Imaging through random media by speckle intensity correlations

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Motivation: Sensor array imaging

• Sensor array imaging (echography in medical imaging, sonar, non-destructive testing, seismic exploration, etc) has two steps:
- data acquisition: an unknown medium is probed with waves; waves are emitted by a source (or a source array) and recorded by a receiver array.
- data processing: the recorded signals are processed to identify the quantities of interest (reflector locations, etc).

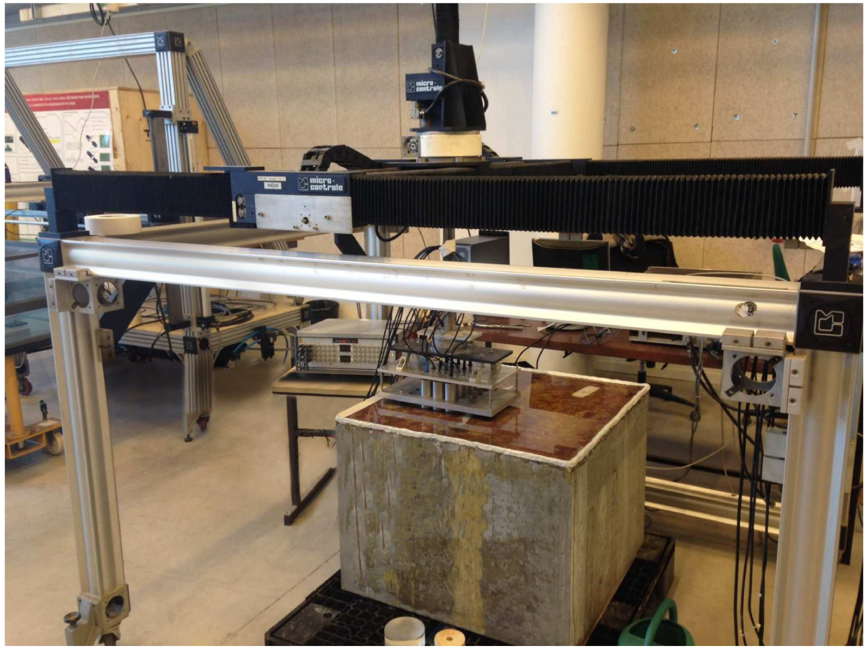
• Example:
Ultrasound echography

$\longrightarrow$

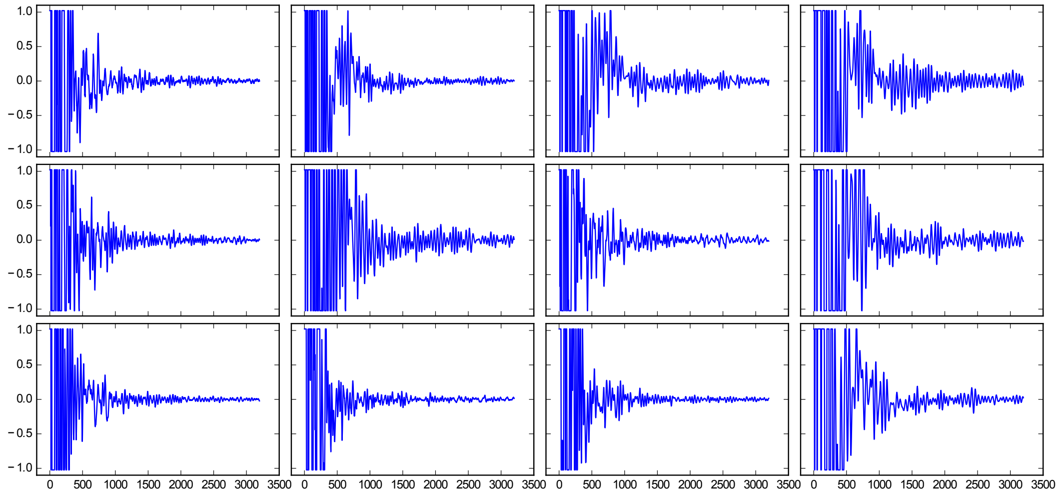
• Standard processing techniques (DAS, Reverse-Time Migration) work well if the background medium is known (ideally, homogeneous medium).

Application 1: Ultrasound echography in concrete

Goal: detect anomalies/cracks/inclusions in concrete.



$\longrightarrow$



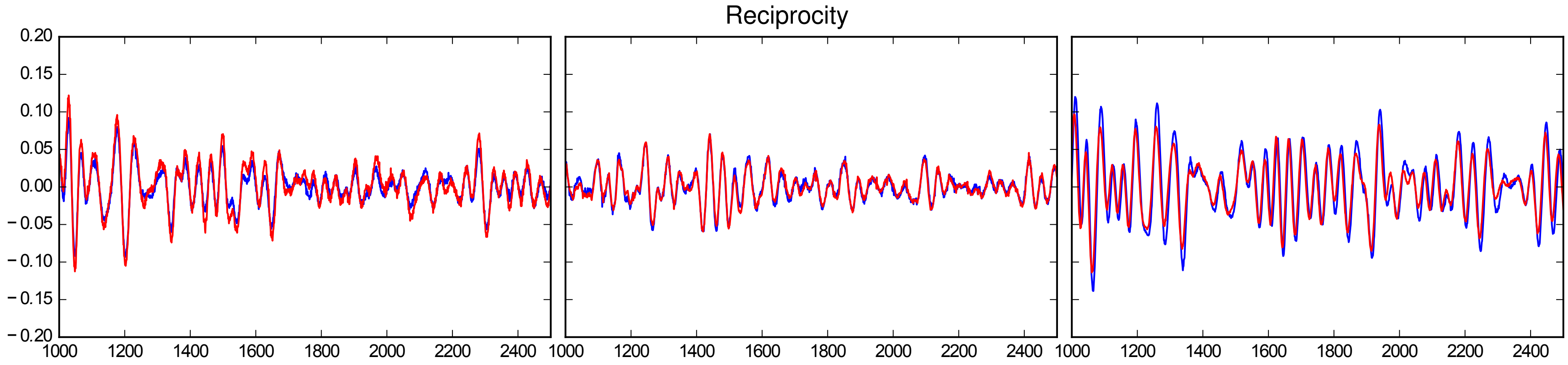
Experimental set-up

Data

The recorded signals look very "noisy".

