Compromising Quality to Stay Relevant

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Work in Progress

Note to Self

Always listen to Yi-Chun

Theoretical Motivation

We study a tractable, dynamic principal-agent setting with the following features:

1. **Adverse selection**: the agent has a private type.
2. **Types differ by rate of information arrival**: good types periodically receive information of varying value to the principal, bad types do not.
3. **Strategic Actions**: The agent chooses whether or not to act on information.
4. **No transfers.**

Framework: Bandit problem with a strategic arm.

- A principal chooses between a costless safe arm and a costly risky arm of privately known quality.
- The risky arm gets utility from being pulled.
- The arm periodically receives information via which it controls the public outcomes (and thereby the principal's payoff) which inform about its quality.

Application: Experts in organizations

Players: A pharma company (principal) hires a scientist (agent) to develop new products.

Agent Type: Her privately known ability determines the frequency with which she receives research ideas of varying quality.

Stage Game

Principal Action: Continue employment (costly) or fire (costless) the scientist.

Agent Information: The quality of idea (if any) she receives.

Agent Action: Based on quality, she decides whether or not to run a drug trial.

Public Outcomes: If the agent runs a trial, its success/failure is observed.

- The likelihood of a success is based on idea quality.

Payoffs: The scientist wants to extend employment and the pharma company wants successful trials.

Application: The "attention economy"

Players: A reader/news aggregator (principal) decides whether to continue visiting/sourcing from a news blog (agent).

Agent Type: Her privately known ability determines the frequency with which she generates breaking news of varying reporting quality.

Stage Game

Principal Action: Click on/source from the website (costly) or not (costless).

Agent Information: The quality of content (if any) she generates.

Agent Action: Based on quality, she decides whether or not to publish content.

Public Outcomes: If content is published, its veracity is revealed.

- The likelihood of truthfulness depends on the how reliably the content is sourced (quality).

Payoffs: The website wants the reader/news aggregator to keep clicking/sourcing and the reader wants truthful content.

Model: Preliminaries

Players: Single principal and single agent (the bandit arm).

Horizon:

- Time $t \in \{1, 2, \dots\}$ is discrete.
- Players discount with common factor δ.

Initial type: Agent's initial type θ is either good $\overline{\theta}$ or bad $\underline{\theta}$.

- Principal's initial belief: $\overline{\theta}$ with probability p_0.
- Type known to agent.

Period-t information: $i_t \in \{i_\phi, i_l, i_h\}$.

- Privately observed by agent.
- i_ϕ corresponds to no information.
- i_l, i_h correspond to low, high quality information respectively.

Model: Stochastic process for information

Information for bad type ($\theta = \underline{\theta}$):

- The bad type never receives information.
- $i_t = i_\phi$ with probability 1.

Information for good type ($\theta = \overline{\theta}$):

- The good type may get information (of either high or low quality).
- $i_t = i_\phi$ with probability $1 - \lambda$.
- $i_t \in \{i_h, i_l\}$ with probability λ.
- Conditional on $i_t \in \{i_h, i_l\}$, it is high (low) quality with probability λ_h/λ (λ_l/λ).

Model: Stage game

Principal Action:

- At the beginning of period t, the principal chooses $x_t \in \{0, 1\}$ which determines her choice of safe or risky arm.
- If $x_t = 0$, the stage game ends and we move to period $t + 1$.
- If $x_t = 1$, the agent chooses an action (described below).
- Action x_t has a cost of $c\, x_t$ to principal.

Agent Actions:

- Agent observes information i_t and chooses $a_t \in \{0, 1\}$ (costless).
- $a_t = 1\,(0)$ corresponds to (not) acting.

Model: Public outcomes

Public Outcomes:

- If the agent acts, an outcome $\overline{s}$ (success) or $\underline{s}$ (failure) is generated with a distribution that depends on the agent's information:

$$\mu(\overline{s} \mid i_t, a_t = 1) = \begin{cases} 1 & \text{if } i_t = i_h, \\ q_l & \text{if } i_t = i_l, \\ 0 & \text{if } i_t = i_\phi, \end{cases} \quad \mu(\underline{s} \mid i_t, a_t = 1) = \begin{cases} 0 & \text{if } i_t = i_h, \\ 1 - q_l & \text{if } i_t = i_l, \\ 1 & \text{if } i_t = i_\phi. \end{cases}$$

Public Outcomes Interpretation:

- Acting based on high-quality information surely generates a success.
- Acting based on low-quality information can generate either a success or failure.
- Acting based on no information surely generates a failure.
- Note: Success is perfectly revealing.

