How to Count Citations If You Must

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by

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Citation Indices

- Citation indices are used (together with other criteria) to rank scholars for various purposes, e.g., the awarding of prizes, grants, fellowships, visiting positions, and even tenure.

- A citation index maps any finite list of nonnegative integers $x_1, x_2, \ldots, x_n$ into a real number, $\iota(x_1, x_2, \ldots, x_n)$.

- The vector $(x_1, x_2, \ldots, x_n)$ describes the record of a particular scholar, where x_i is the number of citations received by the i-th paper (publication) in the list.

- Citation indices, by definition, reduce a scholar's record to a one-dimensional statistic; clearly a loss of information. (field, journal, co-authors, paper age, academic age,…)

Examples of Citation Indices

- The total citation count is the sum of all citations across all papers.
- The i10-index is the number of papers with at least 10 citations.
- The h-index is the largest number, h, of papers with at least h citations.

- These indices are well-intentioned.

  Total is "unbiased;" i10 counts only "significant" papers;

  h-index "guards against" few but highly cited papers.

- But all are *ad hoc* rules of thumb – why not an i20-index? Or the largest number g such that the total citation count of the g most cited papers is at least g^2. (This is an actual index, the "g-index.")

- An axiomatic approach seems appropriate here.

The Rescaling Problem

- Comparing scholars across fields is difficult.
- There is general agreement that some adjustment to the lists of scholars from different fields is needed to "properly" compare them.
- Dividing by the average number of citations per paper in the field appears particularly pertinent. (equalizes citation distributions)
- But, under the h-index, this can reverse the ranking of scholars within the same field.
- E.g. IO_1: 20 papers each with 40 citations h=20

  IO_2: 25 papers each with 15 citations h=15

  Macro: h=22

Macro economists receive 1.8 times as many citations as IO economists; so must multiply IO economists' lists by 1.8 to compare across fields.

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The Axioms/Properties

- A citation index maps any finite list of nonnegative integers $x_1 \geq x_2 \geq \cdots \geq x_n$ into a real number, $\iota(x_1, x_2, \dots, x_n)$.

We seek an index that satisfies the following properties.

P1. (Zero) The index is unchanged if a paper with zero citations is added.

P2. (Monotonicity) The index does not fall if a paper receives more citations.

P3. (Independence) The index's ranking of two lists does not change when a paper with the same number of citations is added to each list.

P4. (Depth Relevance) It is not the case that, for every list, the index weakly increases when any paper in the list is split into two and its citations are divided in any way between them.

P5. (Scale invariance) The index's ranking of two lists does not change when each entry of each list is multiplied by any common positive scaling factor.

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- One can conduct a thought experiment to consider the effect on an index of growth in a field.
- Suppose that there is just a single field and that every scholar in the field is cloned.
- The field now has twice as many scholars. Each scholar has a twin who has written exactly the “same” papers and who cited exactly the same other papers.
- It seems natural that, with any such “balanced” doubling of the field, no scholar’s ranking vis a vis any other scholar should have changed.
- But every scholar now has twice as many citations.
- Hence, for such balanced increases in the size of a field to have no ranking effects, the index would have to satisfy scale invariance.

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P6. (Continuity) If $\iota(x) < \iota(y)$, then $\iota(nx + \mathbf{1}) < \iota(ny)$ for all sufficiently large n.

P7. (Directional Consistency) If $\iota(x) = \iota(y)$ and $\iota(x + r) = \iota(y + r)$, then $\iota(x + \lambda r) = \iota(y + \lambda r)$ for every $\lambda > 1$.

The Euclidean Index

- The Euclidean index assigns to any citation list, $(x_1, x_2, \ldots, x_n)$, its Euclidean length, i.e.,

$$\iota_E(x_1, x_2, \ldots x_n) = \sqrt{x_1^2 + \cdots + x_n^2}\ .$$

- Say that two citation indices are equivalent iff they always agree on the ranking of any two citation lists.

(i.e., iff each one is a positive monotonic transformation of the other)

Theorem.

