Selling Mechanisms for a Financially Constrained Buyer

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UTS Reading Group

Buyers Face Financial Constraints

There is mounting evidence that buyers face financial constraints in both high-stake and low-stake deals.

High-stake deals — value of goods to be allocated is large relative to the buyers' liquid assets:

- professional sports streaming deals;
- privatizations, spectrum licenses, mineral extraction rights, etc;
- housing and other durable goods markets.

Low-stake deals — mental budgeting (accounting) imposes behavioral constraints:

- holiday trips, fund raising events — Thaler [JEP 90];
- on-line commerce — Milkman & Beshears [JEBO 09].

Sellers Know It

Sellers take explicitly into account difference between willingness and ability to pay:

- in Google's keyword auction platform, bidders are required to specify their bids as well as their daily budget limits — Edelman et al [AER 07];
- Amazon's Cloud Computing service asks costumers to create billing alarms to monitor their spending.
  http://docs.aws.amazon.com/AmazonCloudWatch/latest/DeveloperGuide/monitor_estimated_charges_with_cloudwatch.html

Ignoring budget constraints is not inconsequential:

- important theoretical results don't hold when buyers are financially constraint (e.g., revenue equivalence between FPA and SPA — Che & Gale [JET 98];

When financial constraints are private information, available results are mostly restricted to single-item selling mechanisms:

- Che et al [RES 13] — welfare maximization involves random allocation with resale;
- Pai & Vohra [JET 14] — optimal auction;
- Devanur & Weinberg [EC'17] — optimal mechanisms for arbitrary distributions.

2-Item Allocation Problem under Complementarities

The seller possesses two goods to allocate (airfreight routes between major hubs).

- a — allocate a single license;
- a' — allocate both licenses;
- a_0 — exclusion.

The buyer has four different valuations,

	a	a'	a_0
v_1	11	16	0
v_2	10	20	0
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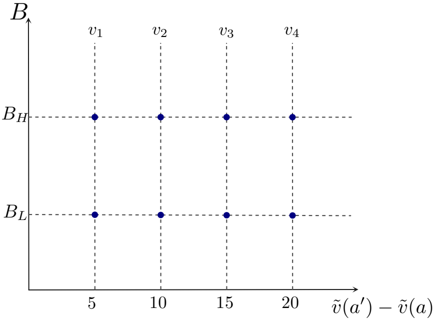

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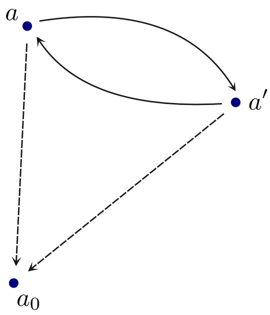
Use an allocation network instead of the 64 incentive and participation constraints.



Formalities

A seller has different items (alternatives) to offer.

- $\mathcal{A}$ is the set of alternatives (finite) with typical elements a, a', etc.

A buyer assigns a valuation $\tilde{v}(a)$ for each of the items a in $\mathcal{A}$.

- $\mathcal{V}(a) \subseteq \mathbb{R}_+$ is set of admissible valuations for item a.
- $\mathcal{V} = \times_{a \in \mathcal{A}} \mathcal{V}(a)$ is the set of valuations with typical elements v, v', etc.

The buyer faces financial constraints.

- $\mathcal{B} \subseteq \mathbb{R}_+$ is the set of admissible budgets, with typical element B.

Hard budget constraints: a buyer with type (v, B) who pays ρ for item a has a utility $v(a) - \rho$ as long as $\rho \leqslant B$, and $-\infty$ otherwise.

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A direct mechanism $\langle f, \rho \rangle$ is called budget feasible for the buyer if it no payment specified by the mechanism exceeds her budget:

$$\rho(v, B) \ \leq \ B, \qquad \text{for all } (v, B) \text{ in } \mathcal{V} \times \mathcal{B}; \tag{BF}$$

It is incentive compatible if any affordable deviation from truth-telling is not profitable:

$$v(f(v, B)) - \rho(v, B) \ \geq \ v(f(v', B')) - \rho(v', B'), \tag{IC}$$

for all (v, B) and (v', B') in $\mathcal{V} \times \mathcal{B}$ such that $\rho(v', B') \leq B$;

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It is individually rational if it never requires the buyer to pay more than her value for an alternative; i.e.,

$$v(f(v, B)) - \rho(v, B) \ \geq \ 0, \qquad \text{for all } (v, B) \text{ in } \mathcal{V} \times \mathcal{B}. \tag{IR}$$

Non-linear Pricing Functions

Let $f^{-1}(a) \subseteq \mathcal{V} \times \mathcal{B}$ be the subset of types that select a under f.

