Structural Rationality in Dynamic Games

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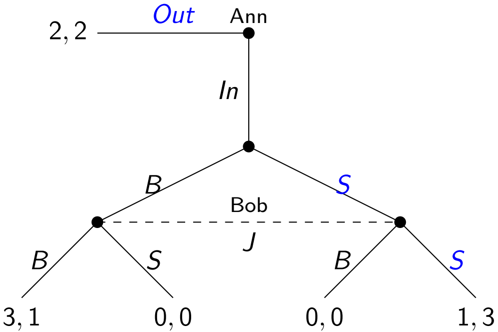
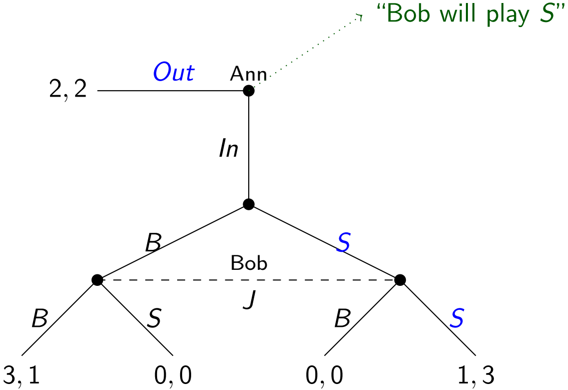
Prelude: Credible Threats



$(\,Out, (S, S)\,)$

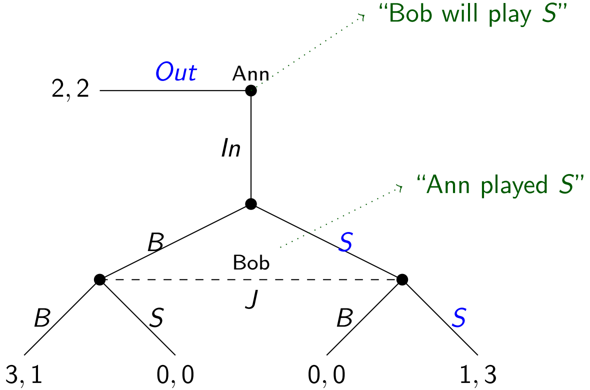
Prelude: Credible Threats



$(\,Out, (S, S)\,)$

- **Threat**: On-path beliefs about off-path play

Prelude: Credible Threats



$$(\,Out, (S, S)\,)$$

- **Threat**: On-path beliefs about off-path play
- **Credible**: Off-path beliefs

This Paper

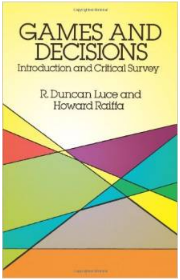
Behavioral content of assumptions on beliefs

Testable implications of solution concepts

in dynamic games

Benchmark: Simultaneous-Move Games

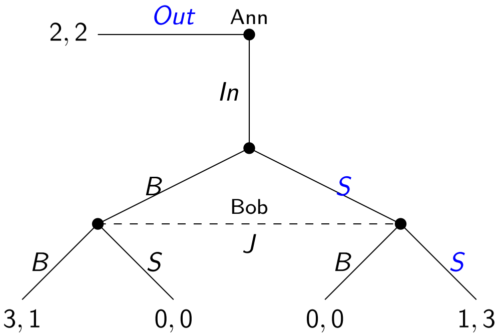
- Luce-Raiffa: elicit beliefs via incentive-compatible **side bets**



- Also practical: e.g. Van Huyck, Battalio, and Beil, 1990; Nyarko and Schotter, 2002. (See also Aumann-Dreze, 2009)

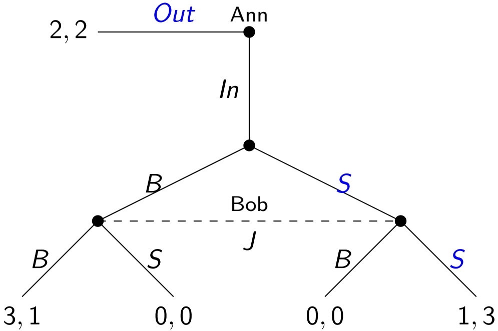
Objective: do the same for **dynamic** games

Eliciting Bob's beliefs in the subgame



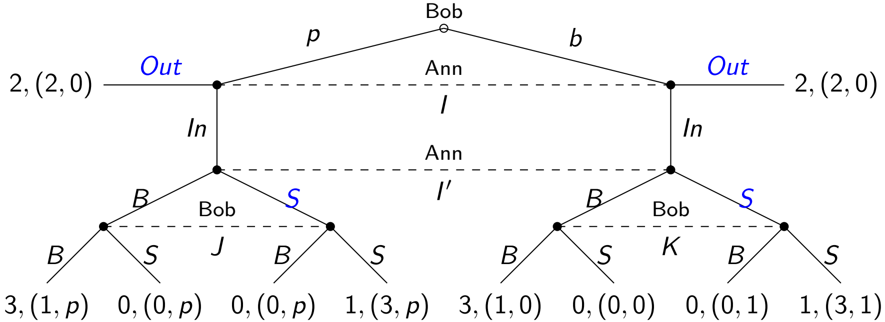
- If subgame reached, could offer side bets on B vs. S
- But in this SPE, the subgame is not reached

Eliciting Bob's beliefs in the subgame



- Could elicit Bob's prior beliefs, then condiiton on "*In*"
- But in this SPE, "*In*" has zero prior probability

