# Optimal Discriminatory Disclosure

Yingni Guo
Northwestern

Li, Hao
UBC

Xianwen Shi
Toronto

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# Disclosure with Privately Informed Buyer

- One seller, one buyer, and one object:
  - buyer has private information $\theta$ about value $\omega$;
  - seller can partially or fully disclose additional signal about $\omega$; disclosure is private: signal realization is observable only to buyer;
  - seller chooses selling mechanism and information policy to maximize the revenue.

- Questions:
  1. does ex post private information generate rent?
  2. can disclosure policy help extract ex ante rent?
  3. can non-discriminatory disclosure policy be optimal?
  4. what is the jointly optimal selling mechanism and disclosure policy?

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# Closely Related Papers

- Eso and Szentes (2007)
  - orthogonal decomposition: ex ante and ex post information.
  - full disclosure attains revenue with observable ex post information under regularity conditions (Krahmer and Strausz 2015: irrelevance theorem fails with discrete ex ante types).
  - $\Rightarrow$ full disclosure is optimal if disclosure is either all or nothing.
  - interpret that full disclosure remains optimal w/o this qualification.

- Li and Shi (2017)
  - allow seller to directly garble her "original" information.
  - show that full disclosure is suboptimal.
  - implicit in ES approach: seller can only garble "orthogonal" shock.
  - do not have characterization of optimal disclosure policy.

- Guo and Shmaya (2017)
  - disclosure to privately informed agent without transfers.
  - optimal discriminatory disclosure has an interval structure.
  - equivalent to non-discriminatory disclosure.

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# Main Results

- Optimal disclosure is a pair of (nested) intervals.
- Optimal disclosure is often discriminatory.
- Information discrimination has to interact with price discrimination in order to be effective in extracting rent.

# Related Literature

- Private disclosure without ex ante private information: Lewis and Sappington (1994), Che (1996), Ganuza (2004), Anderson and Renault (2006), Johnson and Myatt (2006), Ganuza and Penalva (2010), Bergemann and Pesendorfer (2007)
- Private disclosure with privately informed buyer: Eso and Szentes (2007), Hoffman and Inderst (2011), Bergemann and Wambach (2015), Krahmer and Strausz (2015), Li and Shi (2017), Lu and Ye (2017), Bergemann, Bonatti and Smolin (2018), Zhu (2017), Krahmer (2018), Smolin (2018)
- Bayesian persuasion with privately informed receiver: Kolitilin, Mylovanov, Zapechelnyuk and Li (2017), Guo and Shmaya (2017)

# Model Setup

- Buyer's true valuation for the object: $\omega \in [\underline{\omega}, \overline{\omega}]$.
  - buyer privately observes signal $\theta \in \{\theta_L, \theta_H\}$ about $\omega$.
  - $\omega|\theta_i \sim F_i(\omega)$ and $\phi_i = \Pr\{\theta = \theta_i\}$, $i = L, H$.
  - likelihood ratio order: $f_H(\omega)/f_L(\omega)$ is increasing in $\omega$.
- Seller controls all additional information about $\omega$.
  - seller can release signal $s$ to buyer, without observing its realization.
  - buyer observes realization of $s$, and forms posterior estimate of $\omega$.
- Seller's reservation value for the object is $c \in (\underline{\omega}, \overline{\omega})$.
- Both parties are risk-neutral.

# Disclosure Policy

- Signal structure $\langle S, \rho \rangle$: signal space $S$, mapping $\rho : [\underline{\omega}, \overline{\omega}] \to \Delta S$.
- Examples of signal structures:
  - interval structure: signal space $S = \{s_-, s_+\}$ and mapping

$$\rho(s|\omega) = \begin{cases} 1 & \text{if} \quad s = s_- \text{ and } \omega \notin \left[\underline{k}, \overline{k}\right], \\ 1 & \text{if} \quad s = s_+ \text{ and } \omega \in \left[\underline{k}, \overline{k}\right], \\ 0 & \qquad\qquad \text{otherwise,} \end{cases}$$

    where $\underline{\omega} \leq \underline{k} < \overline{k} \leq \overline{\omega}$.
  - threshold structure: interval structure with $\overline{k} = \overline{\omega}$.
- Disclosure policy $\sigma(\theta) : \{\theta_L, \theta_H\} \to \Sigma$ (set of signal structures).