Lemma *Let $\langle f, \rho \rangle$ be an incentive compatible and budget feasible mechanism. Then for every $a \in \mathcal{A}$, there exists a price $p(a) \in \mathbb{R}$ such that*

$$p(a) \;=\; \rho(v, B), \qquad \textit{for all } (v, B) \in f^{-1}(a).$$

Proof Otherwise find two types, say (v, B) and (v', B'), that pay different amounts for the same item. The type that pays the most has an affordable and profitable deviation. □

f is called implementable without deficits if there exists a pricing function $p \colon \mathcal{A} \to \mathbb{R}$ s.t. the selling mechanism $\langle f, p \rangle$ satisfies (IC), (BF), and (ND).

If in addition $\langle f, p \rangle$ satisfies (IR), it is called an acceptable selling mechanism.

Budget Levels and Incremental Values

Fix an allocation function $f\colon \mathcal{V} \times \mathcal{B} \to \mathcal{A}$. The budget level for alternative a is

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The buyer's restricted incremental value between a and a' is

$$\delta^r(a,a') \;=\; \inf\left\{v(a) - v(a') :\, (v,B) \in f^{-1}(a),\; B \geq \beta(a')\right\}. \tag{3}$$

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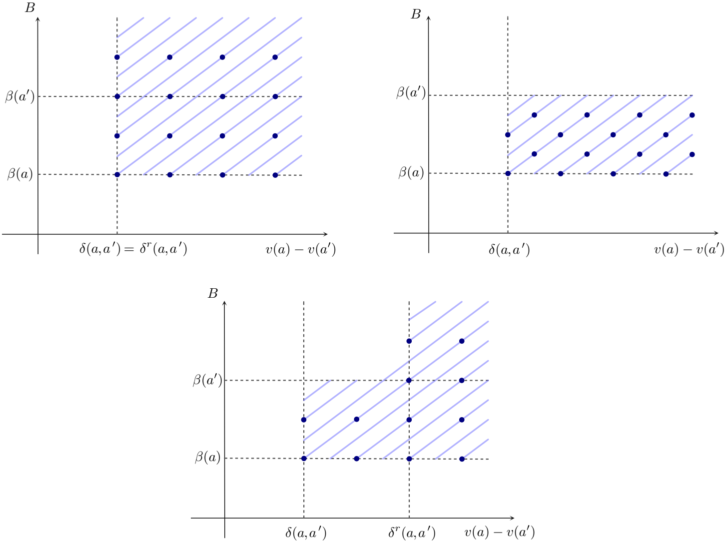
When $\beta(a) < \beta(a')$, all types that are assigned a' by f can 'afford' a and thus

$$\delta(a',a) \;=\; \delta^r(a',a).$$

Budget Levels and Incremental Values



A Preliminary Result

Proposition *Let $\langle f, p \rangle$ be a budget feasible selling mechanism.*

(a) If $\langle f, p \rangle$ is incentive compatible, then $\delta^r(a, a') \geq p(a) - p(a')$ for all $a, a' \in \mathcal{A}$.

(b) If $\delta(a, a') \geq p(a) - p(a')$ for all $a, a' \in \mathcal{A}$, then $\langle f, p \rangle$ is incentive compatible.

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(b) If $\delta(a, a') \geq p(a) - p(a')$ *for all* $a, a' \in \mathcal{A}$*, then* $\langle f, p\rangle$ *is incentive compatible.*

Proof (a) Let $\delta^r(a, a') < \infty$. For $\epsilon > 0$, there is a type (v, B) that is assigned a under f such that $B \geq \beta(a')$ and $v(a) - v(a') \leq \delta^r(a, a') + \epsilon$.

By (BF), we have $p(a') \leq \beta(a')$ and thus this type can afford to buy a'.