Ex-ante conditional bets? (de Finetti)



p close to 1; randomization picks game vs. bet payoff for Bob

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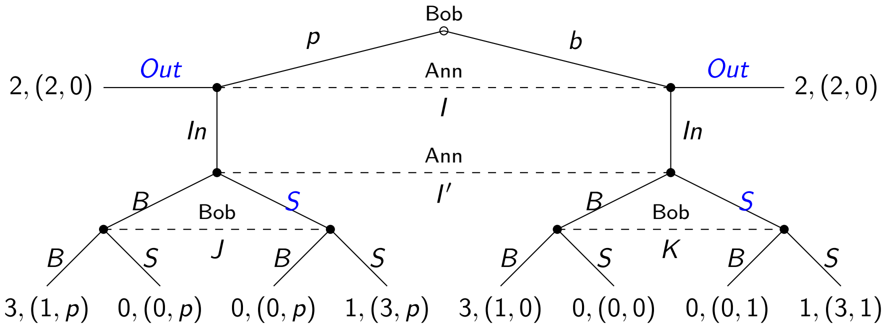

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- Now Bob's bet is always observed

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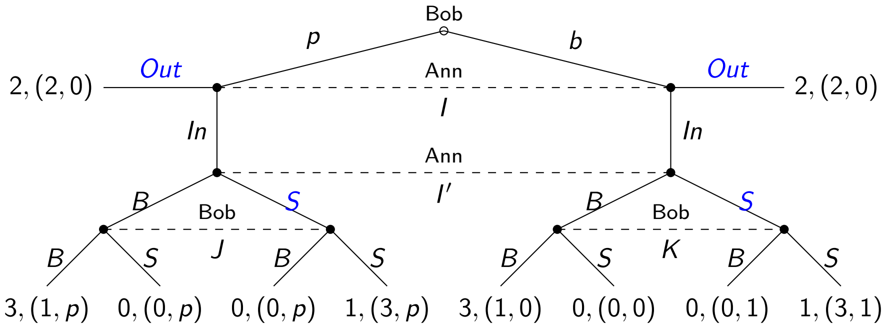

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- Now Bob's bet is always observed
- Sequential rationality: Bob is indifferent between p and b

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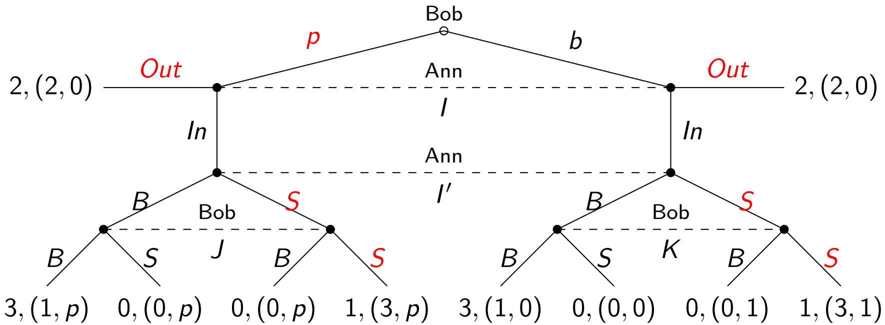
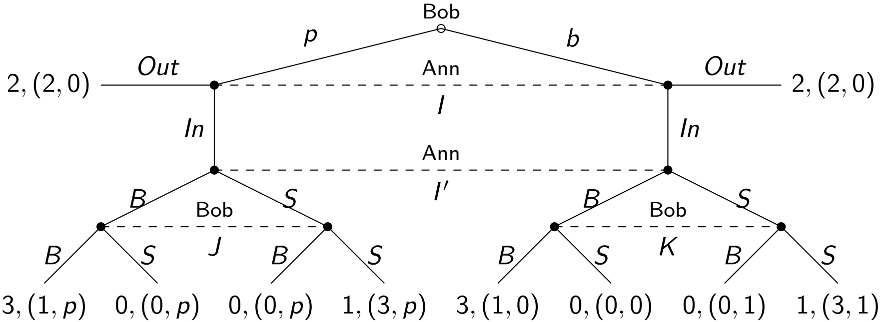


p close to 1; randomization picks game vs. bet payoff for Bob

- Now Bob's bet is always observed
- Sequential rationality: Bob is indifferent between p and b
- $(Out, p, (S, S))$ a sequential equilibrium

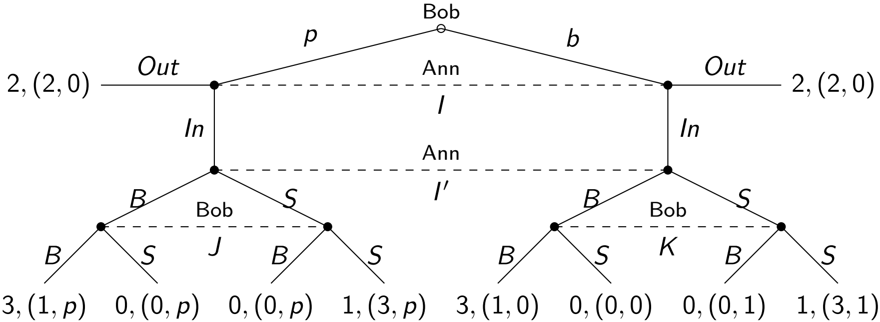
The role of sequential rationality



Sequential rationality: Bob

- reacts optimally to surprises: e.g., if *In*, expect *S* ⇒ play *S*

The role of sequential rationality



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- but need not take into account potential **future** surprises

