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## Dynamic Liquidity-Based Security Design

### Emre Ozdenoren<sup>1</sup> Kathy Yuan<sup>2</sup> Shengxing Zhang<sup>3</sup>

 $^{1}\text{LBS}$  and CEPR,  $^{2}\text{LSE}$  and CEPR,  $^{3}\text{LSE}$ 

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## Motivation

- The determinants of liquidity in a dynamic economy
  - random productivity or endowment shocks
  - adverse selection
  - type of liquidity technology: optimal security design
- The amount of liquidity
  - repo contracts
  - haircuts, interest rates

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## Key Mechanism

- Adverse selection
  - Without collateral borrowers cannot commit to paying back.
  - Productive assets provide liquidity because they can be used as collateral but are subject to adverse selection.
- Inter-temporal feedback
  - Collateral value depends on the re-sale value of the asset.
  - Re-sale value itself depends on the collateral value of the asset.
  - Leads to fragility and volatility in asset price and real output.

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## Main Results

### • Equity contracts: Fragility and self-fulfilling

- Pooling equilibrium: more liquidity and output
- Separating equilibrium: less liquidity and output
- Multiple equilibria
- Security design: liquid repo-debt contract (under monotone payoff constraints)
  - Unique equilibrium: both high and low types issue repo-debt and debt is liquid; low type issues equity
    - Eliminates fragility and improves liquidity
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- Agent O has a CRS z-technology which produces z > 1 units of <u>consumption</u> goods with one <u>intermediate</u> good (capital, equipment) from Agent I
- Agent I produces intermediate goods 1-to-1 from labor
- Both have a basic technology that produces <u>consumption</u> good 1-to-1 from labor
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Introduction	Model	Equilibrium	Results
Utilities			

- Utility in period t is  $U_t(x, l) = x l$
- x is the consumption good
- / is labor
- Discount rate between periods  $\beta$ , with  $0 < \beta < 1$ .

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## Timing

### • Three dates in each period.

### • Date 1: Intermediate good is produced

- perishes at the end of the period
- no direct utility

### • Date 2: Consumption good is produced

- via the productive or basic technology.
- Date 3: Consumption takes place.
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Model

Equilibrium

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- Long lived asset pays *s* units of dividend as consumption good at date 3.
- Fixed supply of the asset is A.
- With prob.  $\lambda$  dividend distribution is  $F_L$  and  $1 \lambda$  it is  $F_H$ .

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- $F_L, F_H \in \Delta[s_L, s_H], \ 0 \le s_L < s_H$
- $F_H$  first order stochastically dominates  $F_L$
- Quality  $Q \in \{H, L\}$  is i.i.d. over time
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## Collateral Asset and Adverse Selection

- Agent O uses the asset as collateral to borrow intermediate goods from agent I.
- Agent *O* privately observes asset quality *O* at the beginning of each period.
  - Adverse selection is within the period
  - Shown later, agent *O* purchases all collateral assets in equilibrium
  - Privately informed about the quality because of
    - opportunity to temper with quality
    - incentive to acquire private information

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## Trading Environment: Two Markets

### $\bullet\,$ Markets for intermediate goods at date 1

- An agent O randomly meets at least two agent Is
- in decentralized market(s)
- intermediate goods are traded for asset-based securities
- i.e, borrowing against some forms of securities takes place
- Market for the collateral asset at date 3
  - After state is realized, asset price,  $\phi_t$ , is determined
  - in a *centralized* market
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## Security Design

- Security Design is conducted ex ante before types are realised.
- An asset-backed security  $y^{j}(s)$  is a state-contingent promise of consumption goods at date 3.
- Two cases of interest:
  - Equity  $y(s) = s + \phi_t, \forall s \in [s_L, s_H]$
  - Set of monotone securities  $\mathscr{I}_t(\phi_t) = \{y : y(s) \text{ increasing in } s, y(s) \le s + \phi_t, \forall s \in [s_L, s_H] \}$

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## Equilibrium in Security *j*'s Market

- Security trading occurs at date 1:
  - bilaterally between agent O and multiple agent Is
  - in dedicated sub-markets for each available security.
- Suppose agent I bids per-unit price  $q_t^j$  for security j.
- If highest bid, agent O offers him  $a_t^Q(j)$  units of security j for  $q_t^j a_t^Q(j)$  intermediate goods.
- In equilibrium, winning bid  $q_t^j$ 
  - agent I: zero expected gain due to Bertrand Competition
  - IC for agent O: profitable for an informed O agent type

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#### Model

Equilibrium

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## Equilibrium in Security *j*'s Market