By (IC), we have $p(a) - p(a') \leq v(a) - v(a') \leq \delta^r(a, a') + \epsilon$.

When $\delta^r(a, a') = +\infty$, the result follows trivially. □

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Proof (b) Suppose on the contrary that $\langle f, p \rangle$ violates (IC).

There exist two alternatives a, a' and a type (v, B) that selects a under f but has a profitable and affordable deviation to a'.

It follows that $-\infty < v(a) - p(a) < v(a') - p(a')$, where the first inequality holds as $\langle f, p \rangle$ satisfies (BF).

Immediately, $\delta(a, a') \leq v(a) - v(a') < p(a) - p(a')$. □

A Preliminary Result

Proposition *Let* $\langle f, p\rangle$ *be a budget feasible selling mechanism.*

(a) If $\langle f, p\rangle$ *is incentive compatible, then* $\delta^r(a, a') \geq p(a) - p(a')$ *for all* $a, a' \in \mathcal{A}$.

(b) If $\delta(a, a') \geq p(a) - p(a')$ *for all* $a, a' \in \mathcal{A}$*, then* $\langle f, p\rangle$ *is incentive compatible.*

(c) If the buyer's financial constraint is publicly known, then $\langle f, p\rangle$ *is incentive compatible if, and only if, for all* $a, a' \in \mathcal{A}$ *one has*

$$+\infty \ > \ \delta(a, a') \ \geq \ p(a) \ - \ p(a') \ \geq \ -\delta(a', a) \ > \ -\infty.$$

Remarks

- Working only with restricted incremental values is not sufficient.
- Working only with unrestricted incremental values is not necessary.
- We have counter examples in the paper.

Example 1

The seller has two items, $\mathcal{A}_1 = \{a, a'\}$. The buyer has a single budget, $\mathcal{B}_1 = \{5\}$, and two private valuations, $\mathcal{V}_1 = \{v, v'\}$,

	a	a'
v	20	10
v'	10	0

f_1 assigns $f_1(v, 5) = a$ and $f_1(v', 5) = a'$. Immediately

$$\delta_1(a, a') = \delta_1^r(a, a') = 10 \quad \text{and} \quad \delta_1(a', a) = \delta_1^r(a', a) = -10.$$

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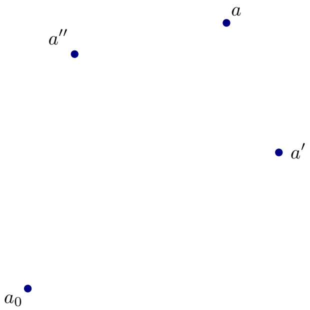
f_1 is not implementable without deficits.

Indeed, from (IC) we must have $10 \geq p_1(a) - p_1(a') \geq 10$.

But $p_1(a)$ must be less than 5 by (BF), whereas $p_1(a')$ is non-negative by (ND).

Flow Network Approach to Implementability without Deficits



The allocation network $H = (N, E)$ associated to f has a set nodes $N = \mathcal{A} \cup \{a_0\}$ and a set of directed edges $E = E_1 \cup E_2 \cup E_3$, where

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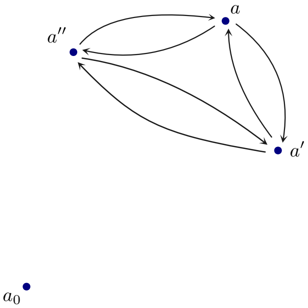

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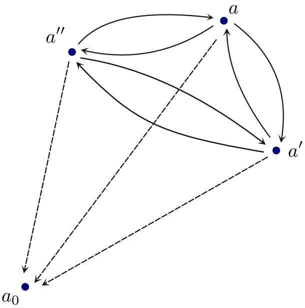

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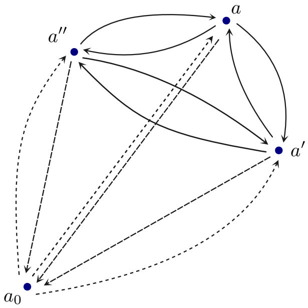