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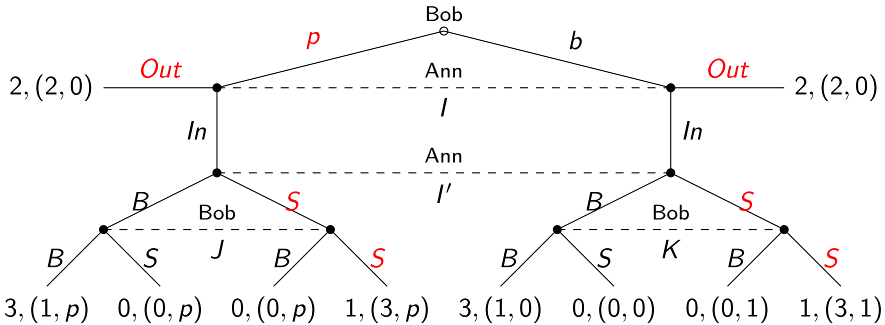

Sequential rationality: Bob

- reacts optimally to surprises: e.g., if *In*, expect $S \Rightarrow$ play S
- but need not take into account potential **future** surprises

e.g., p sequentially rational despite Bob's beliefs following *In*

Structural Rationality

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Every action choice

- takes into account beliefs at all unexpected events
- in a principled way

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Loosely inspired by evidence on strategy method (Selten, 1967)

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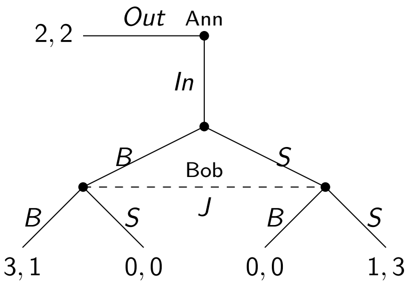
Loosely inspired by evidence on strategy method (Selten, 1967)

Results:

- Implies sequential rationality (generically equivalent)
- Coincides with EU in simultaneous-move games
- Justifies the elicitation of all conditional beliefs
- Characterization via "minimally invasive" trembles

Dynamic games with perfect recall



- Information sets (or nodes): $I, J... \in \mathcal{I}_i$. Root: ϕ, in every $\mathcal{I}_i$
- Strategies $S_a = \{OutB, OutS, InB, InS\}$; $S_b = \{B, S\}$
- Payoff function: $U_i(s_i, s_{-i})$; usual linear extension to $\Delta(S_{-i})$
- Ann's strategies allowing J: $S_a(J) = \{InB, InS\}$;
  $S_a(\phi) = S_a$, $S_a(J)$ are conditioning events

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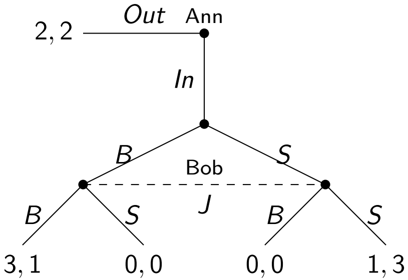

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This talk: "Nested Strategic Information" (paper generalizes)

Beliefs in Dynamic Games

Ann holds beliefs about S_b at each infoset

Definition (Myerson, 1986; Ben-Porath 1997)

A **conditional probability system (CPS)** for i is a collection $\mu = \left\langle \mu\left(\cdot | S_{-i}(I)\right)\right\rangle_{I \in \mathcal{I}_i}$ such that

(1) for all $I \in \mathcal{I}_i$, $\mu\left(\cdot | S_{-i}(I)\right) \in \Delta(S_{-i})$ and $\mu\left(S_{-i}(I) | S_{-i}(I)\right) = 1$
(2) for all $I, J \in \mathcal{I}_i$ and $E \subseteq S_{-i}$ with $E \subseteq S_{-i}(I) \subseteq S_{-i}(J)$:

$$\mu\left(E | S_{-i}(J)\right) = \mu\left(E | S_{-i}(I)\right) \cdot \mu\left(S_{-i}(I) | S_{-i}(J)\right).$$

"Chain rule whenever possible"

Sequential Rationality

Definition (Sequential Rationality à la Reny - Rubinstein)

Fix a CPS μ for player i.

A strategy s_i is sequentially rational (for μ) iff, for all $I \in \mathcal{I}_i$ allowed by s_i, and all t_i that also allow I,

$$U_i\big(s_i, \mu(\cdot|S_{-i}(I))\big) \geq U_i\big(t_i, \mu(\cdot|S_{-i}(I))\big).$$

Structural Rationality

Basic beliefs

Chain rule: if $S_{-i}(I) \subset S_{-i}(J)$ and $\mu(S_{-i}(I)|S_{-i}(J)) > 0$, beliefs at I derived from beliefs at J

Definition

Fix a CPS μ for i.

$I \in \mathcal{I}_i$ is μ-**basic** if $\mu(S_{-i}(I)|S_{-i}(J)) = 0$ for all $J \in \mathcal{I}_i$ with $S_{-i}(J) \supset S_{-i}(I)$

Belief $\mu(\cdot|S_{-i}(I))$ not derived from "earlier" beliefs

Basic beliefs

Chain rule: if $S_{-i}(I) \subset S_{-i}(J)$ and $\mu(S_{-i}(I)|S_{-i}(J)) > 0$, beliefs at I derived from beliefs at J

Definition

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Belief $\mu(\cdot|S_{-i}(I))$ not derived from "earlier" beliefs

$S_{-i}(J) \supset S_{-i}(I)$, $\mu(S_{-i}(I)|S_{-i}(J)) = 0$ also suggest J infinitely more likely than I

Structural Preferences

Definition (Structural Preferences over strategies)

