Mechanism Design with Financially Constrained Agents and Costly Verification

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Motivation

- Governments distribute valuable resources to financially constrained agents.
  - Housing and development board (HDB) in Singapore
  - Medicaid in the U.S.
- One justification for this role is that competitive market fails to maximize social surplus.
  - Some high valuation agents will not obtain the resources while low valuations agents with access to cash will.
- Governments face a mechanism design problem.
  - Agents have private information about their preferences and financial constraints.

Costly verification

Previous work *focuses* on mechanisms with only monetary transfers and *ignores* the role of costly verification.

- Government relies on agents' report of their ability to pay and can verify this information.
  - eligibility conditions on age, family, income, etc.
- An agent who makes a false statement can be punished.
  - fine or imprisonment
- Verification is costly for the government.

This paper: What is the best way to allocate resources in the presence of costly verification?

Preview of model

I characterize the optimal mechanism when ...

- The principal has a limited supply of indivisible goods.
- There is a unit mass of continuum of agents.
- Each agent has two-dimensional private information:
  - value $v \in [\underline{v}, \overline{v}]$, and
  - budget $b \in \{b_1, b_2\}$ with $b_1 < b_2$
- Monetary transfer and costly verification of budget.
  - Principal can verify an agent's budget at a cost and impose an exogenous penalty.
- The principal is also subject to a budget balance constraint.

Main results

Characterization of the optimal (revelation) mechanism.

- Agents who report low budgets receive more cash and in-kind subsidies.
  - In-kind subsidies: provision of goods at discounted prices
- Only those who report low-budgets are randomly verified.
- Verification probability is increasing in reported value.

Comparative statics (via numerical experiments)

Implementation via a two-stage mechanism

1st

- Agents *report* their budgets and *receive*
  - budget-dependent cash subsidies; and
  - the opportunity to participate in a lottery at budget-dependent prices.
- Randomly assign the goods among all lottery participants.
- Randomly inspect low-budget agents.

2nd

- Resale market opens and agents can trade with each other.
- Sellers face budget-dependent sales taxes.
- Randomly inspect low-budget agents who keep their goods.

Example

Main results (Cont'd)

Effects of verification

- w/o verification: equally subsidized, priced and taxed.
- w/: higher cash subsidies, lower prices and higher taxes for low-budget agents.

Intuition

- Higher cash subsidies and lower prices relax low-budget agents' budget constraints.
- Higher taxes discourage low-budget low-valuation agents from arbitrage.

Housing and development board (HDB) in Singapore

This exhibits some of the features of HDB.

Types of flats	Minimum Occupation Periods	
	sell	sublet
Resale flats w/ Grants	5-7 years	5-7 years
Resale flats w/o Grants	0-5 years	3 years

Feature

- More initial subsidies $\rightarrow$ more restrictions on resale/sublease

Technical contribution

Technical difficulties

- One cannot anticipate a priori the set of binding incentive compatibility constraints.
- IC constraints between distant types can bind.

Method

- Focus on a class of allocations rules (step functions) that
  - allow one to keep track of binding ICs; and
  - approximate a general allocation rule well.
- The optimal mechanism is obtained at the limit.

Literature

Mechanisms with financially constrained buyers

- *Known budgets:* Laffont and Robert (1996), Maskin (2000), Malakhov and Vohra (2008)
- *Private budgets:* Che and Gale (2000), Che, Gale and Kim (2013), Richter (2013), Pai and Vohra (2014)
- Difference: Costly verification

Costly verification

- *Single agent:* Townsend (1979), Gale and Hellwig (1985), Border and Sobel (1987), Mookherjee and Png (1989)
- *Multiple agents:* Ben-Porath, Dekel and Lipman (2014), Mylovanov and Zapechelnyuk (2015), Li (2016)
- Difference: Two-dimensional private information

Model

- A unit mass of continuum of risk neutral agents
- A mass $S < 1$ of indivisible goods
- Each agent has
  - a private valuation of the good: $v \in V \equiv [\underline{v}, \overline{v}]$, and
  - a privately known budget: $b \in B \equiv \{b_1, b_2\}$.
- Agent's type: $t = (v, b)$, and the type space: $T = V \times B$
- v and b are independent.
  - $\mathbb{P}(b_1) = 1 - \pi$ and $\mathbb{P}(b_2) = \pi$, and $b_1 < b_2$.
  - v is distributed with CDF F and density f.

Costly verification

- Principal can verify an agent's budget at cost $k \geq 0$, and impose an exogenous non-monetary penalty $c > 0$.
- Verification perfectly reveals an agent's budget.
- The cost to an agent to have his report verified is zero.
- An agent is punished if and only if he is found to have lied.

Mechanism

- A direct mechanism (a, p, q) consists of Details
  - an allocation rule $a : T \to [0, 1]$,
  - a payment rule $p : T \to \mathbb{R}$,
  - a verification rule $q : T \to [0, 1]$.
- The utility of an agent who has type $t = (v, b)$ and reports $\hat{t} = (\hat{v}, \hat{b})$:

$$u(\hat{t}, t) = \begin{cases} a(\hat{t})v - p(\hat{t}) & \text{if } \hat{b} = b \text{ and } p(\hat{t}) \leq b \\ a(\hat{t})v - p(\hat{t}) - q(\hat{t})c & \text{if } \hat{b} \neq b \text{ and } p(\hat{t}) \leq b \\ -\infty & \text{if } p(\hat{t}) > b \end{cases}$$