# (Deterministic) Mechanism and Timing

- Disclosure policy $(\sigma(\theta))$ and a menu of contracts $(a(\theta), p(\theta))$:
  - $\sigma(\theta)$: signal structure assigned for reported type $\theta$.
  - $a(\theta)$: advance payment for reported type $\theta$.
  - $p(\theta)$: strike price for reported type $\theta$.
- Period 1:
  - $\omega$ is realized, and buyer privately observes $\theta$.
  - seller commits to $(\sigma(\theta))$ together with $(a(\theta), p(\theta))$.
  - buyer reports $\tilde{\theta}$, pays $a(\tilde{\theta})$, and is assigned $\sigma(\tilde{\theta})$.
- Period 2:
  - buyer observes realized signal $s$ from $\sigma(\tilde{\theta})$.
  - buyer forms posterior estimate and decides whether to buy at $p(\tilde{\theta})$.

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# Restriction to Binary Signal Structures

- Deterministic mechanism $\Rightarrow$ buyer's period 2 decision is binary.
- No loss to focus on binary signal structures with

$$S = \{\text{“buy”}, \text{“not buy”}\}.$$

- Denote information structure for $\theta_i$ by $\rho_i(\omega) \in [0,1]$, $i = L, H$
  - probability of recommending reported type $\theta_i$ to buy at state $\omega$.

# Classical Sequential Screening

- Information about $\omega$ is fully released to the buyer in period 2.
- Under full disclosure, IR$_L$ and IC$_H$ bind in optimum, and revenue is

$$\underbrace{\phi_H \int_{p_H}^{\overline{\omega}} (\omega - c) dF_H(\omega)}_{\text{trading surplus of type } \theta_H}$$

$$+$$

$$\underbrace{\phi_L \int_{p_L}^{\overline{\omega}} (\omega - c) dF_L(\omega)}_{\text{trading surplus of type } \theta_L} - \underbrace{\phi_H \int_{p_L}^{\overline{\omega}} (F_L(\omega) - F_H(\omega)) d\omega}_{\text{information rent of type } \theta_H}$$

- $p_L$ plays a dual role: determining terms of trade and allocation

# Full Disclosure Not Optimal

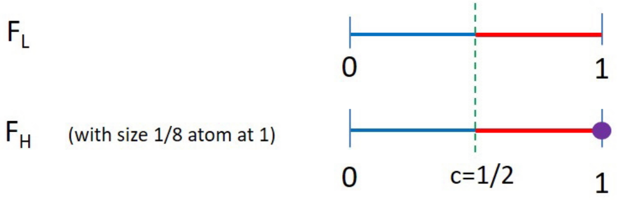
1. Take optimal menu $(a_i, p_i)$ under full disclosure.
2. Consider alternative disclosure policy and menu of contracts:
   - for type $\theta_H$, release $\omega$ fully and retain strike price $p_H$;
   - for type $\theta_L$, offer threshold structure with cutoff $p_L$, raise strike price to $\widehat{p}_L = p_L + \varepsilon$ with $\varepsilon > 0$, and choose $\widehat{a}_L$ to bind IR$_L$.
3. Deviation payoff for type $\theta_H$:

$$-\widehat{a}_L + \underbrace{(1 - F_H(p_L))}_{> \theta_L\text{'s trading prob. } 1 - F_L(p_L)} \times \left(\mathbb{E}_H[\omega | \omega \geq p_L] - \widehat{p}_L\right)$$

   $\Rightarrow$ price hike hurts deviating $\theta_H$ more than $\theta_L$ $\Rightarrow$ raise $a_H$.
4. Same allocation (hence trading surplus), but lower rent.

# Example: Threshold Structure Not Optimal

- Binary types: $F_L$ is uniform but $F_H$ has an atom at $1$




- Strict interval structure (for type $\theta_L$) extracts full surplus:

$$\rho_L(\omega) = \begin{cases} 1 & \text{if} \quad \omega \in \left(\frac{1}{2}, 1\right), \\ 0 & \text{if} \quad \omega \in \left[0, \frac{1}{2}\right] \text{ or } \omega = 1. \end{cases}$$

- Multiplicity: any $p_L \in \left[\frac{5}{12}, \frac{3}{4}\right]$ is optimal ; possible for $p_L < c$.

