## On the Langlands Program: Endoscopy and Beyond – Research Conference (7 – 11 January 2019)

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# Isogeny classes of PPAVs, orbital integrals, and the Steinberg-Hitchin base

## Ali Altuğ

Boston University, USA

#### ABSTRACT

I will report on the recent (partially ongoing) work, joint w/ Achter, Garcia, and Gordon, where we investigate the number of members in an isogeny class of an ordinary principally polarized abelian variety (PPAV) with commutative endomorphism ring of arbitrary genus g over a finite field  $\mathbb{F}_q$ . Following Gekeler (2003), I will describe a simple probabilistic model for the count for g = 1 and describe how it gives the correct number, by first relating the count to certain orbital integrals (via Langlands-Kottwitz-Rapoport), and then relating the orbital integrals to the model through the Steinberg-Hitchin base of GL(2). I will then describe the problem for general genus and the extra ingredients that go into the proof.

# Multiplicities and Plancherel formula for the symmetric space $U(n) \setminus GL_n(E)$

RAPHAËL BEUZART-PLESSIS

Institut de Mathématiques de Marseille, Aix-Marseille Université, France

#### ABSTRACT

Let E/F be a quadratic extension of p-adic fields and  $U(n) \subseteq GL_n(E)$  a unitary group of rank n relative to this extension. Jacquet and Feigon-Lapid-Offen have studied the irreducible representations  $\pi$  of  $GL_n(E)$  which are U(n)-distinguished, i.e. admitting a realization in a space of functions on  $U(n) \setminus GL_n(E)$ , as well as the multiplicities with which they appear. They obtain a complete solution to this problem for generic representations except when  $\pi$  belongs to the ramification locus of the base-change map from  $GL_n(F)$  to  $GL_n(E)$ . In this talk, I shall explain how to treat the missing cases. I will also discuss the related problem of finding an explicit Plancherel decomposition for  $U(n) \setminus GL_n(E)$  and, if time permits, link this to the factorization of unitary periods of automorphic forms in the spirit of general speculations of Sakellaridis-Venkatesh.

# Automorphic reciprocity and families of L-functions

## VALENTIN BLOMER

University in Göttingen, Germany

## ABSTRACT

I will discuss various reciprocity laws for families of higher rank automorphic L-functions along with applications to subconvexity and non-vanishing.

# Jacquet-Mao's metaplectic fundamental lemma

VIET CUONG DO

Hanoi University of Science, Vietnam National University, Vietnam

#### ABSTRACT

Jacquet conjectured that automorphic representations of  $\operatorname{GL}_r$  distinguished by an orthogonal subgroup can be classified by automophic representations of its metaplectic cover. Jacquet and Mao have suggested a program to prove this conjecture by establishing a comparison between two relative traces formulas: one on the group  $\operatorname{GL}_r$  and the other one on its metaplectic cover. One of the steps of this program is precisely a fundamental lemma. In this talk, I shall report on the proof of this fundamental lemma.

# Kazhdan-Lusztig representations and Whittaker space of some genuine representations

## Fan Gao

National University of Singapore, Singapore

#### ABSTRACT

The high multiplicity of Whittaker models for genuine representations of a covering group has been well-known since the work of Kubota, Deligne, Kazhdan-Patterson etc. It gives obstacles to developing certain aspects of the representation theory of covering group in parallel with that of a linear reductive group, for example the Langlands-Shahidi method. However, it also gives rise to new theories of great interest and importance, for instance the theory of Weyl group multiple Dirichlet series pioneered by Brubaker, Bump, Chinta, Friedberg, Hoffstein and Gunnells. In this talk, we will discuss about a conjectural formula for the dimension of Whittaker functionals of irreducible constituents of regular unramified genuine principal series of covering groups. The formula explicitly relates such dimension to the Kazhdan-Lusztig representations associated with certain right cells of the Weyl group. We sketch the proof in several important cases.