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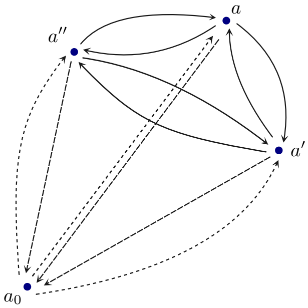

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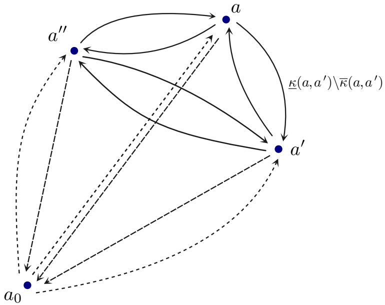

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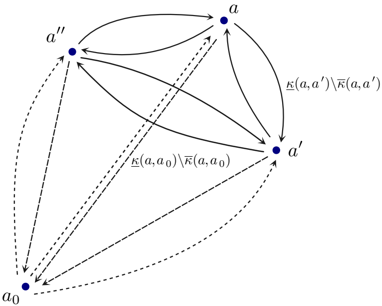

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- for all $(a, a_0) \in E_2$,

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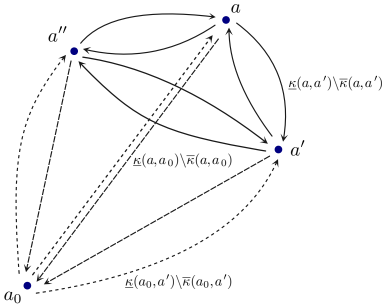

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- for all $(a, a_0) \in E_2$,

  $$\underline{\kappa}(a, a_0) := \beta(a) =: \overline{\kappa}(a, a_0);$$

- for all $(a_0, a) \in E_3$,

  $$\underline{\kappa}(a_0, a) := 0 =: \overline{\kappa}(a_0, a).$$

Allocation Network in Example 1

The seller has two items, $\mathcal{A}_1 = \{a, a'\}$. The buyer has a single budget, $\mathcal{B}_1 = \{5\}$, and two private valuations, $\mathcal{V}_1 = \{v, v'\}$,

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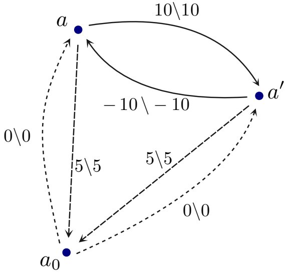
$$\underline{\kappa}(a, a') = \delta_1(a, a') = 10 = \delta_1^r(a, a') = \overline{\kappa}(a, a'),$$
$$\underline{\kappa}(a'a) = \delta_1(a', a) = -10 = \delta_1^r(a', a) = \overline{\kappa}(a', a).$$



f_1 is not implementable without deficits.

Implementability without Deficits

Proposition—necessity *If the allocation function f is implementable without deficits, then the corresponding allocation network $H = (N, E)$ contains no cycle of negative maximal capacity.*

Proposition—sufficiency *If the allocation network $H = (N, E)$ contains no cycles of negative minimal capacity, then there is a pricing function p that implements f without deficits.*

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Remarks

- With a public budget, maximal and minimal capacities coincide.
- With private budgets, the gap between sufficiency and necessity cannot in general be breached — we have counter examples in the paper.
- The difference between minimal and maximal capacities —i.e., between unrestricted and restricted incremental values— is an design instrument.
- A single modification of the allocation network $H = (N, E)$ —the capacities of the edges in E_2— allow us to obtain similar results for acceptable selling mechanisms.

Charges as Prices

The minimal charge $\underline{c}(x,y)$ between nodes x and y in $H = (N, E)$ is the minimal capacity of the path with lowest minimal capacity connecting x and y.

The maximal charge $\overline{c}(x,y)$ between nodes x and y in $H = (N, E)$ is the maximal capacity of the path with lowest maximal capacity connecting x and y.

Charges are the flow network analogue of marginal prices.

- They capture the smallest possible cumulative valuation difference between purchasing a and staying out.