# Seller's Optimization Problem

$$\max_{(\rho_i(\omega), a_i, p_i)} \sum_{i=H,L} \phi_i \left( a_i + (p_i - c) \int_{\underline{\omega}}^{\overline{\omega}} \rho_i(\omega)\, dF_i(\omega) \right),$$

subject to IC constraints:

$$-a_i + \int_{\underline{\omega}}^{\overline{\omega}} (\omega - p_i)\, \rho_i(\omega)\, dF_i(\omega)$$

$$\geq -a_j + \max \left\{ \int_{\underline{\omega}}^{\overline{\omega}} (\omega - p_j)\, \rho_j(\omega)\, dF_i(\omega),\ \underbrace{\mathbb{E}_i[\omega] - p_j}_{\text{double deviation}} \right\}, \qquad (\text{IC}_i)$$

IR constraints

$$-a_i + \int_{\underline{\omega}}^{\overline{\omega}} (\omega - p_i)\, \rho_i(\omega)\, dF_i(\omega) \geq 0, \qquad (\text{IR}_i)$$

and bounds on $p_i$ so truthful types buy iff upon observing "buy":

$$\mathbb{E}_i[\omega | \text{“not buy”}, \rho_i] \leq p_i \leq \mathbb{E}_i[\omega | \text{“buy”}, \rho_i]. \qquad (\text{PB}_i)$$

# Solution Procedure

- Drop IC$_L$ (but retain IR$_H$) to form a relaxed problem
  - here IR$_H$ does not follow from IR$_L$ and IC$_H$.
- At any solution to the relaxed problem:
  1. IR$_L$ and IC$_H$ must bind;
  2. Strike price $p_L \geq \mathbb{E}_H[\omega|\rho_L(\omega) = 0]$;
  3. IC$_L$ is satisfied if $p_L \geq \mathbb{E}_H[\omega|\rho_L(\omega) = 0]$.
- Use IR$_L$ and IC$_H$ to rewrite the relaxed problem, with constraints:
  1. IR$_H$ (non-negative rent);
  2. revised bounds on $p_L$: $\mathbb{E}_H[\omega|\rho_L(\omega) = 0] \leq p_L \leq \mathbb{E}_L[\omega|\rho_L(\omega) = 1]$.
- Allocation and information $\rho_H(\cdot)$ to type $\theta_H$ are efficient.

# Optimal Policy for Low Type

- Optimal $(\rho_L(\cdot), p_L)$ maximizes

$$\phi_L \int_{\underline{\omega}}^{\overline{\omega}} (\omega - c)\rho_L(\omega)\, dF_L(\omega) - \phi_H \int_{\underline{\omega}}^{\overline{\omega}} (\omega - p_L)(f_H(\omega) - f_L(\omega))\rho_L(\omega)\, d\omega$$

subject to:

$$\int_{\underline{\omega}}^{\overline{\omega}} (\omega - p_L)(f_H(\omega) - f_L(\omega))\rho_L(\omega)\, d\omega \geq 0 \qquad (\text{IR}_H)$$

$$\mathbb{E}_H[\omega | \rho_L(\omega) = 0] \leq p_L \leq \mathbb{E}_L[\omega | \rho_L(\omega) = 1]. \qquad (\text{BP}_L)$$

- Note that the objective increases (decreases) in $p_L$ if

$$\underbrace{\int_{\underline{\omega}}^{\overline{\omega}} f_H(\omega)\rho_L(\omega)\, d\omega}_{\text{type } \theta_H \text{ off-path trading prob.}} \geq (\leq) \underbrace{\int_{\underline{\omega}}^{\overline{\omega}} f_L(\omega)\rho_L(\omega)\, d\omega}_{\text{type } \theta_L \text{ on-path trading prob.}}.$$

# Two Cases: Regular vs Irregular

- Regular case:
  - deviating $\theta_H$'s trading probability $\geq$ truthful $\theta_L$'s trading probability.
  - optimal strike price $p_L$ hits the upper-bound $\mathbb{E}_L\left[\omega|\rho_L(\omega)=1\right]$.
- Irregular case:
  - deviating $\theta_H$'s trading probability $\leq$ truthful $\theta_L$'s trading probability.
  - optimal strike price $p_L$ hits the lower-bound $\mathbb{E}_H\left[\omega|\rho_L(\omega)=0\right]$.