# On triple product L functions for GL(3)

## JAYCE GETZ

Duke University, USA

#### ABSTRACT

I will define a multiple Dirichlet series (MDS) attached to three automorphic representations of GL(3) and a quadruple of Hecke characters. Joint work with B. Liu on summation formulae for triples of quadratic spaces implies that the MDS admits meromorphic continuations in all variables and satisfies several functional equations. I will then explain work in progress relating one Dirichlet series in the MDS to the triple product L function attached to the three automorphic representations and how it can be used to deduce new information about this L-function.

# Supercuspidal L-packets

## TASHO KALETHA

University of Michigan, USA

#### ABSTRACT

I will describe work, partly in progress, on the explicit construction of the local Langlands correspondence for supercuspidal parameters (discrete parameters with trivial monodromy) when the residual characteristic is prime to the order of the Weyl group. It relies on a classification of semi-simple supercuspidal representations that is based on a formula for their characters due to Adler-DeBacker-Spice and its close resemblance of the character formula for real discrete series representations due to Harish-Chandra.

# Doubling constructions: from the linear case to covering groups

## Eyal Kaplan

Bar-Ilan University, Israel

#### ABSTRACT

In a recent work we constructed an integral representation for pairs of cuspidal automorphic representations of classical groups and general linear groups. This construction extended the classical doubling method of Piatetski-Shapiro and Rallis in several aspects. One of the main applications of the theory of these integrals was a proof of a global functorial lift of cuspidal automorphic representations to the natural general linear group, by virtue of the Converse Theorem.

In this talk I will briefly discuss the application to functoriality, and report on the extension of these ideas to covering groups.

This talk is based on a joint work with Yuanqing Cai, Solomon Friedberg and David Ginzburg.

# On unipotent representations of real classical groups

## JIA-JUN MA

Shanghai Jiao Tong University, China

#### ABSTRACT

In this talk, I will present a recent work with Binyong Sun and Chengbo Zhu on unipotent representations of real classical groups (real symplectic groups, real orthogonal groups, quaternionic orthogonal groups or quaternionic symplectic groups). Unipotent representations are certain irreducible admissible representations characterized by their associated varieties and infinitesimal characters. They suppose to form the unipotent Arthur packet and are related to the quantization of nilpotent orbits. Barbasch and Vogan established the theory of special unipotent representations for complex classical groups and unitary groups. They also made conjectures for the general case, including a conjecture that unipotent representations attached to special nilpotent orbits are unitarizable.

In 90's, thanks to many peoples work, it becomes clear that iterated theta lifting could be an effective way to construct unipotent representations of real classical groups. In our work, we constructed all unipotent representations attached to quasidisdistinguished nilpotent orbits utilizing algebraic and analytic properties of theta lifts. The unitarity of these representations follows from the construction, thanks to Jian-shu Li, Hongyu He and Harris-Li-Sun's results on matrix coefficients integral.

The construction of unipotent representations attached to a general special unipotent is working in progress (joint with Barbasch).

## Effective limit multiplicities in $SL(2, \mathbb{R}^r \times \mathbb{C}^s)$

JASMIN MATZ

The Hebrew University of Jerusalem, Israel

#### ABSTRACT

For a lattice  $\Gamma$  in a semisimple Lie group G one can define a natural measure  $\mu_{\Gamma}$  on the unitary dual  $\hat{G}$  of G which counts the multiplicities with which representations appears in the discrete part of  $L^2(\Gamma \setminus G)$ . When  $\Gamma$  varies over a family of lattices with  $\operatorname{vol}(\Gamma \setminus G) \to \infty$ , one expects in most cases  $\mu_{\Gamma}$  to tend to the Plancherel measure on  $\hat{G}$ . This has been proven to be true in many situations in which the lattices are either commensurable with each other, uniform in G, or  $G = \operatorname{SL}(2, \mathbb{R})$ ,  $\operatorname{SL}(2, \mathbb{C})$ . In my talk I want to discuss this problem for the natural family of non-commensurable lattices  $\Gamma = \operatorname{SL}(2, \mathcal{O}_F)$  in  $G = \operatorname{SL}(2, F \otimes \mathbb{R})$  when F runs over number fields with a fixed archimedean signature (r, s) and  $\mathcal{O}_F$  is the ring of integers in F. In this case we also obtain a bound on the rate of convergence.