A citation index satisfies zero, monotonicity, independence, depth relevance, scale invariance, continuity, and directional consistency if and only if it is equivalent to the Euclidean index.

- RePEc now includes the Euclidean index as a ranking tool (https://ideas.repec.org/top/)

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Radicchi, F., S. Fortunato, and C. Castellano (2008): “Universality of citation distributions: Toward an objective measure of scientific impact,” *Proceedings of the National Academy of Sciences*, 105 (45):17268-17272.

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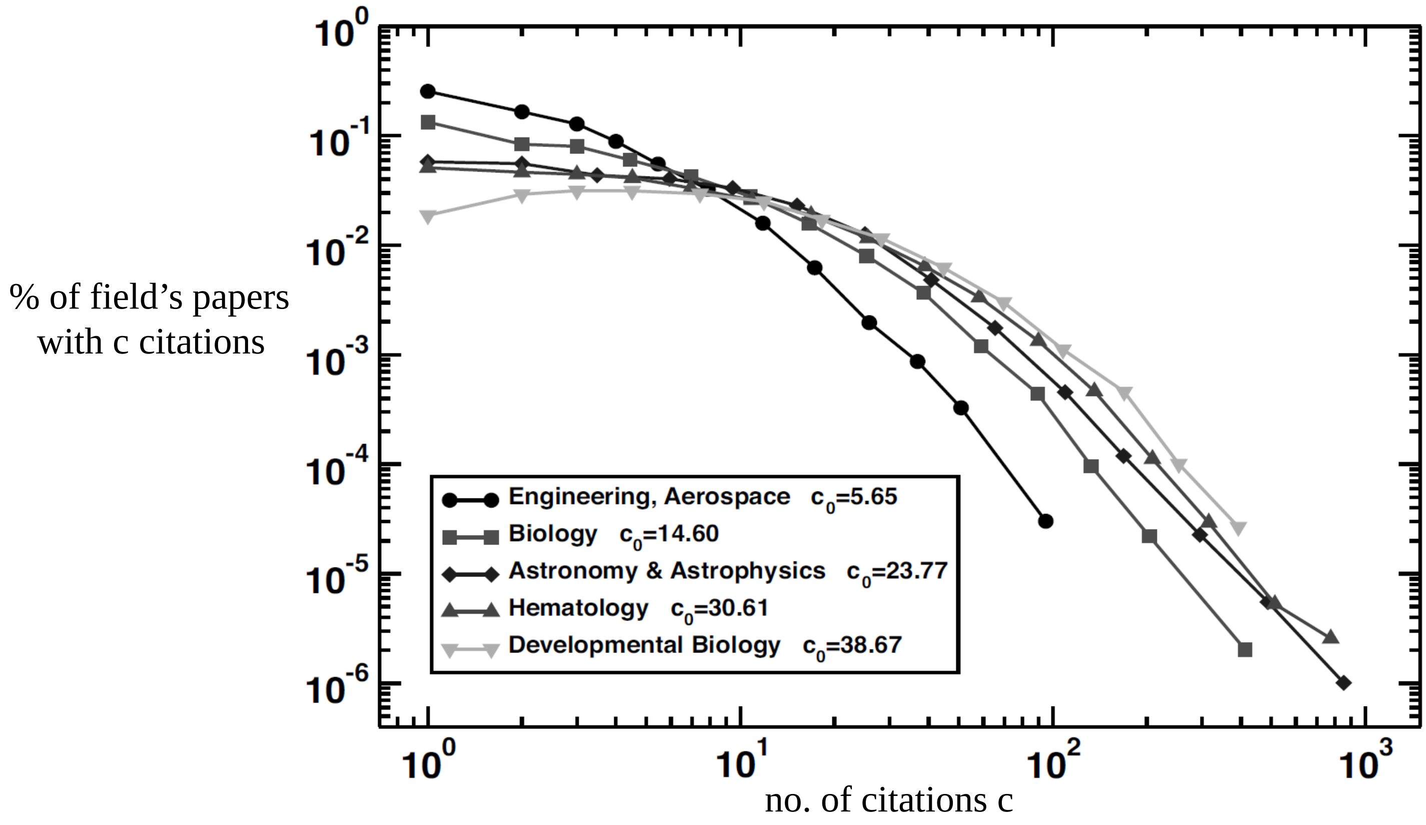

Fig. 1: Histograms of raw citations across fields in 1999, where c_0 is the average number of citations per paper published in 1999 in that field. (From Radicchi et. al., 2008).