• Adverse selection index: higher  $R_t^j$ , lower adverse selection

$$R_t^j \equiv \frac{E_L y_t^j}{E_H y_t^j}$$

• Expected value of security j when both O types participate

$$\overline{q}^{j} = \lambda E_{L} y_{t}^{j} + (1 - \lambda) E_{H} y_{t}^{j}$$

• High O type participates if adverse selection is low:

$$z\overline{q}^{j}-E_{H}y_{t}^{j}\gtrless$$
 0iff  $R\gtrless \zeta$ 

where  $\zeta\equiv 1-(z-1)/\lambda z$ 

Equilibrium

Results

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## Equilibrium Price in Security *j*'s Market

Security Design: Objective

Before learning asset quality, agent O chooses security design  $\mathscr{J}_t(\phi_t) \subseteq \mathscr{I}_t(\phi_t)$  to maximize

$$\begin{aligned} \mathcal{V}_{o,t}(a) &= \lambda \int \left( \sum_{j \in \mathscr{J}_t(\phi_t)} a_t^L(j) \left[ z q_t^j - y_t^j(s) \right] \right) dF_L(s) \\ &+ (1 - \lambda) \int \left( \sum_{j \in \mathscr{J}_t(\phi_t)} a_t^H(j) \left[ z q_t^j - y_t^j(s) \right] \right) dF_H(s) \\ &+ \int a(s + \phi_t) d \left[ \lambda F_L(s) + (1 - \lambda) F_H(s) \right] \end{aligned}$$

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Introduction		Model	Equilibrium	Results
Security	Design:	Constraints		

- Each O type optimally chooses how much to supply:
  - Low type O agent always sells all since  $E_L y_t^j(s) \leq E_H y_t^j(s)$

$$a_t^L(j) = a ext{ and } a_t^H(j) = \begin{cases} a & ext{if } R_t^j \ge \zeta \\ 0 & ext{if } R_t^j < \zeta \end{cases}$$

• The security design must be overall feasible

$$\sum_{j \in \mathscr{J}_t(\phi_t)} y_t^j(s) d\mu_{o,t} \le s + \phi_t$$

• Price is determined via Bertrand competition

$$q_t^j = \begin{cases} \lambda E_L y_t^j + (1 - \lambda) E_H y_t^j & \text{if } R_t^j \ge \zeta \\ E_L y_t^j & \text{if } R_t^j < \zeta \end{cases}$$

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## Dynamic Security Design Equilibrium

A stationary dynamic equilibrium consists of

- $\mathscr{J}_t(\phi_t)$  solves the security design problem
- security price  $q_t^j$  satisfies the submarket Bertrand competition
- asset price  $\phi_t$  solves the Euler equation given by:

$$\phi_t = \beta \left[ z \left( \sum_{j \in P_t} q_t^j + \lambda \sum_{j \in \mathscr{J}_t(\phi_t) \setminus P_t} q_t^j \right) + (1 - \lambda) \sum_{j \in \mathscr{J}_t(\phi_t) \setminus P_t} E_H y_t^j \right]$$

where  $j \in P_t$  iff  $R_t^j \ge \zeta$ .

## Benchmark: Dynamic Lemons Market

• Collateral asset is the only security: No security design

- security price depends on  $\frac{E_L s + \phi_t}{E_H s + \phi_t} \gtrsim \zeta$  :
  - Pooling:  $q_t^P = \phi_t + \lambda E_L s + (1 \lambda) E_H s$
  - Separating:  $q_t^S = \phi_t + E_L s$  otherwise.
- asset price depends on  $\frac{E_L s + \phi_t}{E_H s + \phi_t} \stackrel{\geq}{=} \zeta$  :
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• Separating:  $\phi^{S} = \beta \left[ z\lambda q_{t}^{S} + (1-\lambda) \left( \phi^{S} + E_{H}s \right) \right] = \frac{\beta \left[ \lambda zE_{L}s + (1-\lambda)E_{H}s \right]}{1-\beta (\lambda z+1-\lambda)}$ •  $\phi^{P} > \phi^{S} > PV = \frac{\beta \left[ \lambda E_{L}s + (1-\lambda)E_{H}s \right]}{1-\beta}$ 

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## Fragility of Dynamic Lemons Market

### • There can be multiple equilibria in a dynamic lemons market.

# • Occurs when $\frac{E_L s + \phi^S}{E_H s + \phi^S} < \zeta \le \frac{E_L s + \phi^P}{E_H s + \phi^P}$ .

- Plugging for  $\phi_S$  and  $\phi_P$  we obtain the condition for multiplicity as  $\frac{\zeta - \beta}{1 - \beta} < \frac{E_L s}{E_{LS}} < \frac{\zeta - \beta \left[1 - (1 - \lambda) (z - 1)\right]}{1 - \beta \left[1 - (1 - \lambda) (z - 1)\right]}$
- Easy to see that for intermediate values of  $E_{LS}/E_{HS}$  both equilibria exist.