$\hookrightarrow$ Where does the noise come from ?

Application 1: Ultrasound echography in concrete



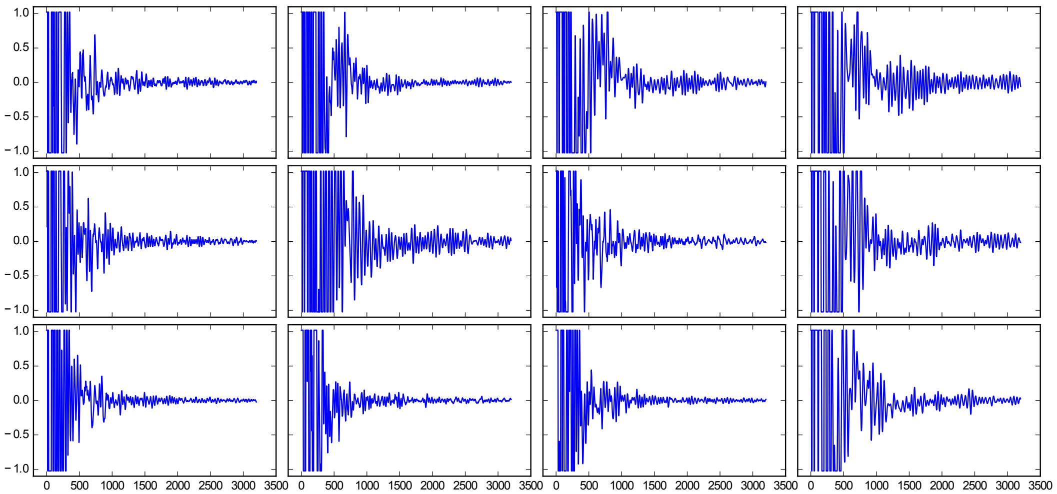
Reciprocity: The signal transmitted by A and recorded by B should be the same as the signal transmitted by B and recorded by A.

$\rightarrow$ This holds true in the data for (almost) all pairs (A, B) !

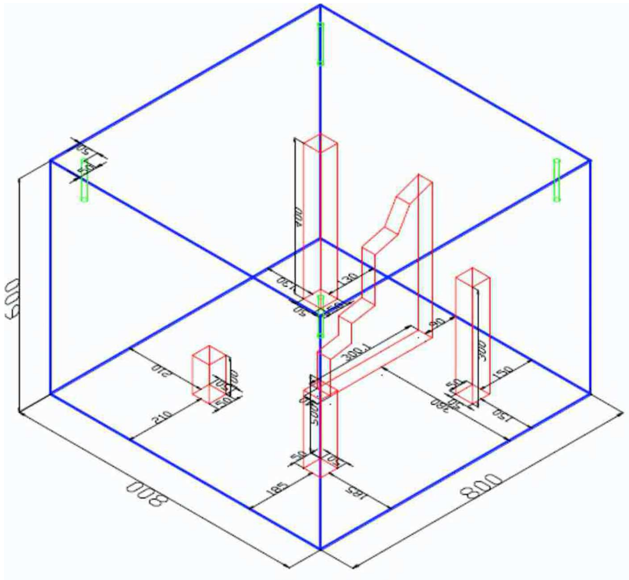
$\hookrightarrow$ The "noise" is not measurement noise !

Application 1: Ultrasound echography in concrete

Goal: detect anomalies/cracks/inclusions in concrete.



$\xrightarrow{?}$



Data | Real configuration

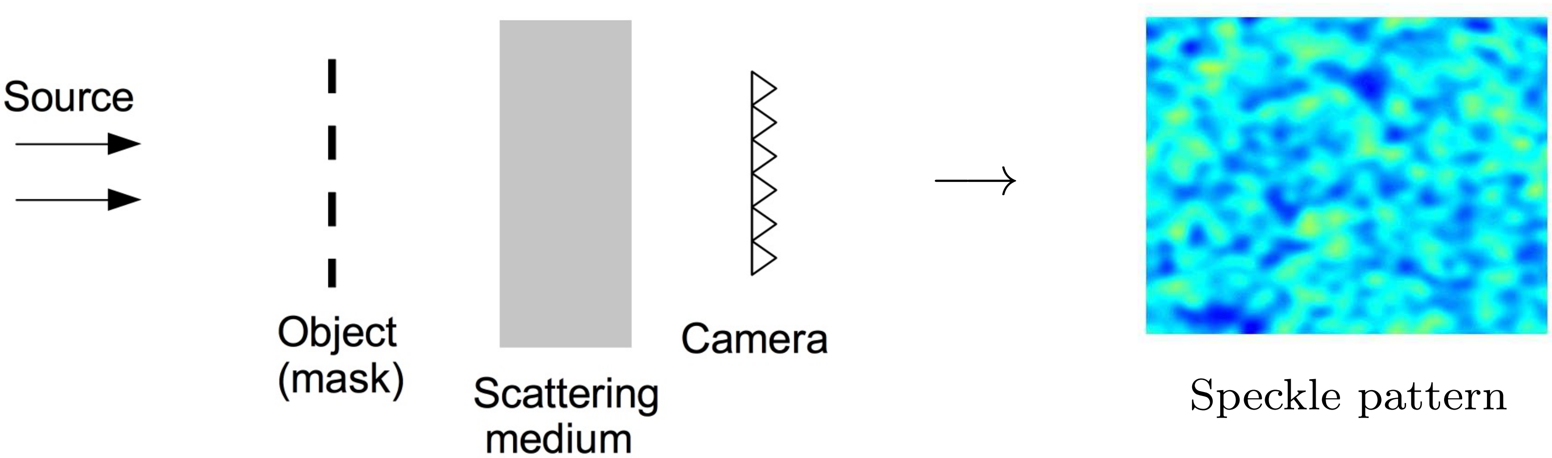
The recorded signals are very "noisy" due to scattering.