Fix a CPS μ for i. Strategy s_i is **structurally (weakly) preferred** to strategy t_i ($s_i \succcurlyeq^{\mu} t_i$) if, for every μ-basic $I \in \mathcal{I}_i$ with

$$U\big(s_i, \mu(\cdot|S_{-i}(I))\big) < U\big(t_i, \mu(\cdot|S_{-i}(I))\big),$$

there is another μ-basic $J \in \mathcal{I}_i$ with $S_{-i}(J) \supset S_{-i}(I)$ and

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"s_i infinitely more likely to be better than to be worse vs. t_i"

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"Break ties along each path"

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"s_i infinitely more likely to be better than to be worse vs. t_i"

"Break ties along each path"

"Extensive-form analog of lexicographic preferences"

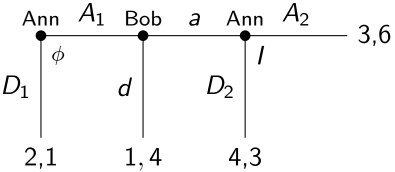
Structural Rationality

Definition (Structural Rationality)

Strategy s_i is **structurally rational for** μ if there is no strategy t_i such that $t_i \succ^\mu s_i$ (that is, $t_i \succsim^\mu s_i$ and not $s_i \succsim^\mu t_i$).

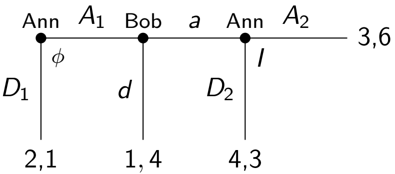
$\succsim^\mu$ possibly incomplete, but transitive: existence guaranteed.

Structural preferences in action



s_a	$[\phi]$	$[I]$
D_1	2	2
$A_1 D_2$	1	4
$A_1 A_2$	1	3

Structural preferences in action



s_a	$[\phi]$	$[l]$
D_1	2	2
$A_1 D_2$	1	4
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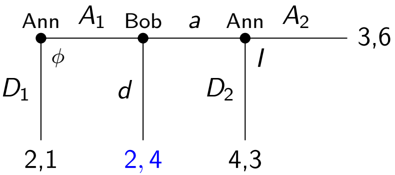
Centipede. $D_1 \succ^{\mu} A_1 D_2 \succ^{\mu} A_1 A_2$

D_1 also unique sequential best reply to μ

Structural preferences in action



s_a	$[\phi]$	$[I]$
D_1	2	2
$A_1 D_2$	2	4
$A_1 A_2$	2	3

Extra power!. $A_1 D_2 \succ^\mu A_1 A_2 \succ^\mu D_1$

Both D_1 and $A_1 D_2$ sequential best replies to μ

"Extensive-form analog of lexicographic preferences"

Features of beliefs	Lexicographic	Structural
Representation	LPS	CPS
Ordering of probabilities	arbitrary	set inclusion
Richness of ordering	complete	partial
Related to extensive form?	no	yes (CPS, basic events)

Main Result 1: Structural implies Sequential

Theorem

Fix a CPS μ for player i. If $s_i \in S_i$ is structurally rational for μ, then it is sequentially rational for μ.

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In static games, structural preferences coincide with EU.
Aligned with experimental evidence!

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In static games, structural preferences coincide with EU.
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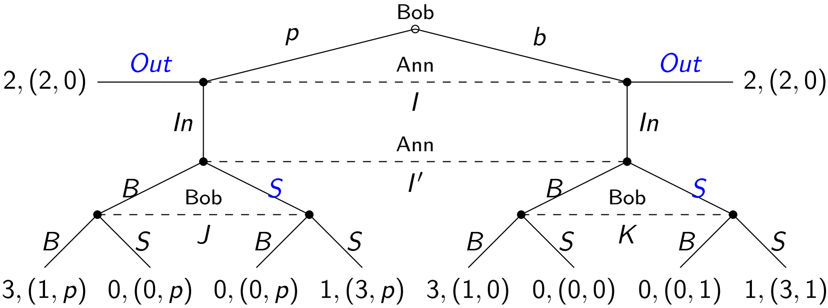
Generic equivalence with sequential rationality

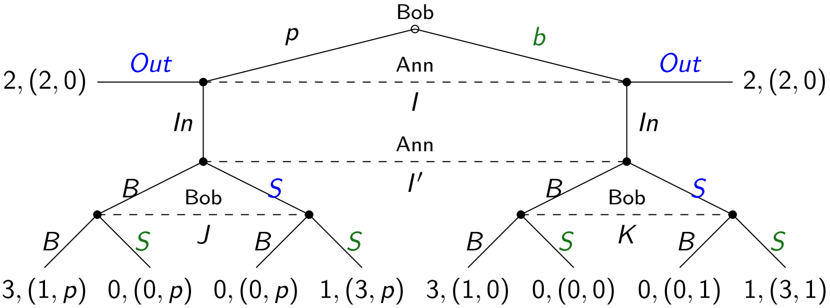
Main Result 2

Elicitation

Back to the Battle of the Sexes



Back to the Battle of the Sexes



s_b	S_a	$S_a(J) = S_a(K)$
pB	$\frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 0$	$\frac{1}{2} \cdot 0 + \frac{1}{2} \cdot p$
pS	$\frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 0$	$\frac{1}{2} \cdot 3 + \frac{1}{2} \cdot p$
bB	$\frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 0$	$\frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 1$
bS	$\frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 0$	$\frac{1}{2} \cdot 3 + \frac{1}{2} \cdot 1$

Main Result 2: Eliciting Off-Path Beliefs (Bob)

Theorem (Elicitation – Bob's beliefs in the subgame)

Fix Ann's CPS μ and Bob's CPS ν in the original game.