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$$\frac{E_L s + \phi^S}{E_H s + \phi^S} < \zeta \leq \frac{E_L s + \phi^P}{E_H s + \phi^P}$$
.

- Plugging for  $\phi_S$  and  $\phi_P$  we obtain the condition for multiplicity as  $\frac{\zeta - \beta}{1 - \beta} < \frac{E_L s}{E_H s} < \frac{\zeta - \beta \left[1 - (1 - \lambda) \left(z - 1\right)\right]}{1 - \beta \left[1 - (1 - \lambda) \left(z - 1\right)\right]}$
- Easy to see that for intermediate values of  $E_{LS}/E_{HS}$  both equilibria exist.

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## Intuition for Dynamic Multiplicity

- There is a dynamic feedback loop.
- If agents anticipate the asset to be traded in a pooling eqm in the decentralized market, then price is high.
- In turn, when the price is high, the H-type O agent is willing to pool.
- If agents anticipate the asset to be traded in a separating eqm in the decentralized market, price is low.
- In turn, when the price is low, the H-type keeps the asset.

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- We call a security traded in a pooling equilibrium in the decentralized market a liquid security.
- First we show that Agent *O* is weakly better-off selling only one liquid security.
- This is because if two securities are both liquid, combination is also liquid and generates at least as much value for Agent O.
- Also if security design is optimal, the feasibility constraint is binding.
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Equilibrium

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## Optimality of Debt

#### Proposition

Assume that  $\frac{f_L(s)}{f_H(s)}$  is decreasing in *s*. The optimal security is a unique standard debt contract  $y_D$  such that

$$y_D(s) = \phi + \min(s, s^*),$$

for some  $s^* \in (s_L, s_H)$ .

## Characterizing the Debt Contract

### Proposition

Assume that 
$$\frac{f_L(s)}{f_H(s)}$$
 is decreasing in s.

• If 
$$\frac{E_L s}{E_H s} < 1 - \frac{z-1}{z} \frac{1}{\lambda(1-\beta)}$$
,  
• ie, the separating region in the dynamic lemons market  
• a unique equilibrium and non-trivial tranching with  
 $D \in (s_L, s_H)$  and  $\phi$  solve:  
 $\phi = \frac{z}{z-1} \lambda \int_{s_L}^{D} \left[ \widetilde{F}_H(s) - \widetilde{F}_L(s) \right] ds - \int_{s_L}^{D} \widetilde{F}_H(s) ds - s_L$  (1)  
 $\phi = \frac{\beta}{1-\beta z} \left\{ z [\lambda E_L s + (1-\lambda) E_H s] - (1-\lambda)(z-1) \int_{D}^{s_H} \widetilde{F}_H(s) ds \right\}$ 
(2)

• Otherwise, a "pass-through security" that promises the entire value of the asset and replicates the pooling equilibrium in dynamic lemons market

• 
$$D = s_H$$
 and  $\phi = \frac{\beta}{1-\beta z} z [\lambda E_L s + (1-\lambda)E_H s]$ .  $\Rightarrow \quad e \Rightarrow \quad$ 

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## Discussions on Liquidity and Fragility

- We show that security design equilibrium Pareto dominates all equilibria of the case in dynamic lemons market
  - more liquidity, more real output and less fragility
  - even if only issue a "pass-through security" that mimics equity

     replicate the pooling

Model

Equilibrium

Results

## Eliminates Low Liquidity Equilibrium



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## Discussions on Fragility and Robustness

- Unravelling results when security design option is introduced.
  - Suppose low asset price,
  - tranche a small senior liquid debt, asset price ↑, which allows more liquid tranching D↑, which leads to asset price ↑, ... converges to optimal.

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## Repo Features

- Face value:  $D + \phi$
- Repo rate:  $\frac{D+\phi-q_D}{q_D}$
- Haircut: equity tranch,  $q_E$

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## Conclusion

Optimal security design in a dynamic lemons market

- Unique equilibrium: both high and low types issue repo-debt and debt is liquid; low type issues equity
- Eliminates fragility and improves liquidity
- Improves social welfare relative to the separating equilibrium under equity contract
- Endogenous amplification of shocks to asset quality and productivity