→ Concrete is highly scattering for ultrasonic waves !

↪ Standard echography fails and new imaging methods are needed.

Application 2: Optics in strongly scattering media

Goal: retrieve the shape of the mask from the recorded intensity.



Speckle pattern

Due to scattering the intensity recorded by the camera is a speckle pattern.

Wave propagation in random media

- Wave equation:

$$\frac{1}{c^2(\vec{\boldsymbol{x}})}\frac{\partial^2 u}{\partial t^2}(t,\vec{\boldsymbol{x}}) - \Delta_{\vec{\boldsymbol{x}}} u(t,\vec{\boldsymbol{x}}) = F(t,\vec{\boldsymbol{x}}), \qquad \vec{\boldsymbol{x}} = (\boldsymbol{x}, z) \in \mathbb{R}^2 \times \mathbb{R}$$

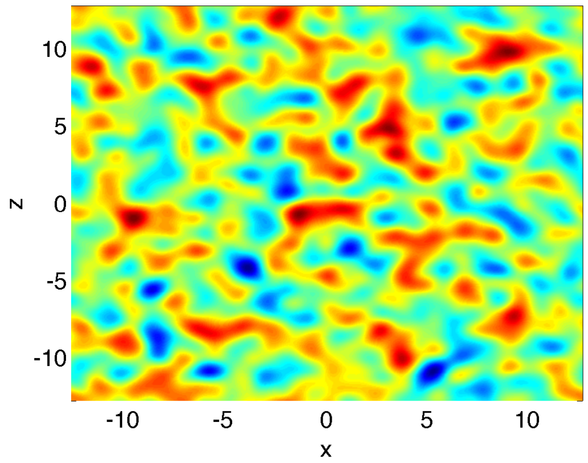
- Time-harmonic source in the plane $z = 0$: $F(t,\vec{\boldsymbol{x}}) = \delta(z) f(\boldsymbol{x}) e^{-i\omega t}$.

- Random medium model:

$$\frac{1}{c^2(\vec{\boldsymbol{x}})} = \frac{1}{c_o^2}\big(1 + \mu(\vec{\boldsymbol{x}})\big)$$

c_o is a reference speed,

$\mu(\vec{\boldsymbol{x}})$ is a zero-mean random process.



- Question: how to characterize the statistical properties of the wave field u ?

$\rightarrow$ Multiscale analysis [1,2].

[1] G. Papanicolaou, *SIAM J. Appl. Math.* **21** (1971) 13. [2] J.-P. Fouque et al., Springer, 2007.

Wave propagation in random media

- In the paraxial regime "$\lambda \ll \ell_c, r_o \ll L$", the envelope $\hat{\phi}(\boldsymbol{x}, z)$:

$$u(t, \boldsymbol{x}, z) = \frac{ic_o}{2\omega} \hat{\phi}(\boldsymbol{x}, z) e^{-i\omega(t - \frac{z}{c_o})}$$

satisfies the Itô-Schrödinger equation [1]

$$d\hat{\phi} = \frac{ic_o}{2\omega} \Delta_{\boldsymbol{x}} \hat{\phi} dz + \frac{i\omega}{2c_o} \hat{\phi} \circ dB(\boldsymbol{x}, z)$$

starting from $\hat{\phi}(z = 0, \boldsymbol{x}) = f(\boldsymbol{x})$, with $B(\boldsymbol{x}, z)$ Brownian field

$$\mathbb{E}[B(\boldsymbol{x}, z) B(\boldsymbol{x}', z')] = \gamma(\boldsymbol{x} - \boldsymbol{x}') \min(z, z'),$$

$\gamma(\boldsymbol{x}) = \int_{-\infty}^{\infty} \mathbb{E}[\mu(\mathbf{0}, 0)\mu(\boldsymbol{x}, z)] dz$.
$\hookrightarrow$ Makes it possible to use Itô-Stratonovich's stochastic calculus [2].

Sketch of proof. Consider the paraxial regime:

$$\omega \to \frac{\omega}{\varepsilon^4}, \qquad \mu(\boldsymbol{x}, z) \to \varepsilon^3 \mu\Big(\frac{\boldsymbol{x}}{\varepsilon^2}, \frac{z}{\varepsilon^2}\Big), \qquad f(\boldsymbol{x}) \to f\Big(\frac{\boldsymbol{x}}{\varepsilon^2}\Big),$$

and take $\varepsilon \to 0$.

[1] J. Garnier et al., *Ann. Appl. Probab.* **19** (2009) 318. [2] D. Dawson et al., *Appl. Math. Optim.* **12** (1984) 97.

Moment calculations in the paraxial regime

Consider

$$d\hat{\phi} = \frac{ic_o}{2\omega}\Delta_\perp \hat{\phi} dz + \frac{i\omega}{2c_o}\hat{\phi} \circ dB(\boldsymbol{x}, z)$$

starting from $\hat{\phi}(\boldsymbol{x}, z = 0) = f(\boldsymbol{x})$.