Principal's problem

$$\max_{a,p,q} \mathbb{E}_t \left[a(t)v - p(t)\right], \tag{$\mathcal{P}$}$$

subject to

$$u(t,t) \geq 0, \qquad \forall t \in T, \tag{IR}$$

$$p(t) \leq b, \qquad \forall t \in T, \tag{BC}$$

$$u(t,t) \geq u(\hat{t},t), \qquad \forall t, \hat{t} \in T, p(\hat{t}) \leq b, \tag{IC}$$

$$\mathbb{E}_t[p(t) - kq(t)] \geq 0, \tag{BB}$$

$$\mathbb{E}_t\left[a(t)\right] \leq S. \tag{S}$$

(IC) constraints

- Ignore constraints corresponding to over-reporting budget.
- Two categories

$$a(v,b)v - p(v,b) \geq a(\hat{v},b)v - p(\hat{v},b), \qquad \text{(IC-v)}$$

$$a(v,b_2)v - p(v,b_2) \geq a(\hat{v},b_1)v - p(\hat{v},b_1) - q(\hat{v},b_1)c. \qquad \text{(IC-b)}$$

- By the standard argument, (IC-v) holds if and only if
  - (monotonicity) $a(v,b)$ is non-decreasing in v, and
  - (envelope cond) $p(v,b) = a(v,b)v - \int_{\underline{v}}^{v} a(\nu,b)\mathrm{d}\nu - u(\underline{v},b)$.
- Difficulty arises from (IC-b).

▸ No Verification

(IC-b) constraint: (v, b_2) misreports as $(\hat{v}, b_1)$

- (IC-b) Constraint:

$$q(\hat{v}, b_1)c \geq \underbrace{a(\hat{v}, b_1)v - p(\hat{v}, b_1)}_{\text{misreport as } (\hat{v}, b_1)} - \underbrace{[a(v, b_2)v - p(v, b_2)]}_{\text{report truthfully}}.$$

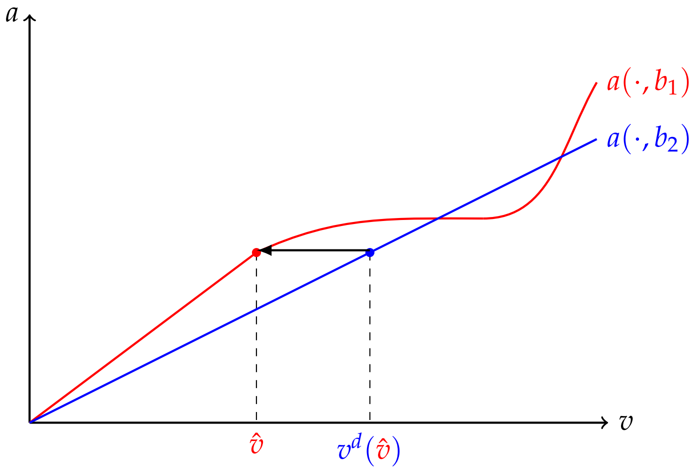
- LHS = Expected punishment
- RHS = Incentive for (v, b_2) to misreport as $(\hat{v}, b_1)$
- Fix $\hat{v}$, RHS is concave in v and maximized at

$$v^d(\hat{v}) \equiv \inf \{v | a(v, b_2) > a(\hat{v}, b_1)\}.$$

  - If $a(\cdot, b)$ is continuous, the $a(v^d(\hat{v}), b_2) = a(v, b_1)$.

▸ Detail

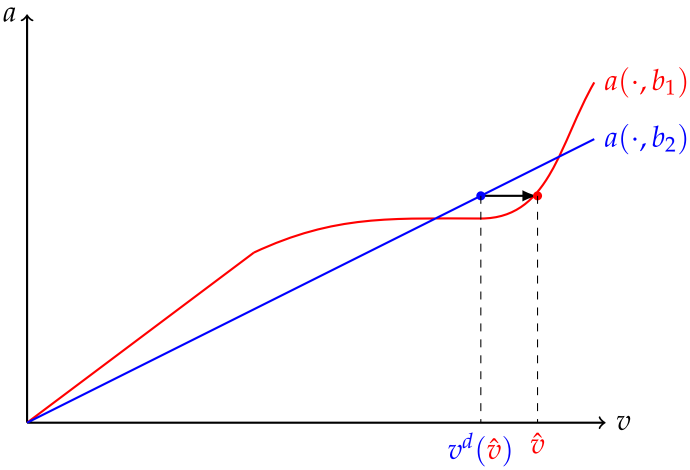
Binding (IC-b) constraints



- Binding (IC-b) constraints: $a(v^d(\hat{v}), b_2) = a(\hat{v}, b_1)$.

Detail

Binding (IC-b) constraints



- Binding (IC-b) constraints: $a(v^d(\hat{v}), b_2) = a(\hat{v}, b_1)$.

Detail

Sketch of the problem-solving strategy

1. Consider the principal's problem $(\mathcal{P}')$ with two modifications:

$$V(M,d) = \max_{a,p,q} \mathbb{E}_t[a(t)v - p(t)], \qquad (\mathcal{P}'(M,d))$$

subject to (IR), (IC-v), (IC-b), (BC), (S),

$$a \text{ is a } M'\text{-step allocation rule for some } M' \leq M,$$

$$\mathbb{E}[p(t) - q(t)k] \geq -d. \qquad \text{(BB-}d\text{)}$$

2. Take $M \to \infty$ and $d \to 0$.

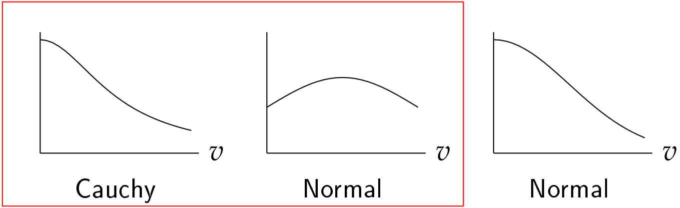
▸ Detail

Regularity conditions

Assumption 1. $\frac{1-F}{f}$ is non-increasing.

Assumption 2. f is non-increasing.

Examples (Banciu and Mirchandani, 2013) uniform, exponential and the left truncation of a normal distribution.