Model: Stage game payoffs

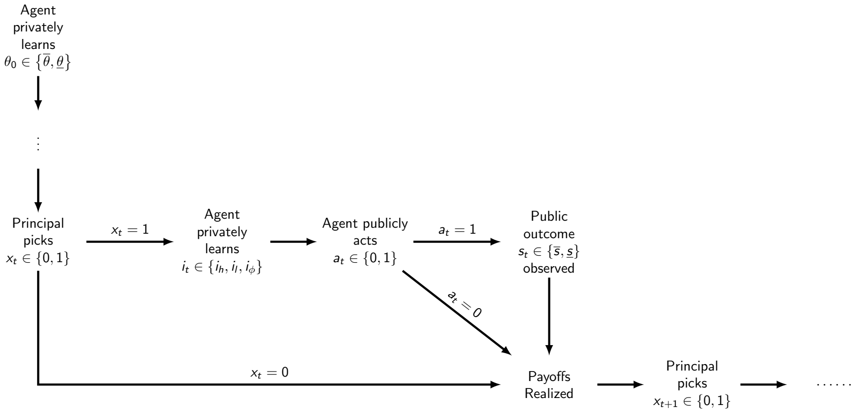
Payoffs:

$$\text{Agent's Payoff} = x_t,$$
$$\text{Principal's Payoff} = \begin{cases} 1-c & \text{if } s_t = \overline{s} \text{ and } x_t = 1, \\ -1-c & \text{if } s_t = \underline{s} \text{ and } x_t = 1, \\ 0 & \text{if } x_t = 0 \text{ or } a_t = 0. \end{cases}$$

Payoffs Discussion:

- The agent wants to maximize the number of risky arm pulls.
- Only the good type ($\theta = \overline{\theta}$) can generate positive payoffs for the principal.
- Action on low-quality information may provide negative expected instantaneous payoff (when $q_l < 1/2$) but could be useful for screening.

Overview of the game



Strategies

Public histories contain past:

- Arm choices.
- Realized outcomes.

The principal's strategy is a (possibly mixed) arm choice as a function of the public history.

- We consider both the cases where the principal can and cannot commit to future policy.

The agent's private history additionally contains past:

- Information received.
- Chosen actions.

The agent's strategy is a (possibly mixed) action choice as a function of her private history.

What we do and find

We aim to understand how the main features of our model (that we highlighted in the introduction) affect the action choices.

- In particular, how does the presence of bad types distort the behavior of good types?
- What is the (qualitative) value of commitment (i.e. how do these distortions differ).

Our main result is that commitment allows for quality control of actions whereas this is never possible without commitment (subject to a minor refinement).

Related literature

Bad reputation: Ely-Välimäki (2003), Ely-Fudenberg-Levine (2008).

Dynamic information disclosure: Acharya-DeMarzo-Kremer (2011), Guttman-Kremer-Skrzypacz (2014).

Career concerned experts: Marinovic-Ottaviani-Sørensen (2013), Rappoport (2015), Backus-Little (2017).

Dynamic mechanism design: Battaglini (2005), Pavan-Segal-Toikka (2014).

...without money: Guo (2016), Guo-Hörner (2016), Mitchell (2017), Deb-Pai-Said (2018).

Adverse Selection + Dynamic Moral Hazard: Halac-Kartik-Liu (2017), Deb-Stewart (2018).

Benchmark: First-best

The first-best is the following:

- If the agent is the good (bad) type, the principal always (never) pulls the risky arm.
- The good type always chooses to take action when high quality information available.
- The good type always (never) chooses the action when low quality information is available iff $q_l > (<) 1/2$.

Agent's actions are costless, so there are no frictions once type uncertainty is resolved.

Commitment: Direct mechanisms

The revelation principle applies, so we need only consider direct mechanisms.

A **direct mechanism** $(\mathbf{x}, \mathbf{a})$ determines:

1. the arm choice $\mathbf{x} : \mathcal{H}_t \to \Delta\{0, 1\}$,
2. an action recommendation $\mathbf{a} : \mathcal{H}_t \times i_t \to \Delta\{0, 1\}$,

where $\mathcal{H}_t$ denotes a public history at the beginning of period t. This contains:

- The initial reported type θ and all subsequently reported information $\{i_{\tilde{t}}\}_{\tilde{t}<t}$.
- All past arm choices and recommended actions.
- All past public outcomes.

The optimal mechanism will be subject to **incentive constraints**: the agent should report her type, information honestly and choose the recommended action.

Commitment: Incentives

The principal needs to balance the quality of actions she recommends with the incentive to screen.

Intuitively, action under no information should not be used for screening.

- Both types can costlessly choose these actions.

Recommending the action with low information has a tradeoff:

- Benefit from screening: success is perfectly.
- Instantaneous payoff loss: low actions may provide a negative expected payoff.