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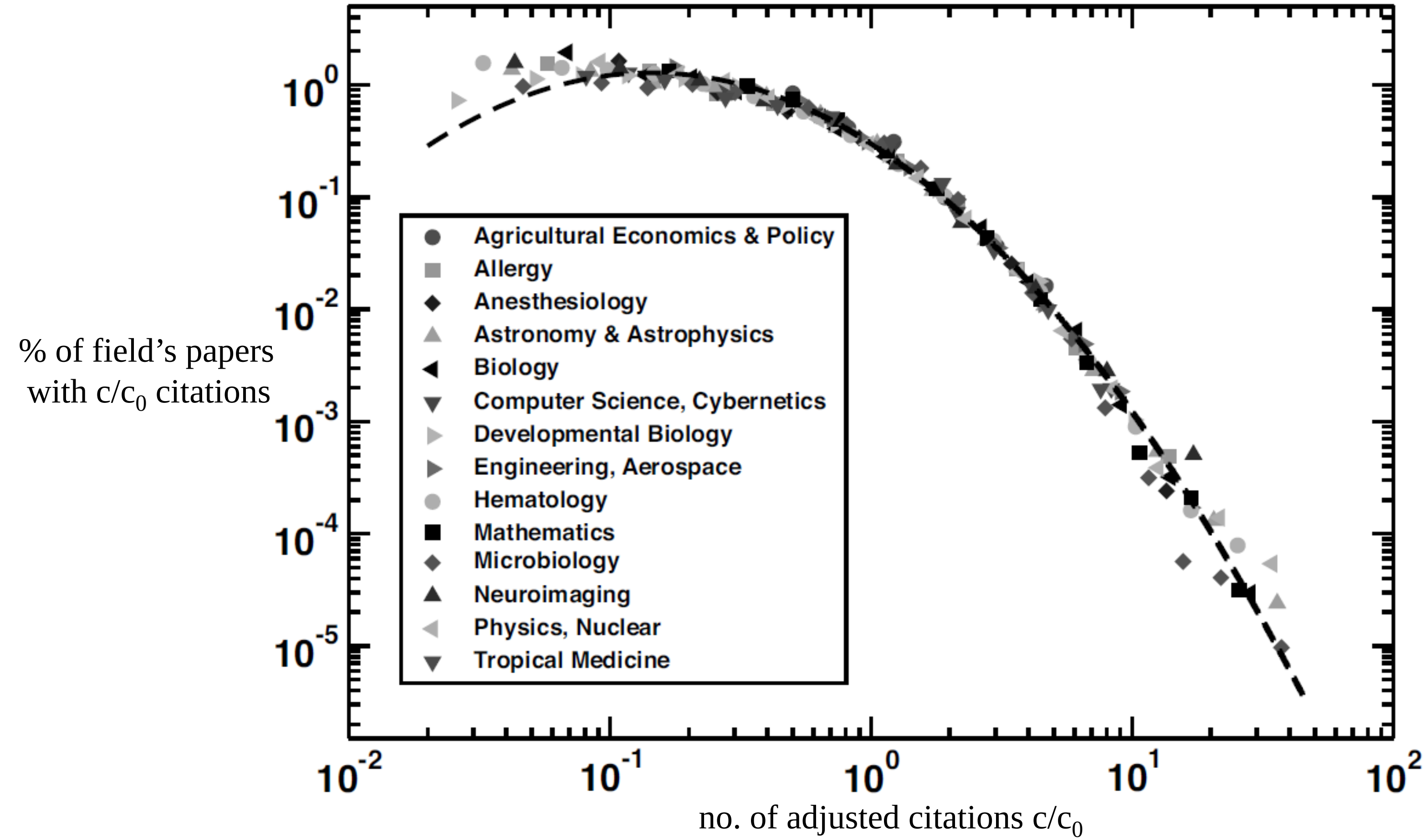

Fig. 2: Histograms of adjusted citations across fields in 1999. (From Radicchi et. al., 2008).