Back

Optimal mechanism

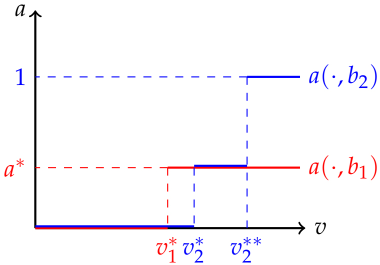
Theorem

Under the regularity conditions, there exists $v_1^* \leq v_2^* \leq v_2^{**}$, $u_1^* \geq u_2^*$ and $0 \leq a^* \leq 1$ such that in the optimal mechanism of $\mathcal{P}$

1. The allocation rule is

$$a(v, b_1) = \begin{cases} 0 & \text{if } v < v_1^* \\ a^* & \text{if } v > v_1^* \end{cases},$$

$$a(v, b_2) = \begin{cases} 0 & \text{if } v < v_2^* \\ a^* & \text{if } v_2^* < v < v_2^{**} \\ 1 & \text{if } v > v_2^{**} \end{cases},$$



Optimal mechanism

Theorem

Under the regularity conditions, there exists $v_1^* \le v_2^* \le v_2^{**}$, $u_1^* \ge u_2^*$ and $0 \le a^* \le 1$ such that in the optimal mechanism of $\mathcal{P}$

2. The payment rule is

$$p(v,b_1) = \begin{cases} -u_1^* & \text{if } v < v_1^* \\ -u_1^* + a^* v_1^* & \text{if } v > v_1^* \end{cases},$$

$$p(v,b_2) = \begin{cases} -u_2^* & \text{if } v < v_2^* \\ -u_2^* + a^* v_2^* & \text{if } v_2^* < v < v_2^{**} \\ -u_2^* + a^* v_2^* + (1-a^*) v_2^{**} & \text{if } v > v_2^{**} \end{cases}.$$

3. The verification rule is

$$q(v,b_1) = \begin{cases} \frac{1}{c}\left(u_1^* - u_2^*\right) & \text{if } v < v_1^* \\ \frac{1}{c}\left[\left(u_1^* - u_2^*\right) + a^* \left(v_2^* - v_1^*\right)\right] & \text{if } v > v_1^* \end{cases},$$

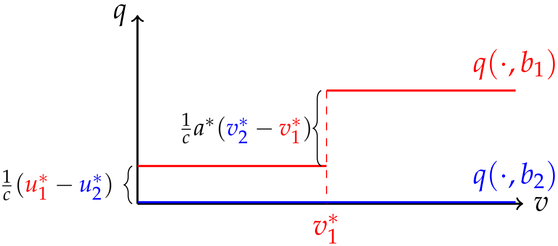
$$q(v,b_2) = 0.$$

Subsidies in cash and in kind

- Subsidies in cash:
  - High-budget: u_2^*.
  - Low-budget: u_1^*.
- Subsidies in kind: provision of goods at discounted prices.
- Use the additional payment made by a high-budget high-value agent as a measure of "price": $p^{\text{market}} = a^* v_2^* + (1 - a^*) v_2^{**}$.
- The amount of in-kind subsidies:
  - High-budget: $a^* \left(p^{\text{market}} - v_2^* \right)$.
  - Low-budget: $a^* \left(p^{\text{market}} - v_1^* \right)$.

Subsidies in cash and in kind (cont'd)

- Verification probability revisited



- Effects of verification cost
  - If $k = 0$, then high-budget agents receive no subsidies: $u_2^* = 0$ and $p^{\text{market}} = v_2^*$.
  - If $k = \infty$, then high-budget agents receive the same subsidies as low-budget agents: $u_2^* = u_1^*$ and $v_2^* = v_1^*$.

Implementation via a two-stage mechanism

1st

- Agents *report* their budgets and *receive*
  - budget-dependent cash subsidies; and
  - the opportunity to participate in a lottery at budget-dependent prices.
- Randomly assign the goods among all lottery participants.
- Randomly inspect low-budget agents.

2nd

- Resale market opens and agents can trade with each other.
- Sellers face budget-dependent sales taxes.
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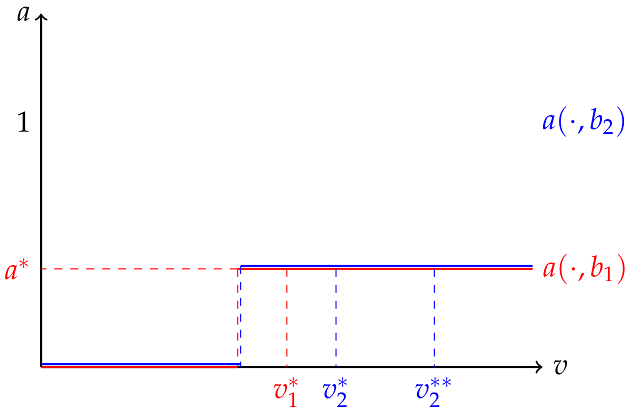
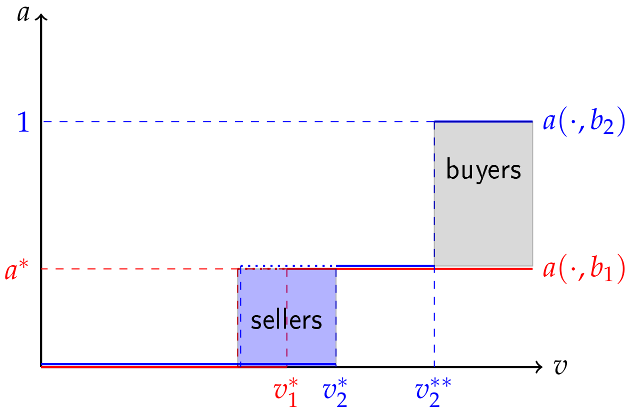
Implementation (cont'd)