# Equivalent definitions of Arthur-packets for real classical groups

## Paul Mezo

Carleton University, Canada

#### ABSTRACT

In his most recent book, Arthur defines A(rthur)-packets for classical groups using techniques from harmonic analysis. For real groups an alternative approach to the definition of A-packets has been known since the early 90s. This approach, due to Adams-Barbasch-Vogan, relies on sheaf-theoretic techniques instead of harmonic analysis. We will report on work in progress, joint with N. Arancibia, in proving that these two different definitions for A-packets are equivalent for real classical groups.

# On the combinatorics of A-packets for classical groups

COLETTE MOEGLIN

Institut de Mathématiques de Jussieu-Paris Rive Gauche, France

#### ABSTRACT

In this talk, I will explain how recent results of Bin Xu can be used to solve some problems of irreducibility for induced representations of classical p-adic groups. This is part of an ongoing project with Marko Tadic related to the classification of unitary representations.

# The spectral side of stable local trace formula for real groups

Chung Pang Mok

Institute of Mathematics, Academia Sinica, Taiwan

#### ABSTRACT

Let G be a connected quasi-split reductive group over  $\mathbb{R}$ , and more generally, a quasisplit K-group over  $\mathbb{R}$ . Arthur had obtained the formal formula for the spectral side of the stable local trace formula, by using formal substitute of Langlands parameters. By combining with the works of Shelstad, we give the explicit expression for the spectral side of the stable local trace formula, in terms of Langlands parameters. Joint work with Zhifeng Peng.

# On the Braverman-Kazhdan program

Ngô Bảo Châu

The University of Chicago, USA and Vietnam Institute for Advanced Study in Mathematics, Vietnam

#### ABSTRACT

Back to 2000, Braverman and Kazhdan formulate a program aiming at generalizing the Godement-Jacquet construction of principal L-function. We will recall their program with more precision and some modest progress.

## Modular forms on exceptional groups

### AARON POLLACK

Duke University, USA

### ABSTRACT

Suppose that G is a reductive Q-group with an exceptional Dynkin type, which is either split  $G_2$  or a form of  $F_4, E_6, E_7, E_8$  with real rank 4. Then G supports a very special class of automorphic functions—the so-called "modular forms"—whose study was initiated by Gross-Wallach and Gan-Gross-Savin. I will define these objects and explain what is known about them.

# Cohomological representations of real reductive groups

DIPENDRA PRASAD

Tata Institute of Fundamental Research, India

### ABSTRACT

This lecture which might be just a survey of known results is about L and A parameters of cohomological representations of real reductive groups.

# Motivated cycles and functoriality

## KARTIK PRASANNA

University of Michigan, USA

#### ABSTRACT

The general conjectures on algebraic cycles predict in many cases that functoriality should be realized by algebraic cycles. However, it is typically rather difficult to construct cycles, and in any case one should not expect to construct such cycles explicitly. On the other hand, it appears that a slightly weaker notion, namely that of motivated cycles (due to Andre), is more suited to explicit constructions. I will explain a construction of a motivated cycle that realizes functoriality in the simplest case, namely the Jacquet-Langlands correspondence for GL(2) and its inner forms over a totally real field. In particular, this shows that the Jacquet-Langlands correspondence is absolutely Hodge. I will also briefly outline what one expects in the more general case of unitary Shimura varieties. (Joint work in progress with Atsushi Ichino.)

# Geometry of the moment map and the relative trace formula

## YIANNIS SAKELLADRIS

Rutgers University - Newark, USA

### ABSTRACT

I will explain how the geometry of the moment map for an affine homogeneous spherical variety of rank one leads to a non-standard matching of orbital integrals between its relative trace formula and the Kuznetsov formula for SL(2) or PGL(2). The matching is achieved through an explicit integral Fourier operator, which is determined by the L-value attached to the corresponding spherical period.

## On multiplicativity of $\gamma$ -factors

## FREYDOON SHAHIDI

Purdue University, USA

#### ABSTRACT

One of the important tools in the study of  $\gamma$ -factors from any approach is "Multiplicativity". It relates the  $\gamma$ -factors of an induced representation to the one defined by the inducing data. It maybe regarded as a defining axiom for any theory of  $\gamma$ -factors and *L*-functions.