# Regular Case

### Theorem

*Let* $\omega_o \in (\underline{\omega}, \overline{\omega})$ *denote the crossing point of* $f_H$ *and* $f_L$*. Suppose* $c \geq \omega_o$*. If there exists* $\beta > 0$ *such that*

$$f_H(\omega) - f_L(\omega) \geq \beta(\omega - \omega_o) f_L(\omega),$$

*then the optimal disclosure policy is a pair of nested intervals with*

$$\begin{aligned} \rho_H(\omega) &= 1 \text{ if } \omega \in [c, \overline{\omega}] \text{ and } 0 \text{ otherwise} \\ \rho_L(\omega) &= 1 \text{ if } \omega \in \left[\underline{k}, \overline{k}\right] \text{ and } 0 \text{ otherwise} \end{aligned}$$

*where* $c \leq \underline{k} < \overline{k} \leq \overline{\omega}$*.*

- Threshold structure if $\overline{k} = \overline{\omega}$ and strict interval if $\overline{k} < \overline{\omega}$.

# Intuition for Sufficient Condition

- Recall that optimal $\left(\rho_L(\cdot), p_L\right)$ maximizes

$$\phi_L \int_{\underline{\omega}}^{\overline{\omega}} (\omega - c)\rho_L(\omega)\, dF_L(\omega) - \phi_H \int_{\underline{\omega}}^{\overline{\omega}} (\omega - p_L)\left(f_H(\omega) - f_L(\omega)\right)\rho_L(\omega)\, d\omega$$

- Must be better than not serving type $\theta_L$ at all:

$$\begin{aligned} 0 &< \int_{\underline{\omega}}^{\overline{\omega}} (\omega - c)\rho_L(\omega)\, dF_L(\omega) \\ &\leq \int_{\underline{\omega}}^{\overline{\omega}} (\omega - \omega_o)\rho_L(\omega)\, dF_L(\omega) \\ &\leq \beta \int_{\underline{\omega}}^{\overline{\omega}} (f_H(\omega) - f_L(\omega))\rho_L(\omega)\, dF_L(\omega) \end{aligned}$$

- But $\int_{\underline{\omega}}^{\overline{\omega}} (f_H(\omega) - f_L(\omega))\rho_L(\omega)\, dF_L(\omega) > 0$ implies regular case

# Intuition for Excluding Top States

- Optimal $p_L$ and $\rho_L$ maximize the difference:

$$\underbrace{\phi_L \int_{\underline{\omega}}^{\overline{\omega}} (\omega - c)\, \rho_L(\omega) f_L(\omega)\, d\omega}_{\text{trading surplus of type } \theta_L} - \underbrace{\phi_H \left( \int_{\underline{\omega}}^{\overline{\omega}} (\omega - p_L)\, \rho_L(\omega) f_H(\omega)\, d\omega - a_L \right)}_{\text{information rent of type } \theta_H}$$

- For given $p_L$, type $\theta_L$ is recommended to buy at state $\omega$ if

$$\phi_L(\omega - c) - \phi_H(\omega - p_L)\left(\frac{f_H(\omega)}{f_L(\omega)} - 1\right) \geq 0.$$

- If $f_H(\overline{\omega})/f_L(\overline{\omega}) \gg 1$ and/or $\phi_H/\phi_L$ is high, excluding top states, not costly in trading surplus with $\theta_L$, can substantially reduce rent.