1st stage



Implementation (cont'd)

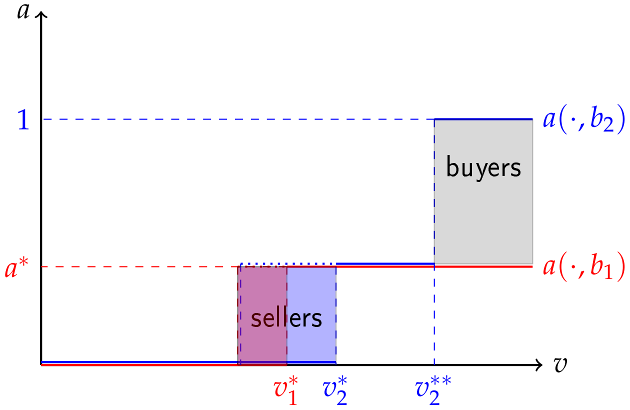
2nd stage



- price $= v_2^{**}$,
- high-budget tax $= a^*(v_2^{**} - v_2^*)$, low-budget tax $= a^*(v_2^{**} - v_1^*)$

Implementation (cont'd)

2nd stage



- price $= v_2^{**}$,
- high-budget tax $= a^*(v_2^{**} - v_2^*)$, low-budget tax $= a^*(v_2^{**} - v_1^*)$

Implementation (cont'd)

Effects of verification

- w/o verification: equally subsidized, priced and taxed.
- w/: higher subsidies, lower price and higher sales taxes for low-budget agents.

Intuition

- Higher subsidies and discounted price relax low-budget agents' budget constraints.
- Higher taxes discourage low-budget low-valuation agents from arbitrage.

Properties of optimal mechanism

1. Who benefits if the supply of goods increases?
2. How does verification cost affect the optimal mechanism's reliance on cash and in-kind subsidies?

Supply (S)

An increase in S improves the total welfare; but its impact on each budget type is not monotonic.

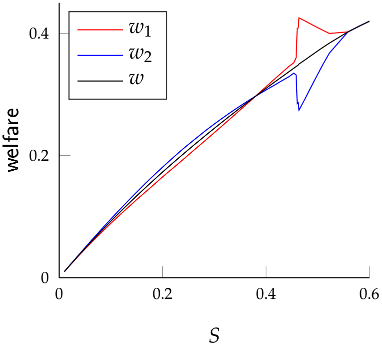
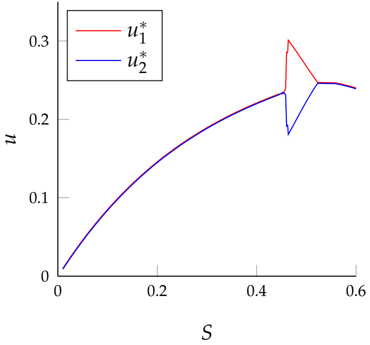


Figure: In this example, $v \sim U[0,1]$, $\rho = 0.08$, $b_1 = 0.2$ and $\pi = 0.5$.

Supply (S)

Low-budget low-valuation agents can get worse off as the amount of cash subsidies to low-budget agents begins to decline for sufficiently large S.



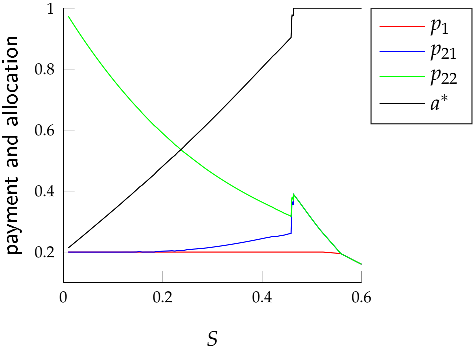


Figure: In this example, $v \sim U[0,1]$, $\rho = 0.08$, $b_1 = 0.2$ and $\pi = 0.5$.

Detail

Supply (S)

High-budget high-valuation agents can get worse off as their payments increase because disproportionately more goods are allocated to low-budget agents.

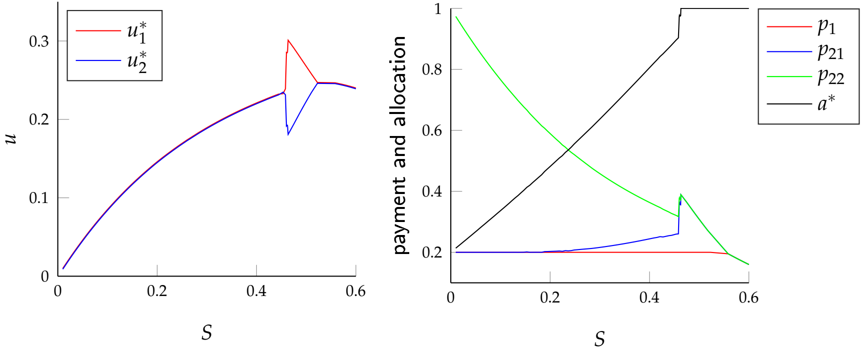


Figure: In this example, $v \sim U[0,1]$, $\rho = 0.08$, $b_1 = 0.2$ and $\pi = 0.5$.

Detail

Verification cost ($\rho = k/c$)

If verification becomes more costly, then agents are inspected less frequently in the optimal mechanism.

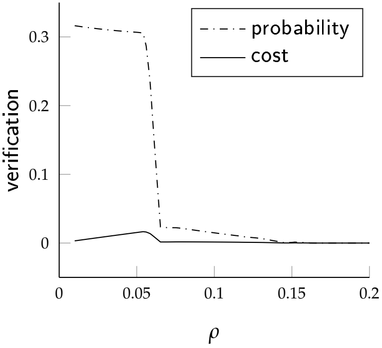


Figure: In this example, $v \sim U[0,1]$, $b_1 = 0.2$, $S = 0.4$ and $\pi = 0.5$.