- By Itô's formula,

$$\frac{d}{dz}\mathbb{E}[\hat{\phi}] = \frac{ic_o}{2\omega}\Delta_\perp \mathbb{E}[\hat{\phi}] - \frac{\omega^2\gamma(\mathbf{0})}{8c_o^2}\mathbb{E}[\hat{\phi}]$$

and therefore

$$\mathbb{E}\big[\hat{\phi}(\boldsymbol{x}, z)\big] = \hat{\phi}_{\text{hom}}(\boldsymbol{x}, z) \exp\Big(- \frac{\gamma(\mathbf{0})\omega^2 z}{8c_o^2}\Big),$$

where $\gamma(\boldsymbol{x}) = \int_{-\infty}^{\infty} \mathbb{E}[\mu(\mathbf{0}, 0)\mu(\boldsymbol{x}, z)]dz$ and $\hat{\phi}_{\text{hom}}$ is the solution in the homogeneous medium.

- Strong damping of the coherent wave.

$\Longrightarrow$ Identification of the *scattering mean free path* $Z_{\text{sca}} = \frac{8c_o^2}{\gamma(\mathbf{0})\omega^2}$ [1].

$\Longrightarrow$ Coherent imaging methods (such as DAS or Reverse-Time migration) fail.

[1] A. Ishimaru, Academic Press, 1978.

Moment calculations in the paraxial regime

- The mean Wigner transform defined by

$$\mathcal{W}(\boldsymbol{r},\boldsymbol{\xi},z) = \int_{\mathbb{R}^2} \exp\big(-i\boldsymbol{\xi}\cdot\boldsymbol{q}\big)\mathbb{E}\Big[\hat{\phi}\big(\boldsymbol{r}+\frac{\boldsymbol{q}}{2},z\big)\overline{\hat{\phi}}\big(\boldsymbol{r}-\frac{\boldsymbol{q}}{2},z\big)\Big]\,d\boldsymbol{q},$$

is the angularly-resolved mean wave energy density.

By Itô's formula, it solves a *radiative transport equation*

$$\frac{\partial \mathcal{W}}{\partial z} + \frac{c_o}{\omega}\boldsymbol{\xi}\cdot\nabla_{\boldsymbol{r}}\mathcal{W} = \frac{\omega^2}{4(2\pi)^2 c_o^2}\int_{\mathbb{R}^2}\hat{\gamma}(\boldsymbol{\kappa})\Big[\mathcal{W}(\boldsymbol{\xi}-\boldsymbol{\kappa}) - \mathcal{W}(\boldsymbol{\xi})\Big]d\boldsymbol{\kappa},$$

starting from $\mathcal{W}(\boldsymbol{r},\boldsymbol{\xi},z=0) = \mathcal{W}_0(\boldsymbol{r},\boldsymbol{\xi})$, the Wigner transform of f.
$\Longrightarrow$ Identification of the *scattering cross section* $\frac{\omega^2}{4c_o^2}\hat{\gamma}(\boldsymbol{\kappa})$ [1].

- The fields at nearby points are correlated and their correlations contain information about the medium.

$\Longrightarrow$ One should use cross correlations for imaging in random media.

[1] A. Ishimaru, Academic Press, 1978.

Stability of the Wigner transform of the field

- The Wigner transform

$$W(\boldsymbol{r},\boldsymbol{\xi},z) := \int_{\mathbb{R}^2} \exp\big(-i\boldsymbol{\xi}\cdot\boldsymbol{q}\big)\hat{\phi}\big(\boldsymbol{r}+\frac{\boldsymbol{q}}{2},z\big)\overline{\hat{\phi}}\big(\boldsymbol{r}-\frac{\boldsymbol{q}}{2},z\big)d\boldsymbol{q}$$

is not statistically stable (i.e. standard deviation > mean).

- Let us consider the smoothed Wigner transform (for $r_{\rm s},\xi_{\rm s}>0$):

$$W_{\rm s}(\boldsymbol{r},\boldsymbol{\xi},z) = \frac{1}{(2\pi)^2 r_{\rm s}^2\xi_{\rm s}^2}\iint_{\mathbb{R}^2\times\mathbb{R}^2} W(\boldsymbol{r}-\boldsymbol{r}',\boldsymbol{\xi}-\boldsymbol{\xi}',z)\exp\Big(-\frac{|\boldsymbol{r}'|^2}{2r_{\rm s}^2}-\frac{|\boldsymbol{\xi}'|^2}{2\xi_{\rm s}^2}\Big)d\boldsymbol{r}'d\boldsymbol{\xi}'.$$

Its coefficient of variation:

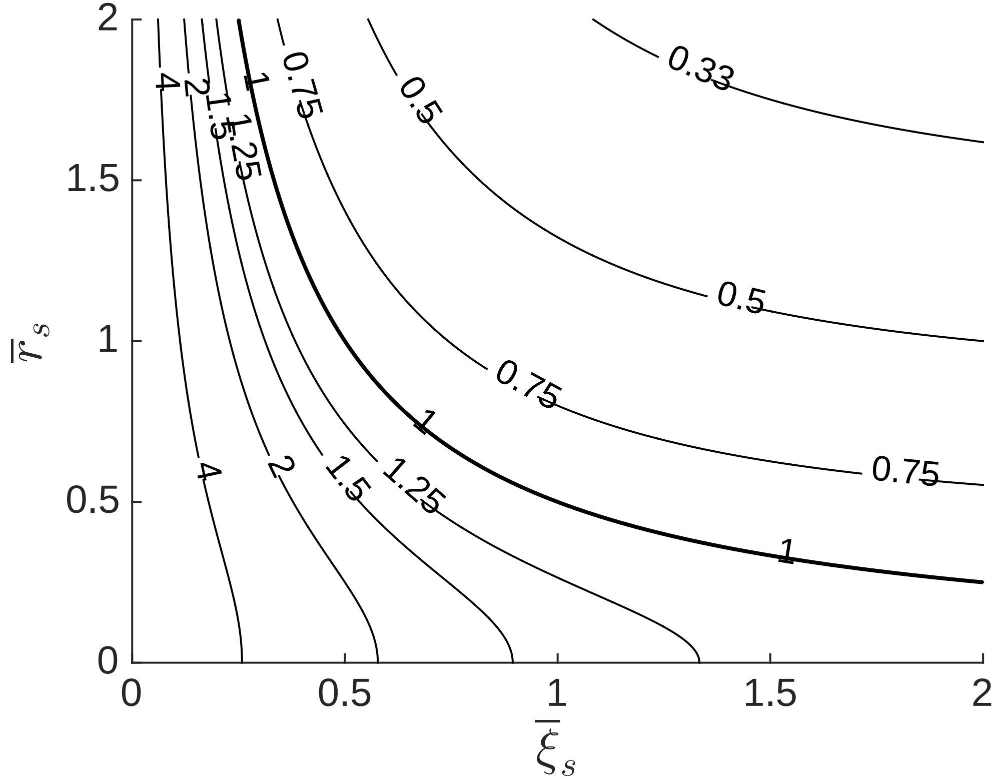
$$C_{\rm s}(\boldsymbol{r},\boldsymbol{\xi},z) := \frac{\sqrt{\mathbb{E}[W_{\rm s}(\boldsymbol{r},\boldsymbol{\xi},z)^2]-\mathbb{E}[W_{\rm s}(\boldsymbol{r},\boldsymbol{\xi},z)]^2}}{\mathbb{E}[W_{\rm s}(\boldsymbol{r},\boldsymbol{\xi},z)]}$$

determines its statistical stability.

$\hookrightarrow$ Analysis of high-order moments of $\hat{\phi}$ [1].

[1] J. Garnier et al., *ARMA* **220** (2016) 37.

Stability of the Wigner transform of the field

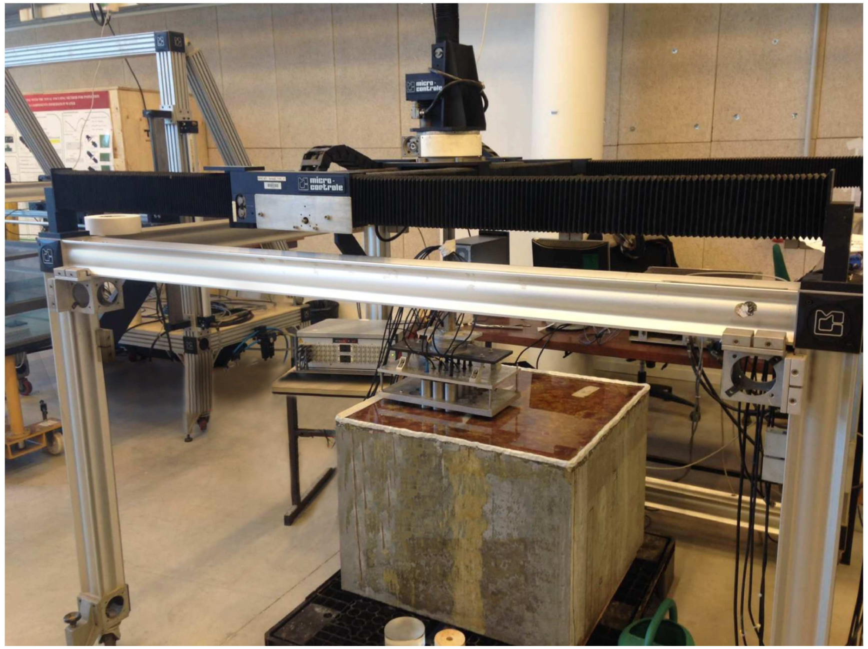


Contour levels of the coefficient of variation of the smoothed Wigner transform. Here $\overline{r}_{\rm s} = r_{\rm s}/\rho$, $\overline{\xi}_{\rm s} = \xi_{\rm s}\rho$, and $\rho = \rho(z;\omega, r_o, \ell_c, Z_{\rm sca})$.

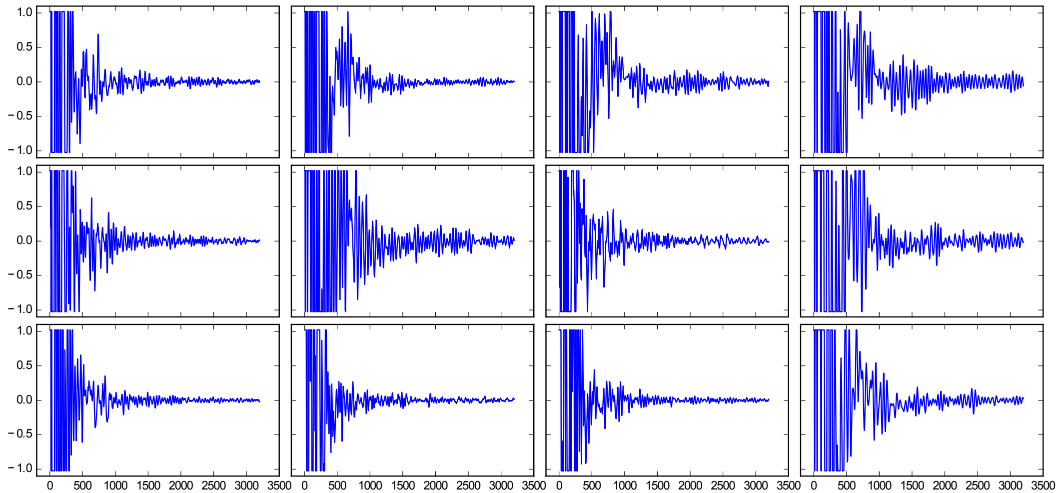
$\to$ This result makes it possible to achieve optimal trade-off between stability and resolution for correlation-based imaging [1,2].