In the elicitation game, assume same beliefs about coplayer, independent of Chance's move. Then, given these beliefs:

- *s_a is structurally rational in the elicitation game iff s_a is structurally rational in the original game*
- *if (s_b, b) [resp. (s_b, p)] is structurally rational, then s_b is structurally rational and $\mu(S|S_{-i}(J)) \geq p$ (resp. $\leq p$)*

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- Initial, simultaneous choices reveal bound on Bob's beliefs.

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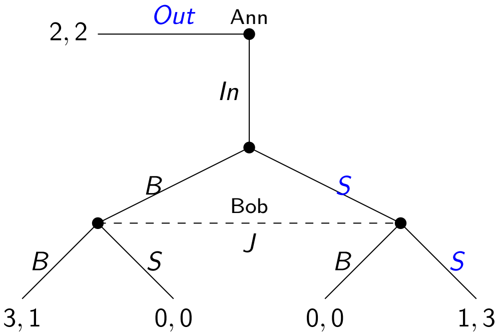
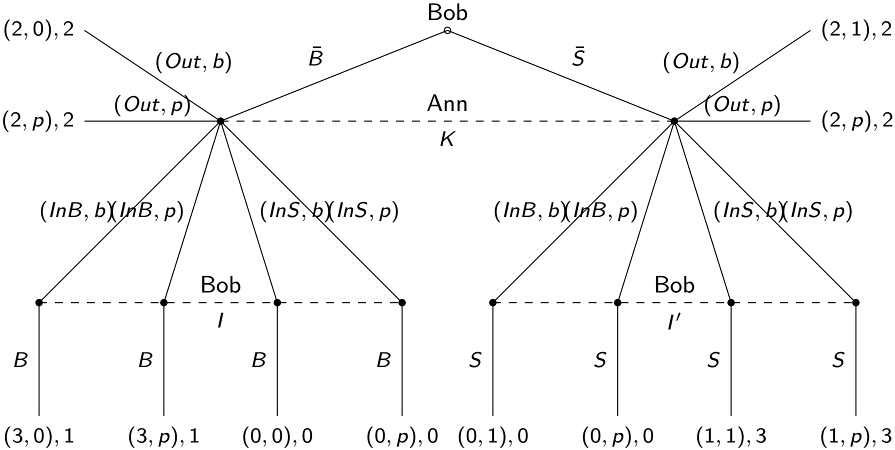
- **Anaologous result in general games**

Eliciting Ann's initial beliefs



- Could offer Ann side bets at ϕ on Bob's choices
- But in this SPE, Ann plays *Out*
- Incentives???

Elicitation and the strategy method



Main Result 2: Eliciting On-path Beliefs (Ann)

Theorem (Elicitation – Ann's initial beliefs)

Fix Ann's CPS μ and Bob's CPS ν in the original game.

In the elicitation game, assume same beliefs about coplayers, independent of Chance's move. Then:

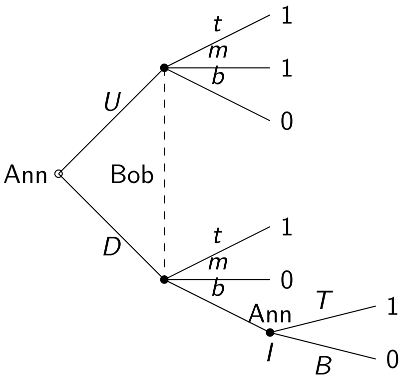
- *s_b is structurally rational in the elicitation game iff s_b is structurally rational in the original game*
- *if (s_a, b) [resp. (s_a, p)] is structurally rational, then s_a is structurally rational and $\mu(S|[[\phi]]) \geq p$ (resp. $\leq p$)*

- Initial, simultaneous choices reveal bound on Ann's beliefs.
- Again, anaologous result for general games

Main Result 3

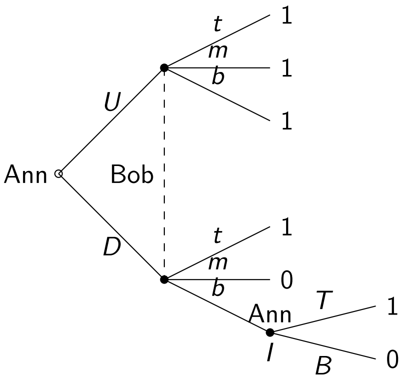
Structural Rationality and Trembles

Perturbations and Spurious Beliefs (1)



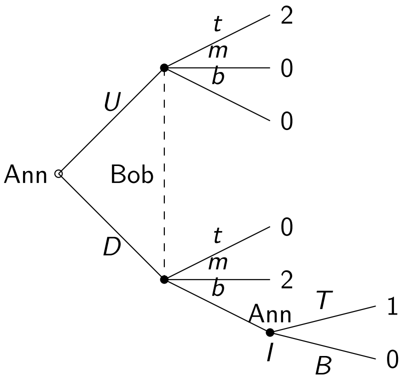
- Ann's CPS: $\mu(t|S_b) = 1$. Then $DT \succ^{\mu} U$.
- Perturbation: $p_\epsilon(t) = 1 - \epsilon - \epsilon^2$, $p_\epsilon(m) = \epsilon$, $p_\epsilon(b) = \epsilon^2$.
- Then $U_a(U, p_\epsilon) > U_a(DT, p_\epsilon)$

Perturbations and Spurious Beliefs (2)