Effective verification cost ($\rho = k/c$)

If verification becomes more costly, then the opt. mechanism relies more on in-kind than cash subsidies to help low-budget agents.

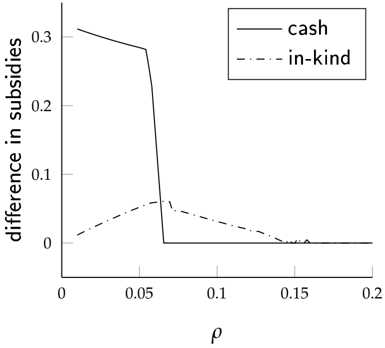


Figure: In this example, $v \sim U[0,1]$, $b_1 = 0.2$, $S = 0.4$ and $\pi = 0.5$.

Effective verification cost ($\rho = k/c$)

If verification becomes more costly, then the opt. mechanism relies more on in-kind than cash subsidies to help low-budget agents.

- Cash subsidy is more efficient because it introduces less distortion into allocation.
- Cash subsidy is more costly because it is attractive to agents with all valuations while in-kind subsidy is attractive to only have-valuation agents.

Extensions

- Ex-post individual rationality
- Costly disclosure

Ex-post individual rationality

- Optimal mechanism may not be ex-post individually rational.
  - Lotteries with positive payments.
- Budget constraint vs. per unit price constraint

$$p(t) \leq b, \qquad \forall t = (v, b), \qquad \text{(BC)}$$

$$p(t) \leq a(t)b, \qquad \forall t = (v, b). \qquad \text{(PC)}$$

- Why study (BC)?
  - Optimal mechanisms in these two settings share qualitatively similar features.
  - For some parameter values, there is no rationing ($a^* = 1$).
  - Rationing is realistic if b_1 is close to zero.

Ex-post individual rationality (cont'd)

$k = \infty$	All results extend. The latter extends Che, Gale and Kim (2013).
$k < \infty$	Multiple levels of in-kind subsidies. Incremental change in diff. in in-kind subsidies is increasing.
$k < \infty$, f is regular	?

Costly disclosure

- Agents also bear a cost of being verified.
- An agent incurs cost c^T from being verified if he reported his budget truthfully and $c^F \geq c^T$ if he lied.
- The utility of an agent who has type $t = (v, b)$ and reports $\hat{t}$ is

$$u(\hat{t}, t) = \begin{cases} a(\hat{t})v - p(\hat{t}) - q(\hat{t})c^T & \text{if } \hat{b} = b \text{ and } p(\hat{t}) \leq b, \\ a(\hat{t})v - p(\hat{t}) - q(\hat{t})\left(c^F + c\right) & \text{if } \hat{b} \neq b \text{ and } p(\hat{t}) \leq b, \\ -\infty & \text{if } p(\hat{t}) > b. \end{cases}$$

Effects of costly disclosure

- Relax an agent's budget constraint:

$$u(v,b) = a(v,b)v - \underbrace{\left[p(v,b) + q(v,b)c^T\right]}_{\text{effective payment, } p^e(v,b)}.$$

- Increase punishment:

$$a(v,b_2)v - p^e(v,b_2) \geq a(\hat{v},b_1)v - q(\hat{v},b_1)(c + c^F - c^T) - p^e(\hat{v},b_1).$$

- Verification is more costly:

$$\mathbb{E}_t\left[p^e(t) - (k + c^T)q(t)\right] \geq 0.$$

Welfare

Proposition

If $\frac{k}{c} \geq \frac{c^T}{c^F - c^T}$, then the presence of disclosure costs improves welfare.

Conclusion

Recap

- Solved a multidimensional mechanism design problem motivated by transfer programs.
- Mechanisms with transfers and costly verification of budget.
- Characterized the surplus-maximizing/optimal mechanism.

Future work

- Interactions between transfers and costly verification.
- Repeated interactions between the principal and agents.

Revelation principle

A general direct mechanism (a, p, q, θ) consists of

- an allocation rule $a : T \to [0, 1]$,
- a payment rule $p : T \to \mathbb{R}$,
- an inspection rule $q : T \to [0, 1]$,
- a punishment rule $\theta : T \times \{b_1, b_2, n\} \to [0, 1]$.
  - $\theta(\hat{t}, n)$: prob. for an agent who reports $\hat{t}$ and is not inspected.
  - $\theta(\hat{t}, b)$: prob. for an agent who reports $\hat{t}$ and whose budget is revealed to be b.

Back

Optimal punishment rule

Lemma

In an optimal mechanism, $\theta((v,b),b) = 0$ and $\theta((v,\hat{b}),b) = 1$.

Punishment without verification

- Relax an agent's budget constraint:

$$u(t) = a(t)v - \underbrace{\left[p(t) + (1 - q(t))\theta(t,n)c\right]}_{\text{effective payment, } p^e(t)}.$$

- But this is costly:

$$\mathbb{E}_t\left[p^e(t) - kq(t) - (1 - q(t))\theta(t,n)c\right] \geq 0.$$

Back

Benchmark: no verification ($k = \infty$)

- Two categories

$$a(v,b)v - p(v,b) \geq a(\hat{v},b)v - p(\hat{v},b), \tag{IC-v}$$

$$a(v,b_2)v - p(v,b_2) \geq a(\hat{v},b_1)v - p(\hat{v},b_1) - q(\hat{v},b_1)c. \tag{IC-b}$$

Back

Benchmark: no verification ($k = \infty$)

- Two categories

$$a(v,b)v - p(v,b) \geq a(\hat{v},b)v - p(\hat{v},b), \qquad \text{(IC-v)}$$

$$a(v,b_2)v - p(v,b_2) \geq a(\hat{v},b_1)v - p(\hat{v},b_1). \qquad \text{(IC-b)}$$