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- But every scholar now has twice as many citations.
- Hence, for such balanced increases in the size of a field to have no ranking effects, the index would have to satisfy scale invariance.
- Note that if fields grow at different rates but in roughly balanced ways, and if fields have the same citation distributions when they are the same size, then rescaling fields by their average number of citations will equate the citation distributions of fields of different sizes. Does this explain Raddicchi et. al.?

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- What if we do not include the directional consistency property?

Theorem. A citation index satisfies zero, monotonicity, independence, depth relevance, scale invariance, and continuity if and only if it is equivalent to an index of the form,

$$(x_1^\sigma + \cdots + x_n^\sigma)^{1/\sigma}, \text{where } \sigma > 1.$$

- Thus, directional consistency pins down the value of σ to $\sigma = 2$.
- How does the Euclidean index perform in practice?

A Rank Correlation Test

- For each pair of scholars, award the index +1 if its ranking of them matches the NRC's ranking of the their departments and award -1 if the rankings are opposed.

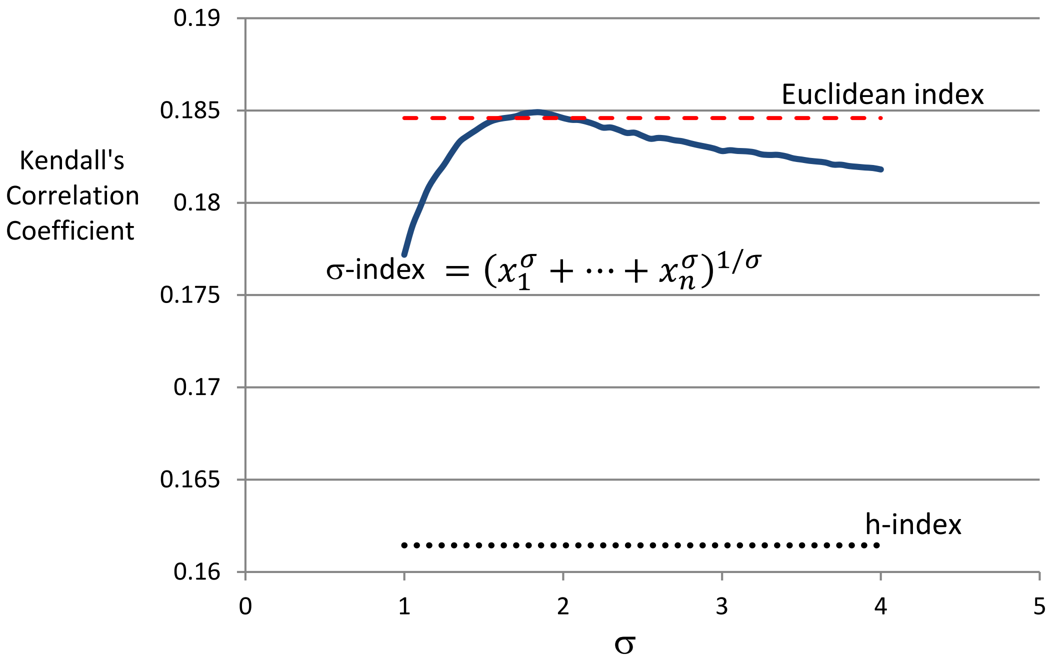

Fig. 3: The Euclidean index outperforms the h-index in matching labor market data.

- So the value $\sigma = 2$ is nearly optimal in this empirical test.