# Threshold vs Strict Interval

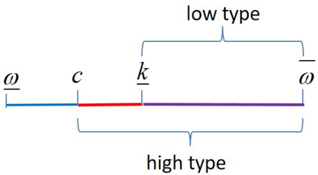
- PROPOSITION Threshold structure is optimal for type $\theta_L$ if it is regular and

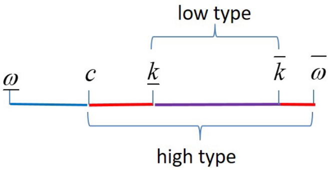
$$\frac{\phi_L}{\phi_H} \geq \frac{f_H(\overline{\omega})}{f_L(\overline{\omega})} - \frac{1 - F_H(c)}{1 - F_L(c)}.$$

- PROPOSITION Strict interval structure is optimal if it is regular, $\phi_L$ is sufficiently small and

$$\left(\frac{f_H(\omega)}{f_L(\omega)}\right)''\Bigg|_{\omega=\overline{\omega}} \geq \frac{3}{\overline{\omega} - c} \left(\frac{f_H(\omega)}{f_L(\omega)}\right)'\Bigg|_{\omega=\overline{\omega}}.$$

# (Non)-Equivalence of Optimal Discriminatory and Nondiscriminatory Disclosure




- Equivalence holds if threshold structure is optimal for $\theta_L$
  - refined partition with $p_H = c$ and $p_L = \mathbb{E}_L\left[\omega | \omega \in [\underline{k}, \overline{\omega}]\right]$.
- Equivalence may fail if strict interval structure is optimal for $\theta_L$
  - non-discriminatory: deviating $\theta_H$ follows recommendation only if

$$\mathbb{E}_H\left[\omega | \omega \in [c, \underline{k}] \cup [\overline{k}, \overline{\omega}]\right] \leq p_L;$$

  - discriminatory: deviating $\theta_H$ follows recommendation only if

$$\mathbb{E}_H\left[\omega | \omega \in [\underline{\omega}, \underline{k}] \cup [\overline{k}, \overline{\omega}]\right] \leq p_L.$$

# Example: Nondiscriminatory Disclosure Not Optimal

- Let $c = \omega_o = 0.8$ and $\phi_L = 0.5$; common support $[0,1]$ with

$$f_L(\omega) = \begin{cases} 1 - 4(\omega - 0.8) & \text{if} \quad \omega \geq 0.8 \\ 1 - \frac{1}{4}(\omega - 0.8) & \text{if} \quad \omega < 0.8 \end{cases}$$

$$f_H(\omega) = \begin{cases} 1 + 4(\omega - 0.8) & \text{if} \quad \omega \geq 0.8 \\ 1 + \frac{1}{4}(\omega - 0.8) & \text{if} \quad \omega < 0.8 \end{cases}$$

- Optimal disclosure: $\underline{k} \approx 0.85$ and $\bar{k} \approx 0.97$

$$\mathbb{E}_H\left[\omega|\omega \in [0,\underline{k}] \cup [\bar{k},1]\right] < p_L \text{ but } \mathbb{E}_H\left[\omega|\omega \in [c,\underline{k}] \cup [\bar{k},1]\right] > p_L.$$

- With nondiscriminatory disclosure $\left\{[0,c], [c,\underline{k}] \cup [\bar{k},1], [\underline{k},\bar{k}]\right\}$, deviating type $\theta_H$ will buy at $\omega \in [c,\underline{k}] \cup [\bar{k},1]$.

# Information Discrimination w/o Price Discrimination

- Suppose price discrimination is not feasible
  - offer the same terms of trade to both types
- Further assume that
  - IC$_H$ are binding under optimal discriminatory disclosure
  - optimal information structure: $[c, \overline{\omega}]$ for $\theta_H$, $[\underline{k}, \overline{k}]$ for $\theta_L$
- Binding IC$_H$ and same terms of trade:
  - deviating $\theta_H$ indifferent at $\omega \in [c, \underline{k}] \cup [\overline{k}, \overline{\omega}]$
  - same rent under $\left\{[\underline{\omega}, c], [c, \underline{k}] \cup [\overline{k}, \overline{\omega}], [\underline{k}, \overline{k}]\right\}$
- Takeaway:
  - information discrimination has to interact with price discrimination to be effective in maximizing revenue.
  - information discrimination is not useful without transfers (Guo and Shmaya, 2017).

# Concluding Remarks

- We study optimal direct disclosure in a binary type setting.
- Two qualitative features of optimal disclosure policy:
  - nested intervals;
  - (often) discriminatory.
- What are natural restrictions on seller's disclosure policies?
- Future work: extension to the continuous type setting.