- It is sufficient to consider two one-dimensional deviations:

$$a(v,b)v - p(v,b) \geq a(\hat{v},b)v - p(\hat{v},b),$$

$$a(v,b_2)v - p(v,b_2) \geq a(v,b_1)v - p(v,b_1).$$

- To see this, note that

$$\begin{aligned} a(v,b_2)v - p(v,b_2) &\geq a(v,b_1)v - p(v,b_1) \\ &\geq a(\hat{v},b_1)v - p(\hat{v},b_1). \end{aligned}$$

Back

(IC-b) constraint: (v, b_2) misreports as $(\hat{v}, b_1)$

- (IC-b) Constraint:

$$q(\hat{v}, b_1)c \geq \underbrace{a(\hat{v}, b_1)v - p(\hat{v}, b_1)}_{\text{misreport as } (\hat{v}, b_1)} - \underbrace{\left[a(v, b_2)v - p(v, b_2)\right]}_{\text{report truthfully}}.$$

- Fix $\hat{v}$, $\partial \text{RHS}/\partial v = a(\hat{v}, b_1) - a(v, b_2)$, which is non-increasing in v.
- Hence, RHS is concave in v and maximized at

$$v^d(\hat{v}) \equiv \inf \{v | a(v, b_2) > a(\hat{v}, b_1)\}.$$

Back

(IC-b) constraint: (v, b_2) misreports as $(\hat{v}, b_1)$

- Using the envelope condition, (IC-b) becomes:

$$q(\hat{v}, b_1)c \geq \underbrace{u(\underline{v}, b_1) + a(\hat{v}, b_1)(v - \hat{v}) + \int_{\underline{v}}^{\hat{v}} a(\nu, b_1)\mathrm{d}\nu}_{\text{misreport as } (\hat{v}, b_1)} - \underbrace{\left[u(\underline{v}, b_2) + \int_{\underline{v}}^{v} a(\nu, b_2)\mathrm{d}\nu\right]}_{\text{report truthfully}}$$

- Fix $\hat{v}$, $\partial \text{RHS}/\partial v = a(\hat{v}, b_1) - a(v, b_2)$, which is non-increasing in v.
- Hence, RHS is concave in v and maximized at

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- Using the envelope condition, (IC-b) becomes:

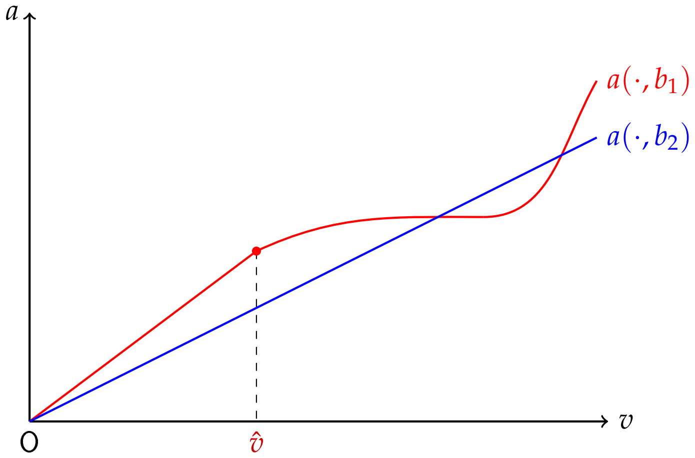
$$q(\hat{v}, b_1)c \geq \underbrace{u(\underline{v}, b_1) + a(\hat{v}, b_1)(v - \hat{v}) + \int_{\underline{v}}^{\hat{v}} a(\nu, b_1)\mathrm{d}\nu}_{\text{misreport as } (\hat{v}, b_1)}$$

$$- \underbrace{\left[u(\underline{v}, b_2) + \int_{\underline{v}}^{v} a(\nu, b_2)\mathrm{d}\nu\right]}_{\text{report truthfully}}$$

- Fix $\hat{v}$, $\partial \text{RHS}/\partial v = a(\hat{v}, b_1) - a(v, b_2)$, which is non-increasing in v.
- Hence, RHS is concave in v and maximized at

$$v^d(\hat{v}) \equiv \inf\left\{v | a(v, b_2) > a(\hat{v}, b_1)\right\}.$$

(IC-b) constraint: (v, b_2) misreports as $(\hat{v}, b_1)$



Back

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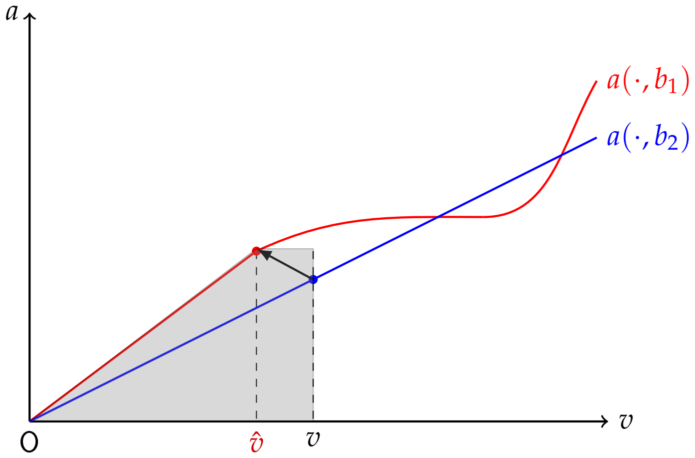