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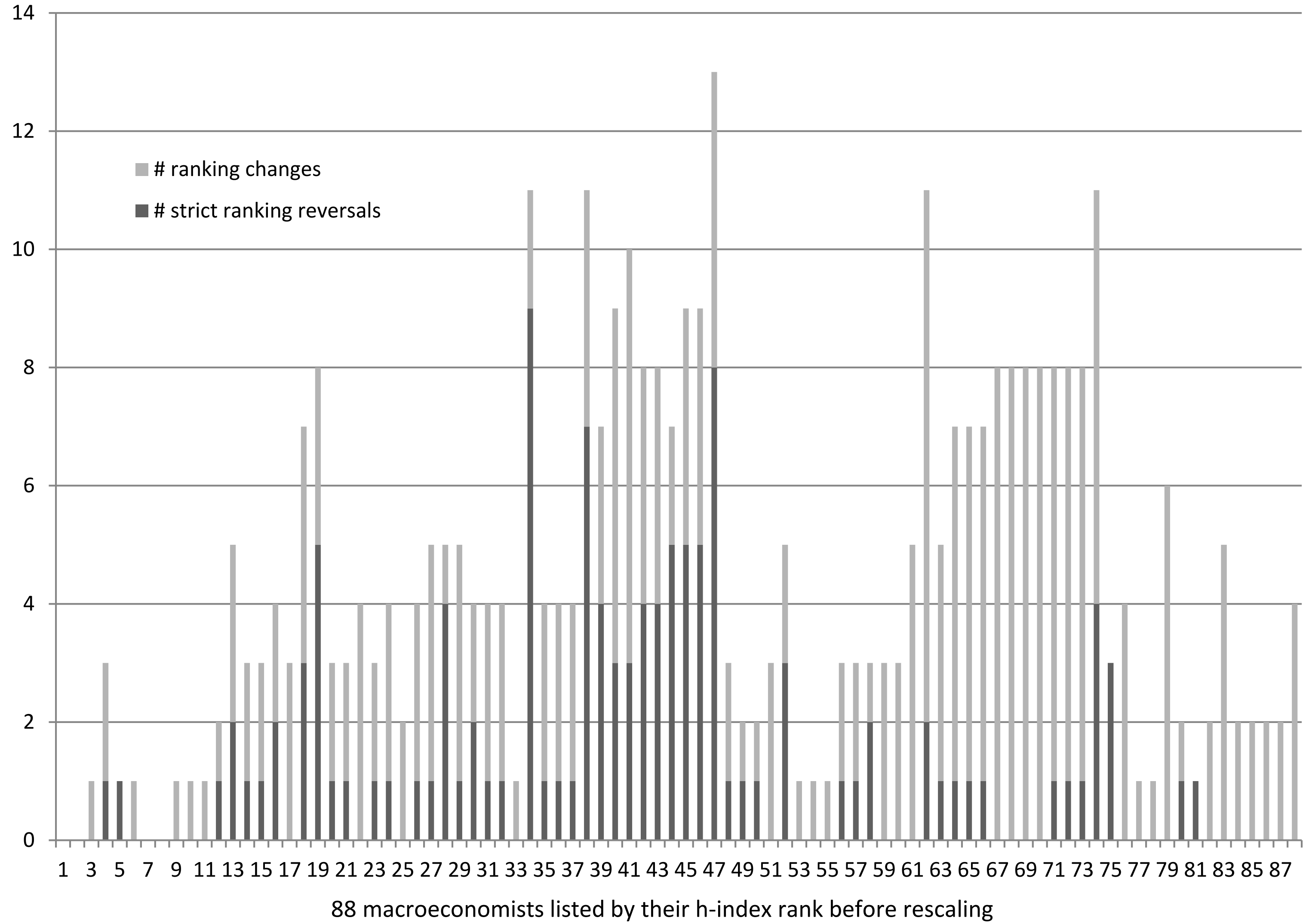

Fig. 4: The h-index is susceptible to ranking changes after rescaling for differences in fields.