Minimal charges are used to construct the pricing function in the proof of our sufficiency result: if $H = (N, E)$ has no cycle of negative minimal capacity, then

$$p(a) \;=\; \underline{c}(a, a_0), \quad \text{for all } a \in \mathcal{A}.$$

Charges as Prices

Proposition—price bounds *If the pricing function p implements f without deficits, then one has*

$$p(a) \;\leq\; \overline{c}(a, a_0), \qquad \textit{for all } a \in \mathcal{A}.$$

Proof Suppose that $P = \{a, a', a'', a_0\}$ is the path with lowest maximal capacity between a and a_0:

$$\begin{aligned} \overline{c}(a, a_0) &= \quad \overline{\kappa}(a, a') \quad + \quad \overline{\kappa}(a', a'') \quad + \overline{\kappa}(a'', a_0) \\ &\geq\; p(a) - p(a') + p(a') - p(a'') + \beta(a''). \qquad \square \end{aligned}$$

In the 2-item case, the upper bound on implementable prices is tight.

This pins down maximal prices for any given allocation function f.

- Also true in the n-item case with some additional assumptions.

2-Item Allocation Problem: Strong Financial Constraints

The seller possesses two goods to allocate (airfreight routes between major hubs).

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- a_0 — exclusion.

The buyer has four different valuations,

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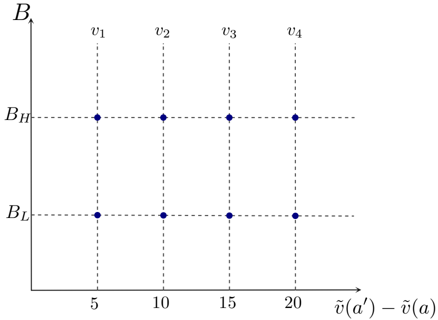
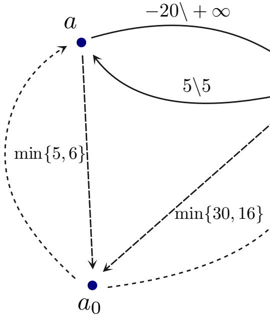
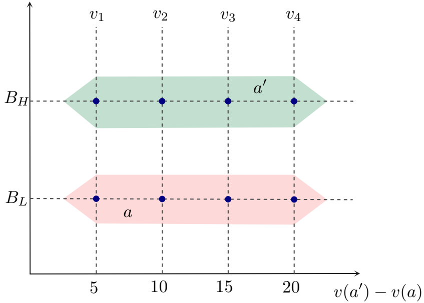
v_4	6	26	0

The buyer has two budgets $\{B_L, B_H\}$, where $B_L = 5$ and $B_H = 30$.

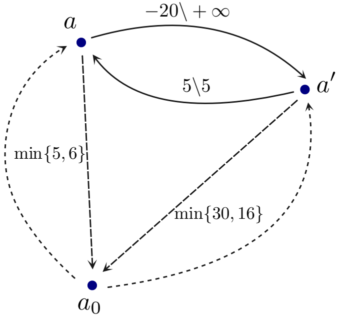


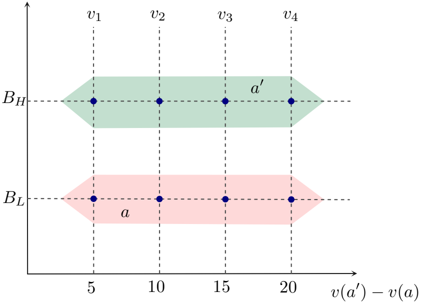
2-Item Allocation Problem: Strong Financial Constraints





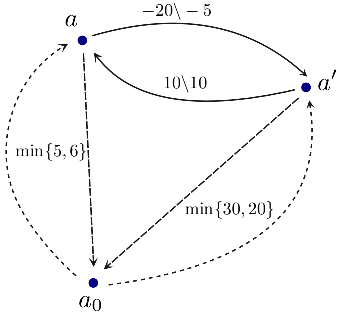
2-Item Allocation Problem: Strong Financial Constraints





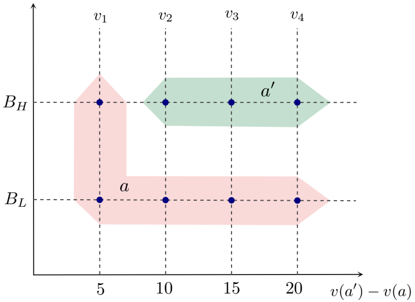
- $p(a) = \bar{c}(a, a_0) = 5$,
- $p(a') = \bar{c}(a', a_0) = 10$,
- $R = 60$ (each type occurs once).