- gain = gray area, loss = blue area

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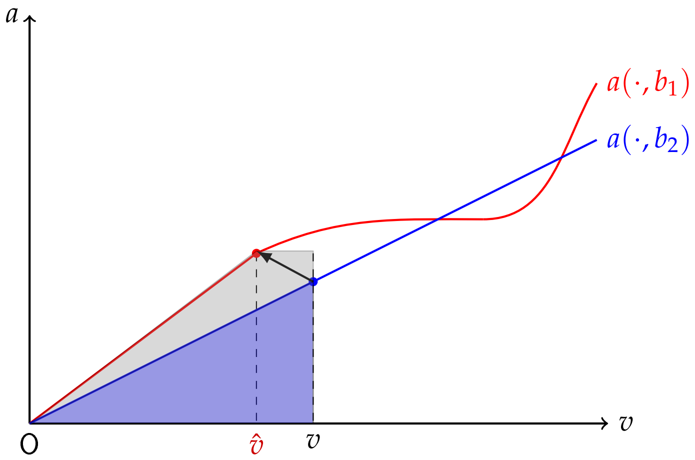

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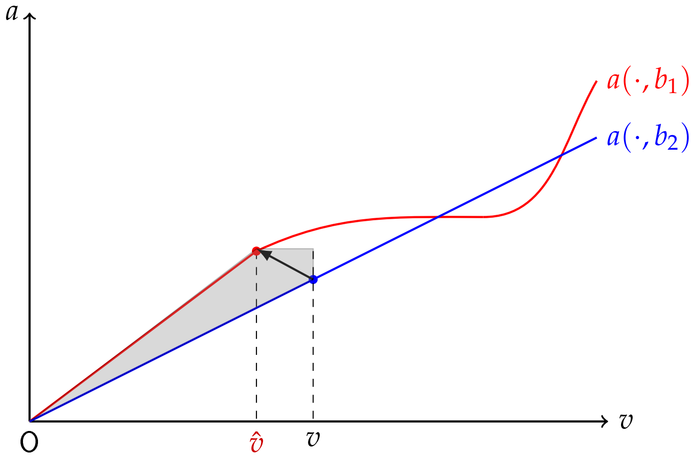

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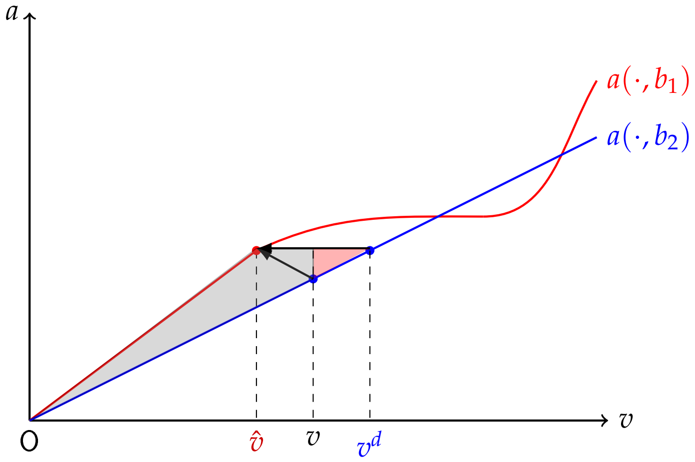
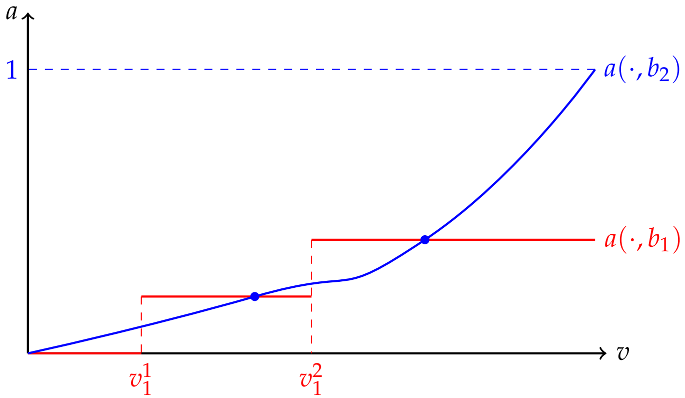


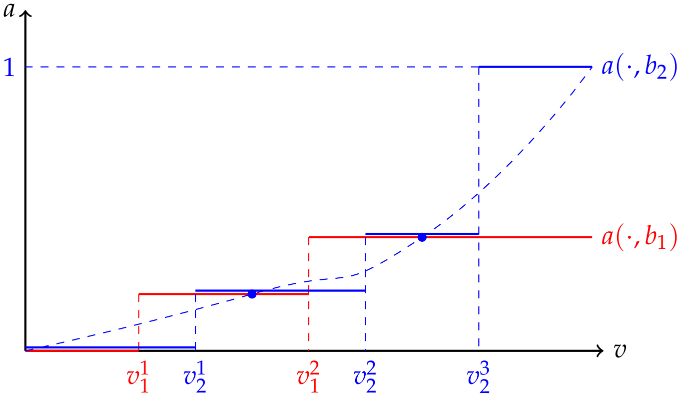
Back

Approximate allocation rules using step functions



- Assume that $a(\cdot, b_1)$ takes M distinct values. $\longrightarrow$ $a(\cdot, b_2)$ takes at most $M + 2$ distinct values.
- M-step allocation rule

Approximate allocation rules using step functions



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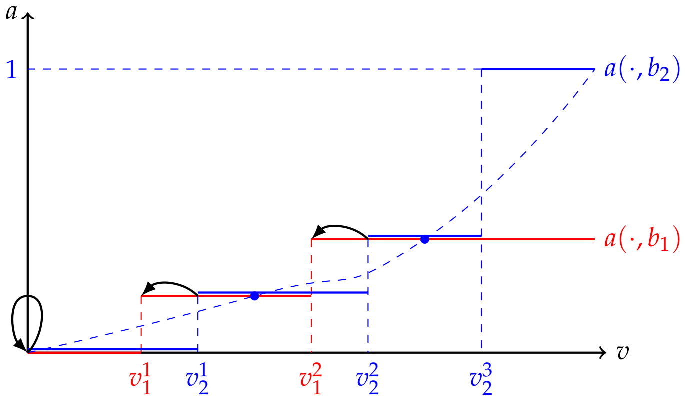