Properties of the optimal mechanism

Lemma

The following properties hold in all (non-trivial) optimal mechanisms.

1. *Agent action is always recommended when high information available.*
2. *Action is never recommended under no information.*
3. *If $q_l \geq 1/2$, action is recommended when low information available.*
4. *There exists $\overline{q_l} > 0$, such that when $q_l \leq \overline{q_l}$, action is never recommended with low information.*

Properties of the optimal mechanism: Implications

Under certain conditions (high or low q_l), commitment can align incentives and result in efficient action choices.

Of course, private information does result in an inefficient arm choices.

- The arm is pulled inefficiently less (more) times when the agent is high (low) type.

Inefficient actions ($q_l < 1/2$) can be used for screening.

- Consider the case where q_l is close to $1/2$.

Optimal mechanism: Intuition

Commitment allows the principal to screen out action under no information.

- Suppose the mechanism recommends action under no information to one of the types at some history.
- The mechanism can be altered to recommend no action instead without affecting incentives.

In addition, action under low information when q_l is very low are also screened out.

- Intuitively, the screening benefit only arises with probability q_l.
- The loss from low quality actions is $1 - 2q_l$.
- For low q_l, the latter outweighs the former.

Incomplete characterization on the intermediate case.

- Usual approach: for any recommended action, characterize the optimal contract. Then figure out optimal action sequence.
- Can tell you properties of the optimal contract for any action sequence.
- Low quality actions may be recommended at some, but not all, histories, can't characterize that yet.

Structure of the optimal mechanism

Theorem

When $q_l \geq 1/2$, the optimal mechanism can be implemented as follows. There is a time T such that the principal:

- *Pulls the risky arm at every $t \leq T$.*
- *Pulls the risky arm forever if a success is realized prior to T.*
- *Switches to the safe arm forever if no success is realized prior to T.*

Structure of the optimal mechanism

Theorem

When $q_l \leq \overline{q}_l$, the optimal mechanism is as follows. At histories where no action has been taken in all periods in the past, there is a time T such that, the principal:

- *Pulls the risky arm at all periods $t \leq T$.*
- *Switches to the safe arm forever with some fixed, time-invariant probability at each $t > T$.*

At the first time where an action is taken

- *If success, she pulls the risky arm forever after.*
- *If failure, she switches to the safe arm forever from there.*

Note that after the first high or low outcome, the mechanism does not depend on subsequent outcomes.

Intermediate q_l

When q_l is smaller than $\frac{1}{2}$ (so acting on low information is inefficient), but larger than $\overline{q}_l$, the principal may wish to have the agent act on low information in some but not all periods.

Theorem

In any period where the principal would like the agent to act only on high quality information:

- *If success, she pulls the risky arm forever after.*
- *If failure, she switches to the safe arm forever from there.*

If would like the agent to act on both high and low quality information:

- *If success, she pulls the risky arm forever after.*
- *If failure, ignore.*

Can't yet tell you what the optimal action sequence looks like. Conjecture: ask the agent to only act on high quality information for the first T_1 periods, either high or low quality information for next T_2, switch to safe arm if no success by then.

Structure of the optimal mechanism: Implications and intuition

Note that the optimal mechanism can be implemented without a menu.

Commitment from firms (via contracts) or news aggregators (via their algorithms) can provide incentives for minimal quality standards.

Successes are rewarded with tenure.

When trying to incentivize action on both types of information, ignore failures.

- Of course, they cannot be rewarded.

When $q_l \leq \overline{q}_l$, the principal wants to prevent low actions.

- She might as well impose the most draconian punishment for failures.
- Additionally, she has to inefficiently pull the risky arm too much.

Equilibrium analysis: Solution concept

Our model is a reputation setting with two long-lived players one of whom has multiple strategic types.

Here, there can be "implausible" equlibria with inefficiencies even in the absence of incentive problems.

- For instance, the principal pulls the safe arm even when she knows the agent is the high type (with probability 1).
- In response, the high type only chooses bad actions.

We follow Ely and Välimäki (2003) and impose a mild "renegotiation-proofness" refinement.

An equilibrium is **renegotiation-proof** if, when the principal believes that the agent is the high type ($\theta = \overline{\theta}$) with probability 1 on path, the following hold:

1. The principal pulls the risky arm forever.
2. The agent only chooses efficient actions.

Equilibrium properties

Theorem

Every (non-trivial) renegotiation-proof Nash equilibrium will see agents act on both low quality and no information (with positive probability) on path.

Note that the result holds for all δ, q_l, λ_h and λ_l.

Equilibrium properties: Implications

Without commitment, a minimal quality standard for the good type can never be enforced at all (on-path) histories.

