_1 wants to report $m_2 m_2 \succ_{w_1} m_1 m_1$ in W-proposing T-DA to enforce μ^A.

Def 4: Rankability (Kennes, Monte and Tumennasan, 2014a)

For $m \in M$, $\succ_m$ satisfies rankability if $\forall x_1, x_2 \in W \cup \{m\}$, $x_1 x_1 \succ_m x_2 x_2 \iff yx_1 \succ_m yx_2$ and $x_1 y \succ_m x_2 y$, $\forall y \in W \cup \{m\}$ and $y \neq x_2$. Similarly we can define $\succ_w$ for $w \in W$.

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Theorem 8: Efficiency and Restricted Strategyproofness

In a two-period matching market $(M, W, \overline{R})$ where everyone's preference exhibits rankability,
(1). All dynamically stable matching with two-sided commitment is Pareto efficient within the set of individually rational matchings.
(2). If all agents can only report preferences that satisfy rankability, then the T-DA is strategy-proof for the proposing side.

- Rankability incompatible with arrivals and departures of agents in period two.

The set of agents $M \cup W$ is divided into three types:

- **Type 1** T_1: Stay on the market for two periods & satisfy rankability.
- **Type 2** T_2: Enter the market in period 2 & all plans where he or she is matched in period 1 are unacceptable;
- **Type 3** T_3: Leave the market in period 2 & all plans where he or she is matched in period 2 are unacceptable;

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Proposition 1: General Two-Stage Stable Mechanism

For any marriage market $(M, W, \overline{R})$, μ^I is a matching where $\mu^I|_{T_1 \cup T_2}$ is dynamically stable with full commitment in the market restricted to $T_1 \cup T_2$ and $\mu^I_1|T_3$ is spot stable in the market restricted to T_3. If μ^* is derived via M-proposing / W-proposing adjustments for plans with separation based on μ^I, then μ^* is dynamically stable with two-sided commitment.

Assumption *A*1

(1). For $m_1 \in T_1$, $w_1 \in T_3$, $w_2 \in T_2$. *If* $w_1 w_2 \succ_{m_1} w_3 w_3$ *for some* $w_3 \neq w_1 \in W$, *then* $w_1 m_1 \succeq_{m_1} w_3 w_3$ *and* $m_1 w_2 \succeq_{m_1} w_3 w_3$.

(2). For $m_1, w_1 \in T_1$, $w_2 \in T_2$. *If* $w_1 w_2 \succ_{m_1} m_1 w_3$ *for some* $w_3 \in W$, *then* $w_1 w_1 \succeq_{m_1} m_1 w_3$, *and* $m_1 w_2 \succeq_{m_1} m_1 w_3$. *If* $w_1 w_2 \succ_{m_1} w_3 w_3$, *then* $w_1 w_1 \succeq_{m_1} w_3 w_3$ *and* $m_1 w_2 \succeq_{m_1} w_3 w_3$.

Assumption $A1$

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Proposition 2

For any two-period matching market $(M, W, \overline{R})$, *denote* T_1, T_2, T_3 *as the sets of three types of agents,*

(1). If $A1$ *holds and* μ *is the outcome of some General Two-Stage Stable Mechanism, then* μ *is Pareto efficient within the set of individually rational matchings.*

(2). If either $(T_2 \cap W) \cup (T_3 \cap M) = \emptyset$ *or* $(T_2 \cap M) \cup (T_3 \cap W) = \emptyset$ *or* $(T_2 \cap M) \cup (T_2 \cap W) = \emptyset$, *then all DSTC matching is Pareto efficient within the set of individually rational matchings.*

Table: Summary of Main Results

Commitment Type	Full	Two-sided	One-sided	No (KK2015a)
Feature	Once-for-all	Mutual Consent	Asymmetric	No Constraint
Stability	DSFC	DSTC	DSOC	DSNC
Algorithm (Preferences)	P-DAFC (General)	T-DA (Rankability+SI)	P-DAA (Rankability+SI)	P-DAA (SIC+SA/RDS)
Efficiency	$\surd$	Rankability (IR matchings)	Rankability (IR matchings)	Inertia (consistent μ)
Strategy-Proofness	$\times$	$\times$	$\times$	$\times$
SP for one side	$\surd$	Rankability (Restricted)	SIC,WREP (Restricted)	SIC (Restricted)
Entries and Exits	$\surd$	Special case	Special case	$\times$

Extensions and Work in progress

Currently focusing on

- Weaker assumptions for existence of a stable matching;
- Welfare comparison among stable matchings with different levels of commitment.

Potential directions for future work

- dynamic setting with incomplete information;
- Monetary transfers and more general commitment like liquidated damages;
- Real-world applications such as unraveling, more specific markets, et al.

Thank you!

(Any Question?)