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Back

Sketch of the problem-solving strategy

1. Consider the principal's problem $(\mathcal{P}')$ with two modifications:

$$V(M,d) = \max_{a,p,q} \mathbb{E}_t[a(t)v - p(t)], \qquad (\mathcal{P}'(M,d))$$

   subject to (IR), (IC-v), (IC-b), (BC), (S),

$$a \text{ is a } M'\text{-step allocation rule for some } M' \leq M,$$

$$\mathbb{E}[p(t) - q(t)k] \geq -d. \qquad \text{(BB-}d\text{)}$$

2. Take $M \to \infty$ and $d \to 0$.

Sketch of the problem-solving strategy (cont'd)

Under the regularity conditions,

1. Consider the principal's modified problem $\mathcal{P}'(M, d)$

2. Take $M \to \infty$ and $d \to 0$.

Sketch of the problem-solving strategy (cont'd)

Under the regularity conditions,

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   Lemma 1: $V(M,d) = V(2,d)$ for all $M \geq 2$ and $d \geq 0$.

   ▸ Proof

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Sketch of the problem-solving strategy (cont'd)

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2. Take $M \to \infty$ and $d \to 0$.

   Lemma 2: $V = V(2,0)$, where V is the value of $\mathcal{P}$.

   ▸ Proof

Optimal mechanism of $\mathcal{P}'(M, d)$

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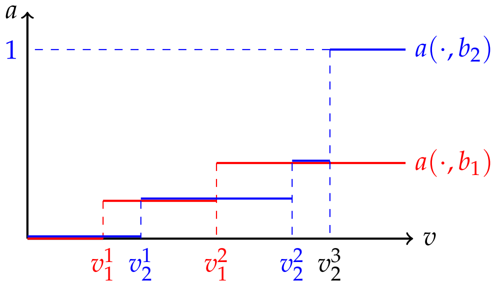
Lemma 1

Let $V(M,d)$ denote the value of $\mathcal{P}'(M,d)$. Then $V(M,d) = V(2,d)$ for all $M \geq 2$ and $d \geq 0$.

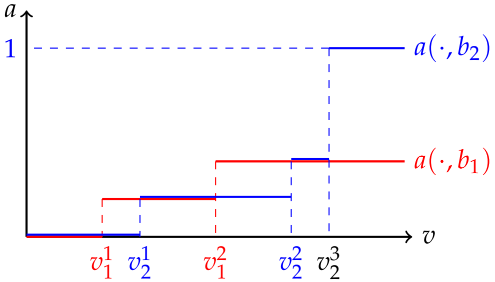
Proof Sketch.



- For each $m = 1, \ldots, M-1$, v_1^m and v_2^m satisfy a set of FOCs.
- If f is "regular", then this set of FOCs has a unique solution.

Intuition of Lemma 1

- Every linear program has an extreme point that is an optimal soln.
- $v_2^m - v_1^m$ is non-negative and increasing.
  - Incremental change in diff. in in-kind subsidies is increasing.
  - The number of active constraints is finite.



- If f is "regular", $V(M,d) = V(5,d)$ for all $M \geq 5$ and $d \geq 0$.

Back

Intuition of Lemma 1

- Every linear program has an extreme point that is an optimal soln.
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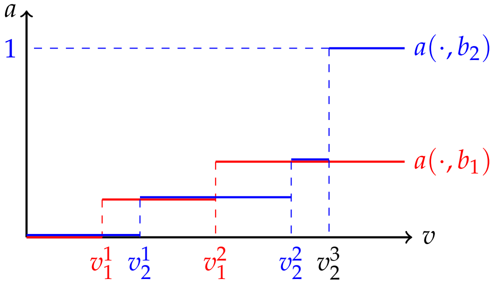

- If f is "regular", $V(M, d) = V(2, d)$ for all $M \geq 2$ and $d \geq 0$.

Back

Optimal mechanism of $\mathcal{P}$ ($M \to \infty$, $d \to 0$)

Lemma 2

Let V denote the value of $\mathcal{P}$. Then $V = V(2,0)$.

Proof sketch.

- $\forall d > 0$, $\exists\ \overline{M}(d) > 0$ such that $\forall M > \overline{M}(d)$

$$V - V(M,d) \leq (1-\pi)\frac{\mathbb{E}[v]}{M}.$$

- Fix $d > 0$ and let $M \to \infty$: $V(2,0) \leq V \leq V(2,d)\ \forall d > 0$.
- $V = V(2,0)$ by the continuity of $V(2,d)$.

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Supply (S)

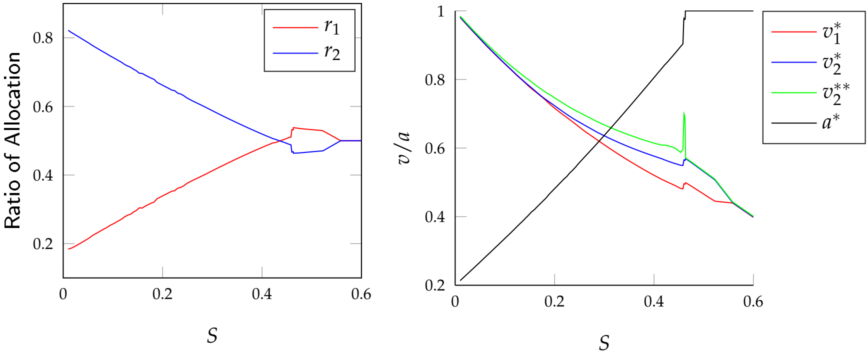

Figure: In this example, $v \sim U[0, 1]$, $\rho = 0.08$, $b_1 = 0.2$ and $\pi = 0.5$.

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