In this talk we discuss this axiom within the approach of Braverman–Kazhdan and Ngo, generalizing Godement–Jacquet. We show that the proper framework to study and formulate multiplicativity in this approach is the Renner's construction of monoids attached to the given representation of  $\hat{G}$ , where G is a split reductive group. We will then formulate and propose a proof of multiplicativity in this approach, assuming the commutativity of the Harish–Chandra transform and corresponding Fourier transforms whose existence we assume. We check this against a number of known and proposed cases. We also observe one important moral of Renner's construction: Every pair of a reductive monoid and a Fourier transform on it should give a theory of  $\gamma$ –factors and L–functions for its group of units (and a representation of its L–group).

# One-dimensional representations are most nontempered

## Sug Woo Shin

University of California, Berkeley, USA

#### ABSTRACT

There may be a few different ways to interpret the title. I will formulate a version in terms of Jacquet modules on p-adic groups, discuss results, and mention an application if time permits. This is joint work with Arno Kret.

# On the Ramanujan conjecture over function fields

## NICOLAS TEMPLIER

Cornell University, USA

### ABSTRACT

Arthur conjectured in 1988 that if an automorphic representation has a local component that belongs to a supercuspidal L-packet, then it is tempered at every unramified place. Over function fields, we establish this conjecture for mgs packets, that is for those packets that arise from compact-induction of characters and remain supercuspidal after unramified base change. Our method relies on combining the l-adic geometry of  $\operatorname{Bun}_G$  and trace formulas. Work with Will Sawin.

# A simple local r-trace formula for the exterior square L-function of GL(2n)

## CHEN WAN

University of Minnesota, USA

#### ABSTRACT

In this talk, I will first discuss the notion of "r-trace formula" in the theory of beyond endoscopy. Then I will talk about a local analogue of the r-trace formula. Finally, by applying the local trace formula for Shalika model, we prove a simple version of the local r-trace formula for the case when G=GL(2n) and r is the exterior square representation of the dual group. This is a joint work with Raphael Beuzart-Plessis.

# Structure of A-packets for p-adic symplectic/orthogonal groups

## Bin Xu

Tsinghua University, China

#### ABSTRACT

The global A-packets appear in Arthur's conjectural description of the discrete automorphic spectrum of a reductive group over the number field. Their local components, also called local A-packets, are conjectured to be finite sets of irreducible admissible representations of the reductive group over the local field. On the other hand, one has the L-packets from the conjectural local Langlands correspondence. So it is a natural problem to understand the relation between the two. In this talk, I will consider non-endoscopic global A-packets of a symplectic/orthogonal group and present the corresponding results for their nonarchimedean components.

## Twisted automorphic descent

## Lei Zhang

National University of Singapore, Singapore

#### ABSTRACT

Applying Arthur's endoscopic classification of discrete automorphic representations for classical groups, we extend Ginzburg-Rallis-Soudry's automorphic descent from quasi-split classical groups setting to their pure inner forms. Combining with the multiplicity-free results of local Gan-Gross-Prasad conjecture, we not just provide a concrete module construction of cuspidal automorphic representations from the sauce representations of general linear groups, but also prove one direction of global Gan-Gross-Prasad conjecture. Moreover, our study leads us to many fundamental questions on the Fourier coefficients of automorphic forms, which connect to the existence of nonvanishing twist central value of L-functions for instance.

# Periods of automorphic forms over reductive groups

## MICHAL ZYDOR

University of Michigan, USA

#### ABSTRACT

Periods of automorphic forms have an important place in the theory of automorphic representations. They are often related to special values of L-functions and have applications to arithmetic geometry and analytic number theory. For an automorphic form on a group G, a period is its integral over a subgroup. If the automorphic form is not cuspidal such integrals are usually divergent. It is nonetheless possible in certain cases to extend the definition of the period to almost all automorphic forms which has direct applications to the study of the given period. In this talk I will describe a general procedure of defining such periods in the case when the subgroup is reductive.