[1] L. Borcea et al., *Inverse Problems* **27** (2011) 085004. [2] J. Garnier et al., *ARMA* **220** (2016) 37.

Application 1: Ultrasound echography in concrete



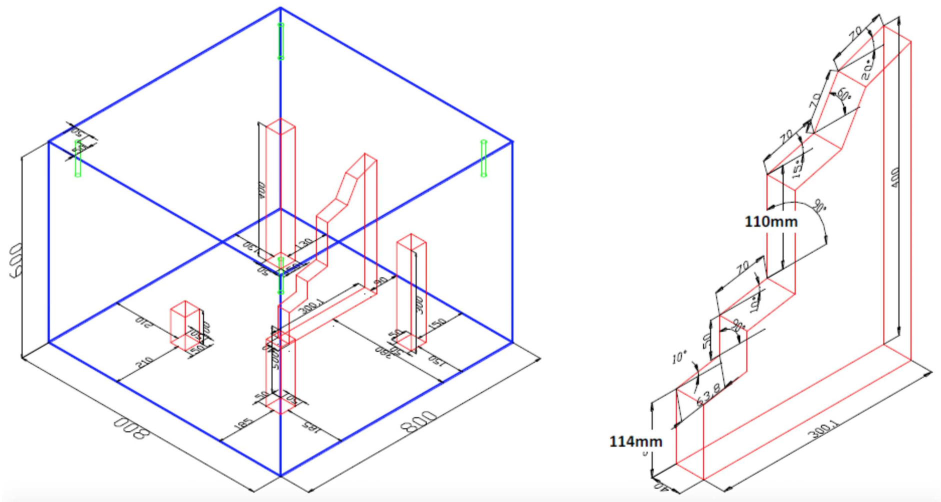
$\longrightarrow$



Experimental set-up Data

Concrete: highly scattering medium for ultrasonic waves.

Application 1: Ultrasound echography in concrete



Real configuration

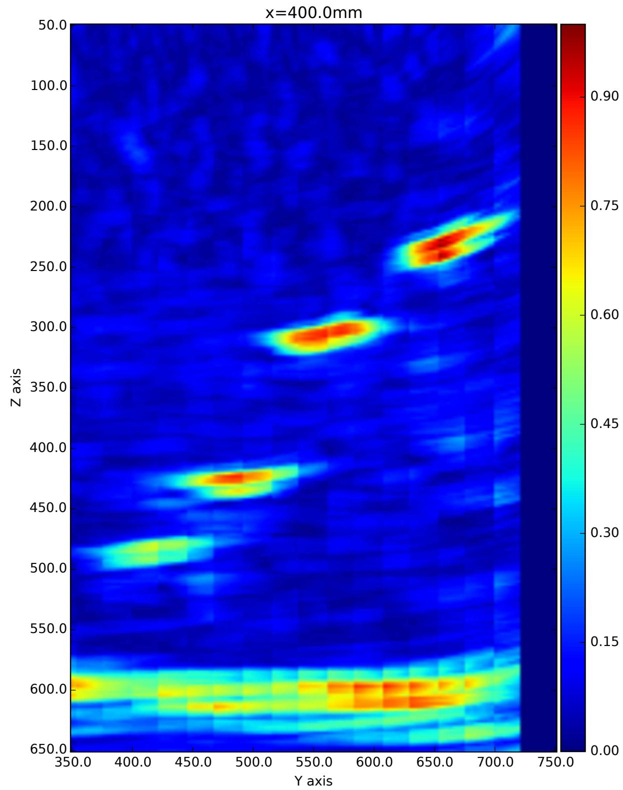


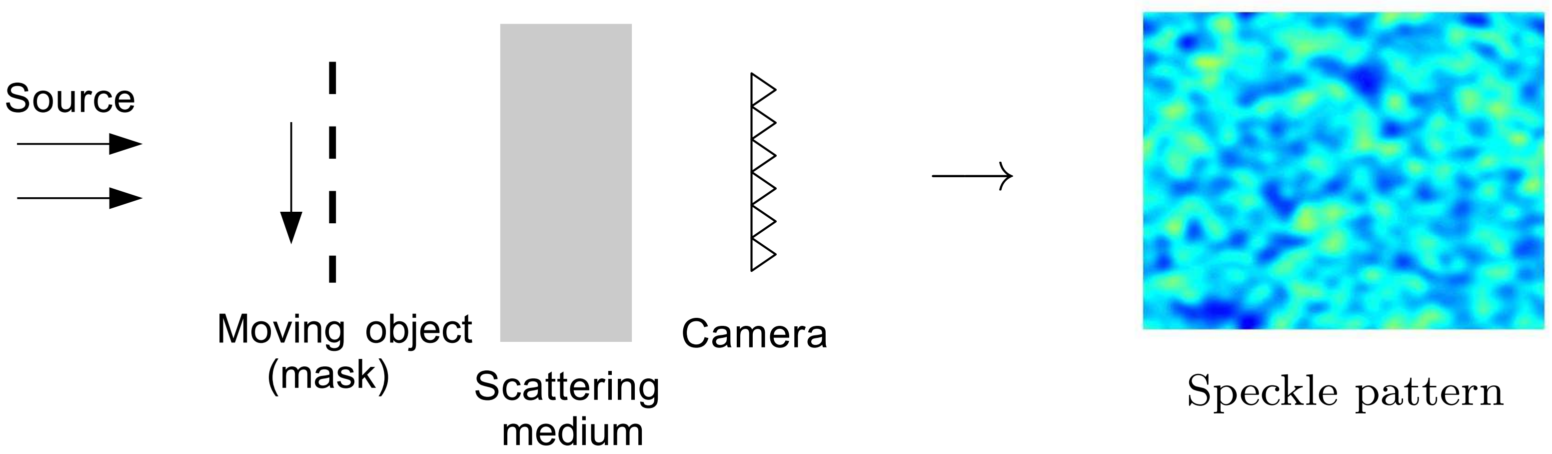
Image (2D slice)

