Self-Evident Events and the Value of Linking

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Introduction

1. Linking independent and identical copies of the same private-information problem makes them easier to solve.
2. Repeated games with imperfect monitoring.
3. Long-term contracting Radner (1985), Fuchs (2007),
4. Repeated hidden-information problems (Jackson and Sonnenschein (2007), Escobar and Toikka (2013))
5. Intuition: the law of large number reduces the degree of information asymmetry between players.

Repeated Games

1. A large literature of repeated games are driven by two observations:
2. Providing incentives is costly when actions are imperfectly monitored (Green and Porter 1984)
3. Linking incentives across periods may reduce efficiency loss (Abreu, Milgrom, and Pearce, 1991)

Abreu, Milgrom, and Pearce (1991)

1. Repeated Prisoners' Dilemma with a noisy public signal
2. Two scenarios: 1. the public signal observed at the end of each period; 2. the public signals in every T period block observed at the end of the block.
3. Linking has no value in Case 1. The second best can be achieved by a stationary trigger-strategy equilibrium.
4. Linking improves efficiency in Case 2. Efficiency loss goes to zero as T goes to infinity.
5. Sannikov and Skrzypacz (2007)—reverse AMP. As information arrives faster and faster, collusion becomes impossible.

Follow up

1. Identify the deviator: Kandori and Matsushima (1998): pair-wise identifiability; Rahman and Obara (2010): attributability (weak identifiability).
2. Endogenous Delay with private signals: Compte (1998) with independent private signals and Obara (2009) with correlated signals
3. Endogenous Delay through correlated strategies: Rahman (2014), Sugaya (2016)

Questions

1. Linking is useful (not useful) if the signals are private and independent (public).
2. What if players observe both private and public signals?

Overview of results

1. Generalize the insights of AMP to general stage games.
2. Show that any efficiency loss results from a logic similar to the public signal case.
3. Provide a tight bound on the per-period efficiency loss in enforcing a particular action profile in a T period contracting game when T becomes large.
4. Characterize the efficiency loss in terms the primitives of the stage game, combining linking with Obara and Rahman (2010).
5. Show that for any strictly enforceable action profile, there is a correlated action profile close to it that can enforced with arbitrarily small long-term efficiency loss. Simila to Rahman (2014), Sugaya (2016) is similar, but weaker (?)

Outline

1. AMP
2. Self-evident events
3. Characterize the long-run per period efficiency loss.

AMP

1. Noisy Prisoners' Dilemma

Actions

	C	D
C	$1, 1$	$-h, 1+d$
D	$1+d, -h$	$0, 0$

Public Signal Dist.

	H	L
CC	p	$1-p$
CD	q	$1-q$

$1 > p > q, h, d > 0$.

2. If players observe the public signal immediately at the end of each period, the average symmetric equilibrium payoff must be less than $(1, 1)$.
3. If players observe the public signals in all previous T periods once every T periods, the average symmetric equilibrium payoff of the best equilibrium approaches $(1, 1)$ at T becomes large.

Stage game

1. $n \geq 2$ players. $A = A_1 \times ... \times A_n$.
2. In each period,
   1. A mediator picks $\widetilde{a}(t)$ according to η and informs player i of $\widetilde{a}_i(t)$.
   2. Each player i chooses $a_i(t)$ from A_i.
   3. A profile of signals $y(t) = (y_1(t), ...y_n(t))$ realized with $p(y|a)$. Player i observes $y_i(t)$.

What do the players know at the end of a stage game?

1. $(A \times Y)(\eta)$ the set of $(\widetilde{a}, y)$ that is possible given η and p.
2. P_i is the information partition of i.
3. For each $(\widetilde{a}_i', y_i') \in (A_i \times Y_i)(\eta)$, $P_i(\widetilde{a}_i', y_i')$ is the subset of $(A \times Y)(\eta)$ consistent with $(\widetilde{a}_i', y_i')$.
4. Player i "knows" E at $(\widetilde{a}_i, y_i)$ if $P_i(\widetilde{a}_i, y_i) \subseteq E$.
5. A subset E of $(A \times Y)(\eta)$ is self-evident if E is common knowledge at any $(\widetilde{a}, y) \in E$.
6. P is the meet of $P_1, ..., P_n$.
7. Each $\omega \in P$ is self-evident. Each proper subset of ω is not.