2-Item Allocation Problem: Strong Financial Constraints

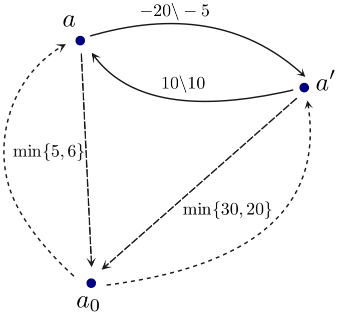


Under full exclusion of low-budget types, $R = 64$ (a' costs 16, a costs 11).

Seller does better by pooling a high budget type with all low budget types.



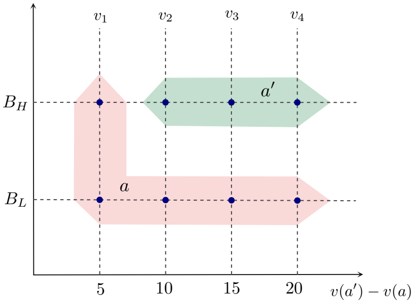
2-Item Allocation Problem: Strong Financial Constraints



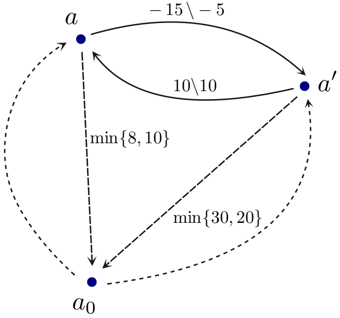
- $p(a) = \bar{c}(a, a_0) = 5$,
- $p(a') = \bar{c}(a', a_0) = 15$,
- $R = 70$ (each type occurs once).

Under full exclusion of low-budget types, $R = 64$ (a' costs 16, a costs 11).

Seller does better by pooling a high budget type with all low budget types.

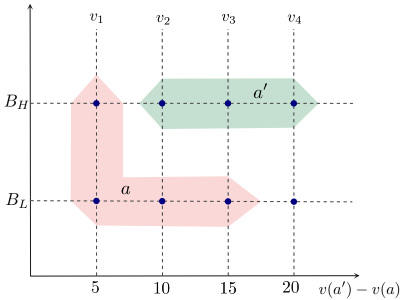


2-Item Allocation Problem: Weak Financial Constraints

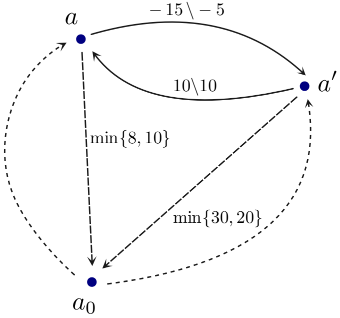


Comparative statics: $B_L = 8$.

Weakening the financial constraints changes the optimal allocation.



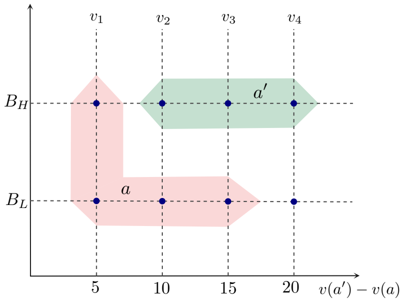
2-Item Allocation Problem: Weak Financial Constraints



- $p(a) = \bar{c}(a, a_0) = 8$,
- $p(a') = \bar{c}(a', a_0) = 18$,
- $R = 86$ (each type occurs once).

Comparative statics: $B_L = 8$.

Weakening the financial constraints changes the optimal allocation.



Concluding Remarks

We study the design of deterministic selling mechanisms in a general setting with private valuations and private budgets.

- A seller interacts with a buyer and designs prior-free selling mechanisms that are budget feasible, incentive compatible and do not raise deficit.

We provide sufficient and necessary conditions for an allocation rule to be implementable without deficits (and, in the paper, acceptable).

- Subtle difference between incremental values is key to our results.
- This difference has economic content and provides a new way to look at the problem.
- These conditions cannot replace each other.

We construct a novel flow network to understand implementability under private financial constraints.

Our approach directly informs the construction of implementable prices.