Image obtained by travel-time migration of *well-regularized* cross correlations of data.

Remark on fourth-order moments

- Fourth-order moments are useful to:
- quantify the statistical stability of correlation-based imaging methods.
- implement intensity-correlation-based imaging methods (when only intensities can be measured, as in optics).

Speckle intensity correlation imaging through a scattering medium

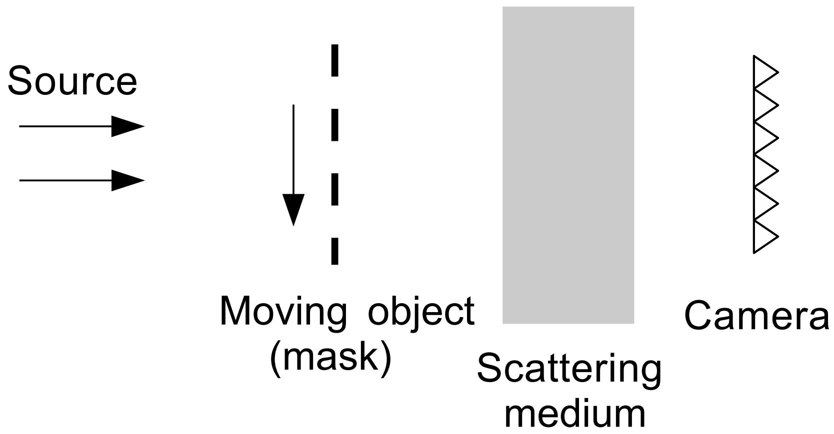


Experimental set-up [1]

- The light source is a time-harmonic plane wave.
- The object to be imaged is a mask that can be shifted transversally.
- For each position of the object the spatial intensity of the transmitted field (speckle pattern) can be recorded by the camera.

[1] J. A. Newmann et al., *Phys. Rev. Lett.* **113** (2014) 263903.

Speckle intensity correlation imaging through a scattering medium



- The field just after the mask (in the plane $z = 0$) is (for a transverse shift $\boldsymbol{r}$):

$$f_{\boldsymbol{r}}(\boldsymbol{x}) = f(\boldsymbol{x} - \boldsymbol{r}),$$

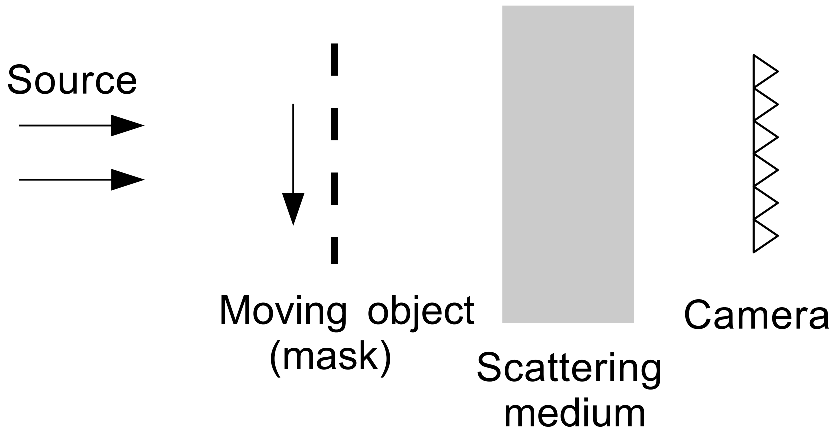
where f is the indicator function of the mask.

- The field in the plane of the camera (in the plane $z = L$) is denoted by $\hat{\phi}_{\boldsymbol{r}}(\boldsymbol{x})$.
- The measured intensity correlation is

$$\begin{aligned} \mathcal{C}_{\boldsymbol{r},\boldsymbol{r}'} &= \frac{1}{|A_0|} \int_{A_0} |\hat{\phi}_{\boldsymbol{r}}(\boldsymbol{x})|^2 |\hat{\phi}_{\boldsymbol{r}'}(\boldsymbol{x})|^2 d\boldsymbol{x} \\ &\quad - \Big(\frac{1}{|A_0|} \int_{A_0} |\hat{\phi}_{\boldsymbol{r}}(\boldsymbol{x})|^2 d\boldsymbol{x} \Big) \Big(\frac{1}{|A_0|} \int_{A_0} |\hat{\phi}_{\boldsymbol{r}'}(\boldsymbol{x})|^2 d\boldsymbol{x} \Big), \end{aligned}$$

where A_0 is the spatial support of the camera.