- Ann's CPS: $\mu(t|S_b) = 1$. Then $DT \sim^{\mu} U$.
- Perturbation: $p_\epsilon(t) = 1 - \epsilon - \epsilon^2$, $p_\epsilon(m) = \epsilon^2$, $p_\epsilon(b) = \epsilon$.
- Then $U_a(U, p_\epsilon) > U_a(DT, p_\epsilon)$

Perturbations and Spurious Beliefs (3)



- Ann's CPS: $\mu(t|S_b) = \mu(m|S_b) = \frac{1}{2}$. Then $DT \succ^{\mu} U$.
- Perturbation: $p_\epsilon(t) = \frac{1}{2}$, $p_\epsilon(m) = \frac{1}{2} - \epsilon$, $p_\epsilon(b) = \epsilon$.
- Then $U_a(U, p_\epsilon) > U_a(DT, p_\epsilon)$

Main Result 3: Structural Rationality and Trembles

Definition

$(p^n)_{n\geq 1} \subset \Delta(S_{-i})$ is a **structural perturbation** of μ if

(i) for all $I \in \mathcal{I}_i$, $p^n(S_{-i})(I)) > 0$ and $p^n(\cdot|S_{-i}(I)) \to \mu(\cdot|S_{-i}(I))$;

(ii) $\mathrm{supp}\ p^n = \bigcup_{I\in\mathcal{I}_i} \mu(\cdot|S_{-i}(I))$; and

(iii) $\frac{p^n(\{s_{-i}\})}{p^n(\{t_{-i}\})} = \frac{\mu(\{s_{-i}\}|S_{-i}(I))}{\mu(\{t_{-i}\}|S_{-i}(I))}$ $\forall I \in \mathcal{I}_i$, $s_{-i}, t_{-i} \in \mathrm{supp}\ \mu(\cdot|S_{-i}(I))$.

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(iii) $\frac{p^n(\{s_{-i}\})}{p^n(\{t_{-i}\})} = \frac{\mu(\{s_{-i}\}|S_{-i}(I))}{\mu(\{t_{-i}\}|S_{-i}(I))}$ $\forall I \in \mathcal{I}_i$, $s_{-i}, t_{-i} \in \mathrm{supp}\ \mu(\cdot|S_{-i}(I))$.

Theorem

$s_i \in S_i$ is structurally rational for μ iff, for every $t_i \in S_i$, there is a structural perturbation (p^n) of μ such that $U(s_i, p^n) \geq U_i(t_i, p^b)$ for all $n \geq 1$.

Conclusions

New optimality criterion: **Structural Rationality**

- Implies sequential rationality: the extensive form matters!
- Allows the elicitation of all conditional beliefs
- Also justifies the strategy method
- As a bonus, sometimes refines sequential rationality
- Characterization via "minimally invasive" **trembles**
- General games: Newcomb paradox, KW consistency
- Easy to add payoff uncertainty and higher-order beliefs

Papers

Now at http://faculty.wcas.northwestern.edu/∼msi661

Sequential Rationality and Elicitation (this talk):
"Structural Preferences and Sequential Rationality"

Axiomatics:
"Foundations for Structural Preferences"

Ask me:

Forward induction
"Structural Preferences in Epistemic Game Theory"

THANK YOU!

Nested Strategic Information (1)

Recall: $S_{-i}(I)$ = strategies of opponents reaching I

Assumption (Nested strategic information)

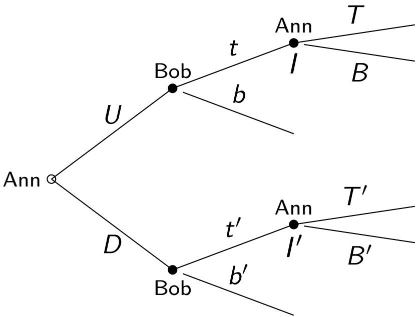
For every real player i and infosets I, J of i,

either $S_{-i}(I) \cap S_{-i}(J) = \emptyset$ or $S_{-i}(I) \subseteq S_{-i}(J)$ or $S_{-i}(J) \subseteq S_{-i}(I)$.

- Signalling games
- Games where a player moves only once on each path
- Games with centipede structure
- Ascending-clock auctions
- Event trees

Nested Strategic Information (2)

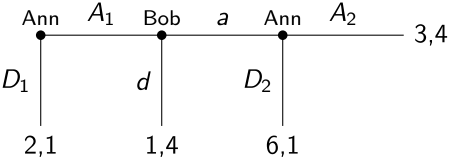
Rules out:



$S_{-i}(I) = \{tt', tb'\}$; $S_{-i}(I') = \{tt', bt'\}$. Not nested.

back

How about trembles? Removing actions?



Mechanical trembles: **no**

- Change the game (a fortiori if remove actions—e.g. D_1)
- Impact strategic reasoning (Reny, Ben-Porath, Bagwell)
- Also: which trembles (Binmore)? Details matter!