Additionally, actions on no information can never be completely eliminated.

This can result in further inefficiency. Formally, there is a set of initial beliefs p_0 such that:

- With commitment, the principal experiments with the agent.
- Without commitment, the principal does not.

Equilibrium properties: Implications

To see this, suppose we alter the principal's payoff

$$\text{Principal's Payoff (gross of cost)} = \begin{cases} 1 & \text{if } s_t = \overline{s} \text{ and } x_t = 1, \\ -\pi & \text{if } s_t = \underline{s} \text{ and } x_t = 1, \\ 0 & \text{if } s_t = \phi \text{ or } x_t = 0, \end{cases}$$

where $\pi > 1$.

For $q_l < \overline{q}_l$, this will not alter the commitment optimal as failures never arise on path.

But, without commitment, this will lead to breakdown as π gets larger.

Equilibrium properties: Intuition

Suppose there was a (renegotiation-proof) equilibrium with no action on low or no information on path.

- There will be a belief $\tilde{p}$ below which the principal will switch to the safe arm.
- By assumption, the agent chooses no action on path with positive probability.
- The belief following no action will drift towards $\tilde{p}$.
- At the last period before belief reaches $\tilde{p}$, there will be a strict incentive (due to renegotiation-proofness) to choose to act if low information.

Equilibrium properties: Intuition

Equilibrium cannot just have actions on low information.

- At any such history where failures by good type are on path, the belief will jump to $p = 1$ following a failures.
- Both types will have an incentive to choose to act on no information.

Equilibrium cannot just have action on no information.

- At histories where actions on no are chosen by the good type, he will have a strict incentive to choose actions on no information.
- At histories where bad actions are only chosen by the bad type, the posterior following low outcomes will drop to $p = 0$.

Markov equilibria

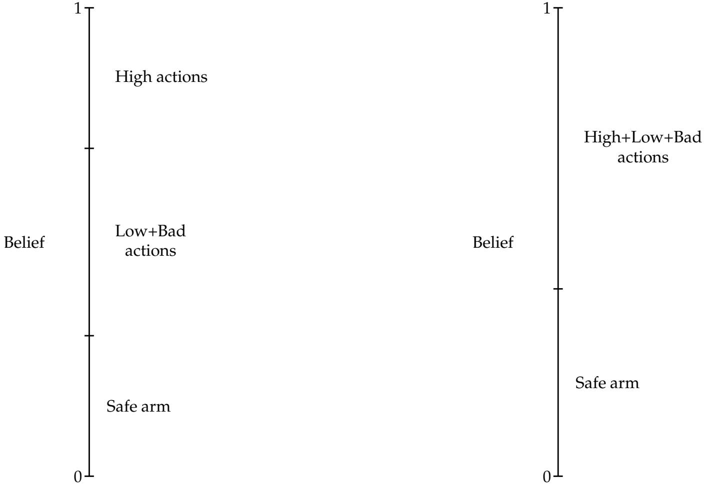
It is straightforward to derive a class of Markov equilibria with the following properties.

1. There may be beliefs where only high actions are chosen.
2. At all other beliefs, the posteriors following failure and no action taken are the same.
3. The good type chooses actions at all beliefs where bad actions are on path.
4. The high type may also choose bad actions.

Intuition:

- Both types are indifferent between bad and no actions at histories where low outcomes are on-path.
- When low outcomes are on-path, the high type strictly prefers to choose the low action.

Examples of Markov equilibria



Markov equilibria: Refinements

The set of Markov equilibria is quite large.

In particular, there are Markov equilibria where the good type may not choose low actions even when $q_l > 1/2$.

We are working on finding an appropriate refinement with bite.

In particular, we are tying to derive the best Markov equilibrium for the principal.

Discussion: Allowing transfers

Suppose, we augment the game to allow for a round of transfers at the beginning of the stage game.

Here it is trivial for the principal to achieve the first-best.

- The good type makes a transfer of $1/(1-\delta)$ to the principal at the beginning of $t = 1$.
- The low type does not.
- The principal chooses the risky arm forever upon receiving a transfer.
- The high type chooses efficient actions.
- Off-equilibrium, principal switches to the safe arm.

Clearly both types are indifferent and the principal best responds.

It is potentially interesting to study a model of paid-advice with one-way transfers (from the principal to agent).

Concluding remarks

We study a novel dynamic model with adverse selection, moral hazard and no transfers.

We show that, without commitment, minimal quality standards cannot be maintained.

Things to do:

- Finish the paper.
- Study what happens without renegotiation-proofness.
- Examine the other corner case where bad outcomes are perfectly revealing.
- Study a version with private monitoring and communication (which models subscription services).
- Consider a market with competition amongst content providers.