Example 1 (pure strategy, public monitoring)

$y_i \in \{H, M, L\}$

	H	M	L
H	$+$	0	0
M	0	$+$	0
L	0	0	$+$

$P = \{(H, H)\}, \{(M, M), (L, M), (L, L)\}$

Example 2 (pure strategy, private monitoring)

$y_i \in \{H, M, L\}$

	H	M	L
H	$+$	0	0
M	0	$+$	0
L	0	$+$	$+$

$$P = \{(H, H)\}, \{(M, M), (L, M), (L, L)\}$$

Example 3 (correlated strategy, public monitoring)

$y \in \{H, L\}, A_1 = A_2 = \{C, D\}$
$(\eta(CC), \eta(CD), \eta(DC), \eta(DD)) = (+, +, +, 0)$

	HC	HD	LC	LD
HC	$+$	$+$	0	0
HD	$+$	0	0	0
LC	0	0	$+$	$+$
LD	0	0	$+$	0

"H" $= \{HCC, HCD, HDC\}$, "L" $= \{LCC, LCD, LDC\}$
$P = \{$"H", "L"$\}$

Stage game incentives

1. At the end of a period, each player i report $\widehat{y}_i$. Mediator reveals $\widetilde{a}$.
2. To enforce η, each player is paid $w_i\left(\widetilde{a}, \widehat{y}\right)$
3. Require: $\sum_{i=1}^{n} w_i \leq 0$. Incentives are costly.
4. Player i's total payoff

$$\sum_{t=1}^{T} g_i(a(t)) + w_i\left(\widetilde{a}, \widehat{y}\right).$$

Decomposition of Incentives

We can decompose any incentives:

$$w_i(\widetilde{a}, \widehat{y}) = w_{i,a}(\widetilde{a}, \widehat{y}) + w_{i,b}(\widetilde{a}, \widehat{y}),$$

where

$$\begin{aligned} w_{i,a}(\widetilde{a}, \widehat{y}) &\equiv w_i(\widetilde{a}, \widehat{y}) - w_{i,b}(\widetilde{a}, \widehat{y}) \\ w_{i,b}(\widetilde{a}, \widehat{y}) &\equiv E\left[w_i\left(\widetilde{a}', \widehat{y}'\right) |\sigma^*, P(\widetilde{a}, \widehat{y})\right] - \max_{\omega \in P} \sum_{i=1}^{n} E\left[w_i(\widetilde{a}, \widehat{y}) | \sigma^*, \omega\right]. \end{aligned}$$

The decomposition divides w_i into a self-evident component, $w_{i,b}$, which depends solely on $P(\widetilde{a}, \widehat{y})$, and a residual private component, $w_{i,a}$.

Decomposition of incentives

Consider a two-player game. Signal distribution under a pure action profile η.

$$\begin{array}{cccc} & h_2 & m_2 & l_2 \\ h_1 & 0.5 & 0 & 0 \\ m_1 & 0 & 0.1 & 0 \\ l_1 & 0 & 0.2 & 0.2 \end{array}, \quad \begin{array}{cccc} & h_2 & m_2 & l_2 \\ h_1 & -2 & * & * \\ m_1 & * & 0 & * \\ l_1 & * & -4 & -6 \end{array}$$

$$P : \{\{(h_1, h_2)\}, \{(m_1, m_2), (l_1, m_2), (l_1, l_2)\}\}.$$

The expected transfer conditional on the first is -2, and that conditional on the second is -4. A player receives 0 if $\{(h_1, h_2)\}$ and $-4 + 2 = -2$ if $\{(m_1, m_2), (l_1, m_2), (l_1, l_2)\}$. We can decompose the incentives into two components:

$$\begin{array}{cccc} & h_2 & m_2 & l_2 \\ h_1 & 0 & * & * \\ m_1 & * & -2 & * \\ l_1 & * & -2 & -2 \end{array}, \quad \begin{array}{cccc} & h_2 & m_2 & l_2 \\ h_1 & -2 & * & * \\ m_1 & * & +2 & * \\ l_1 & * & -2 & -4 \end{array}$$

Result

- The long-term efficiency loss depends only on the self-evident component.
- Efficiency loss associated with the residue can be eliminated through linking.

Intuition

1. The first part extends AMP(1991)
2. The secon part utilizes differential beliefs of the players within a non-reducible self-evident set.
3. Original idea comes from Fong et. al. 2011.
4. Chan and Zhang 2016 extend to full support.
5. This paper extends to any irreducible self-evident set.