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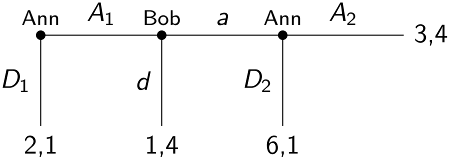

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Belief perturbations (Kreps - Wilson, 1982): **yes!**

- Proposed approach also models infinitesimal probabilities
- Paper: novel (to me) implications of KW-style consistency

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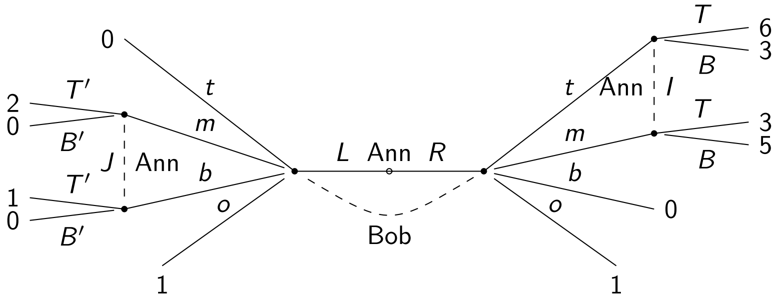
Structural Rationality for General Games

The issue



Non-nested strategic information: $[I] \not\supseteq [J], [J] \not\supseteq [I]$

$\mu(o|S_b) = 1; \quad \mu(t|[I]) = \mu(m|[I]) = \frac{1}{2}; \quad \mu(m|[J]) = \mu(b|[J]) = \frac{1}{2}$

RB is "structurally rational:" see payoff given $\mu(\cdot|[J])$

Yet, *RB* is **not** sequentially rational!

Step 1: Likelihood ordering

$[J] \supset [I]$, $\mu([I]|[J]) = 0$ suggests J "infinitely more likely" than I

Notice $\mu([J]|[I]) > 0$ (indeed, 1) because $[J] \supset [I]$.

Generalize: even if $[I]$, $[J]$ not nested, $\mu([J]|[I]) > 0$, suggests $[J]$ "not infinitely less likely" than $[I]$

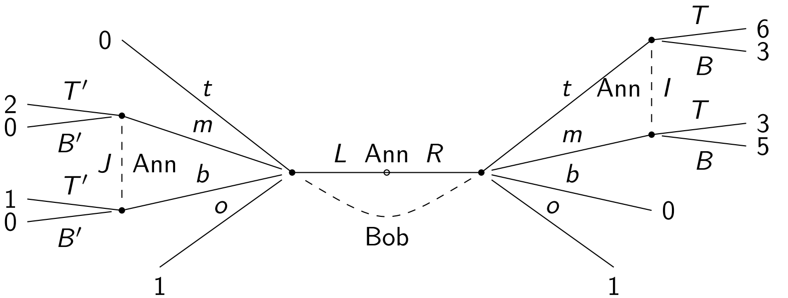
Likelihood should be transitive. Hence:

Definition (Likelihood ordering)

$[J] \geq^{\mu} [I]$ iff there are $I_1, \ldots, I_L \in \mathcal{I}_i$ with $I_1 = I$, $I_L = J$, and

$$\mu([I_{\ell+1}]|[I_\ell]) > 0 \qquad \ell = 1, \ldots, L-1.$$

Step 2: Basic event — back to the example



$\mu(o|S_b) = 1; \quad \mu(t|[I]) = \mu(m|[I]) = \frac{1}{2}; \quad \mu(m|[J]) = \mu(b|[J]) = \frac{1}{2}$

Definition of likelihood implies $S_b >^{\mu} [I] =^{\mu} [J]$. Intuitive!

$\mu(\cdot|[I]), \mu(\cdot|[J])$ are updates of uniform prob on $[I] \cup [J] = \{t, m, b\}$

Take $[I] \cup [J]$ as basic event: prob uniqely identified from μ!

Step 2: Basic events — definition

Definition (CPS on general conditioning events)

Fix a CPS μ for i and consider $\geq^{\mu}$. Let

$$\mathcal{G}_i = \left\{ \cup_{k=1}^{K} [I_k] \; : \; K \in \mathbb{N}, \; , [I_k] =^{\mu} [I_\ell] \;\; \forall \ell, k = 1, \ldots, K \right\}.$$

The **extension** of μ is a CPS ν on S_{-i} with conditioning events $\mathcal{G}_i$ such that

$$\forall I \in \mathcal{I}_i, \quad \nu(\cdot | [I]) = \mu(\cdot | [I]).$$

Note: $[I] \in \mathcal{G}_i$ for all $I \in \mathcal{I}_i$.

Existence and uniqueness of basis: later, or ask me.

Step 3: General Structural Preferences

Definition (Structural Preferences over strategies)

Fix a CPS μ for player i that admits an extension μ. Strategy s_i is **structurally (weakly) preferred** to strategy t_i ($s_i \succcurlyeq^{\mu} t_i$) if, for every $F \in \mathcal{G}_i$ with

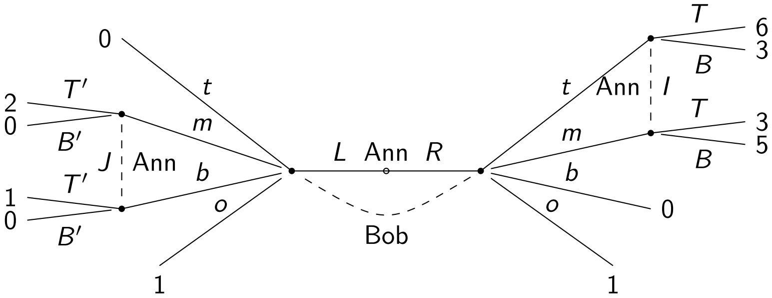
$$\int U(s_i, s_{-i}) d\nu(s_{-i}|F) < \int U(t_i, s_{-i}) d\nu(s_{-i}|F),$$

there is $G \in \mathcal{G}_i$ with $G \geq^{\nu} F$ and

$$\int U(s_i, s_{-i}) d\nu(s_{-i}|G) > \int U(t_i, s_{-i}) d\nu(s_{-i}|G).$$

Same as before, but using extension ν instead of μ

Structural preferences in action



$\mu(o|S_b) = 1; \quad \mu(t|[I]) = \mu(m|[I]) = \frac{1}{2}; \quad \mu(m|[J]) = \mu(b|[J]) = \frac{1}{2}$

Likelihood: $S_b >^\mu [I]$, $S_b >^\mu [J]$, $[I] =^\mu [J]$

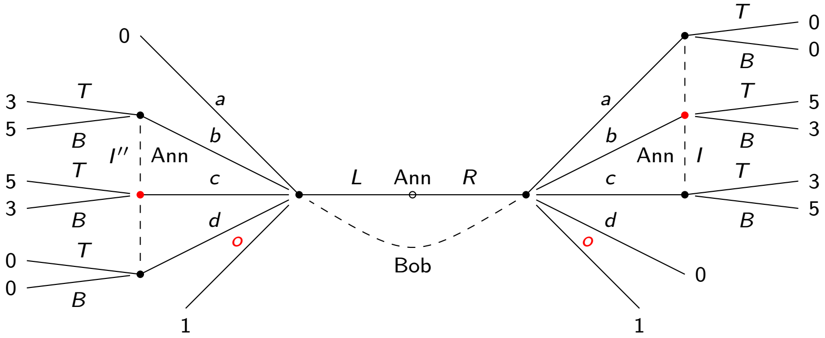
$\mathcal{G}_a = \{S_b, [I], [J], [I] \cup [J]\}$. Extension: $\nu(\cdot|[I] \cup [J])$ uniform

Basic events for ν: $S_b, [I] \cup [J]$

$RT \succ^\mu RB \succ^\mu LT' \succ^\mu LB'$. RT structurally rational; unique

Congruent CPSs and Extensions

A Newcombe Paradox for CPSs



CPS: $\mu(o|S_b) = 1$; $\mu(b|[I]) = 1$, $\mu(c|[I']) = 1$

Set of sequential best replies: LT, RT.

Kreps-Wilson consistency, Myerson complete CPSs:
$\{LT, RT\}$ cannot be the set of sequential best replies

Indeed μ does not admit an extension!

Main Result 3: Congruent CPSs

μ is **congruent** if, for every $(F_m)_{n=1}^{N}$ with $\mu(F_{n+1}|F_n) > 0$, $n = 1, \ldots, N-1$, and every $E \subseteq F_1 \cap F_N$,

$$\mu(E|F_1) \cdot \prod_{n=1}^{N-1} \frac{\mu(F_n \cap F_{n+1}|F_{n+1})}{\mu(F_n \cap F_{n+1}|F_n)} = \mu(E|F_N)$$

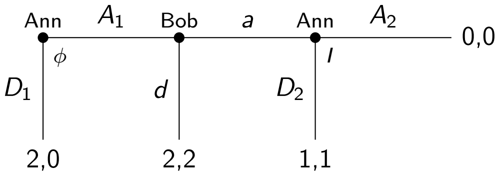
Congruence implies the Chain Rule: take $F_1 \subset F_2$.

Theorem

The following are equivalent:

- μ *is congruent*
- μ *is generated by taking limits of strictly positive probabilities*
- μ *admits an extension, which is unique*

Structural preferences in action: Extra Power!



$$S_b \supset [I] = \{a\};\ \mu(d|S_b) = 1;\ \mu(a|[I]) = 1.$$

s_a	S_b	$[I] = \{a\}$
D_1	2	2
$A_1 D_2$	2	1
$A_1 A_2$	2	0

$$D_1 \succ^{\mu} A_1 D_2 \succ^{\mu} A_1 A_2$$

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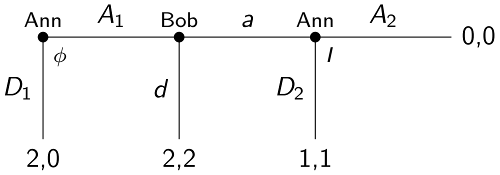
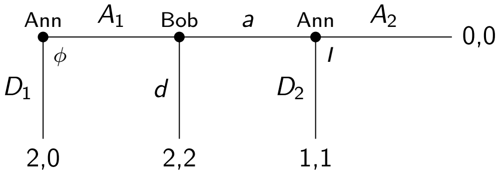


$$S_b \supset [I] = \{a\};\ \mu(d|S_b) = 1;\ \mu(a|[I]) = 1.$$

s_a	S_b	$[I] = \{a\}$
D_1	2	2
$A_1 D_2$	2	1
$A_1 A_2$	2	0

Yet $A_1 D_2$ sequentially rational: at I, no longer care about D_1

Structural preferences in action: Extra Power!



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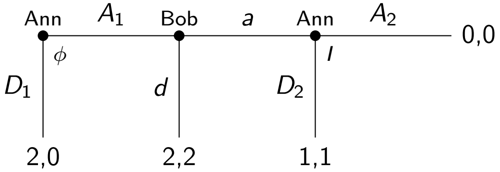
s_a	S_b	$[I] = \{a\}$
D_1	2	2
$A_1 D_2$	2	1
$A_1 A_2$	2	0

$D_1 \succ^{\mu} A_1 D_2$ reflects ex-ante view: at ϕ, can still choose D_1

Structural preferences in action: Extra Power!



$S_b \supset [l] = \{a\}$; $\mu(d|S_b) = 1$; $\mu(a|[l]) = 1$.

s_a	S_b	$[l] = \{a\}$
D_1	2	2
$A_1 D_2$	2	1
$A_1 A_2$	2	0

($D_1 \succ^{\nu} A_1 D_2$ for any CPS ν — not just this μ)