Speckle intensity correlation imaging through a scattering medium

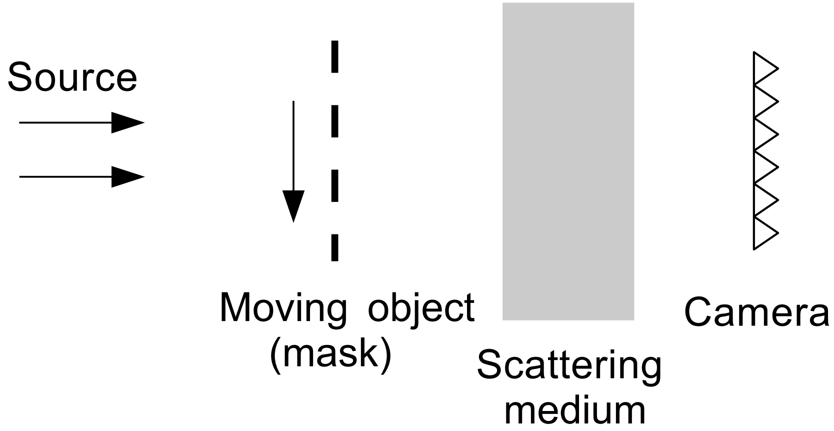


- Result (in the paraxial regime):

$$
\begin{aligned}
\mathbb{E}\big[\mathcal{C}_{\boldsymbol{r},\boldsymbol{r}'}\big] \;=\; & \int_{A_0} d\boldsymbol{X} \int d\boldsymbol{Y} \Big| \frac{1}{(2\pi)^2} \int \Big(\int f\big(\boldsymbol{x} + \frac{\boldsymbol{r}'-\boldsymbol{r}}{2}\big) \overline{f}\big(\boldsymbol{x} - \frac{\boldsymbol{r}'-\boldsymbol{r}}{2}\big) \exp\big(-i\boldsymbol{\zeta}\cdot\boldsymbol{x}\big) d\boldsymbol{x} \Big) \\
& \times \exp\Big(i\boldsymbol{\zeta}\cdot\big(\boldsymbol{X} - \frac{\boldsymbol{r}+\boldsymbol{r}'}{2}\big)\Big) \exp\Big(\frac{\omega^2}{4c_o^2} \int_0^L \gamma\big(\frac{c_o\boldsymbol{\zeta}}{\omega} z - \boldsymbol{Y}\big) - \gamma(\boldsymbol{0}) dz \Big) d\boldsymbol{\zeta} \Big|^2 \\
& - \Big| \frac{1}{(2\pi)^2} \int_{A_0} d\boldsymbol{X} \Big(\int f\big(\boldsymbol{x} + \frac{\boldsymbol{r}'-\boldsymbol{r}}{2}\big) \overline{f}\big(\boldsymbol{x} - \frac{\boldsymbol{r}'-\boldsymbol{r}}{2}\big) \exp\big(-i\boldsymbol{\zeta}\cdot\boldsymbol{x}\big) d\boldsymbol{x} \Big) \\
& \times \exp\Big(i\boldsymbol{\zeta}\cdot\big(\boldsymbol{X} - \frac{\boldsymbol{r}+\boldsymbol{r}'}{2}\big)\Big) \exp\Big(-\frac{\omega^2}{4c_o^2} \gamma(\boldsymbol{0}) L \Big) d\boldsymbol{\zeta} \Big|^2 ,
\end{aligned}
$$

with $\gamma(\boldsymbol{x}) = \int_{-\infty}^{\infty} \mathbb{E}[\mu(\boldsymbol{0},0)\mu(\boldsymbol{x},z)] dz$.

Speckle intensity correlation imaging through a scattering medium



- Result: When $L \gg Z_{\rm sca} := \frac{8c_o^2}{\gamma(\mathbf{0})\omega^2}$ and

$$|A_0| (\sim \text{diam(camera)}^2) \gg \rho_L^2 := \frac{Z_{\rm sca}\ell_c^2}{L}$$

($\rho_L =$ speckle radius), we have

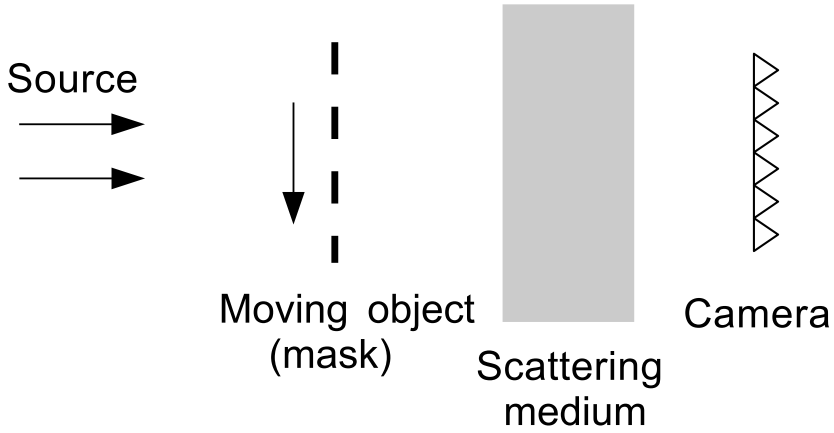
$$\mathcal{C}_{\boldsymbol{r},\boldsymbol{r}'} \simeq \mathbb{E}\big[\mathcal{C}_{\boldsymbol{r},\boldsymbol{r}'}\big] \approx \Big| \int |\hat{f}(\boldsymbol{\kappa})|^2 \exp\big(i\boldsymbol{\kappa}\cdot(\boldsymbol{r}'-\boldsymbol{r})\big) d\boldsymbol{\kappa}\Big|^2,$$

up to a multiplicative constant, where

$$\hat{f}(\boldsymbol{\kappa}) = \int f(\boldsymbol{x}) \exp\big(-i\boldsymbol{\kappa}\cdot\boldsymbol{x}\big) d\boldsymbol{x}.$$

$\hookrightarrow$ It is possible to reconstruct the mask indicator function f.

Speckle intensity correlation imaging through a scattering medium



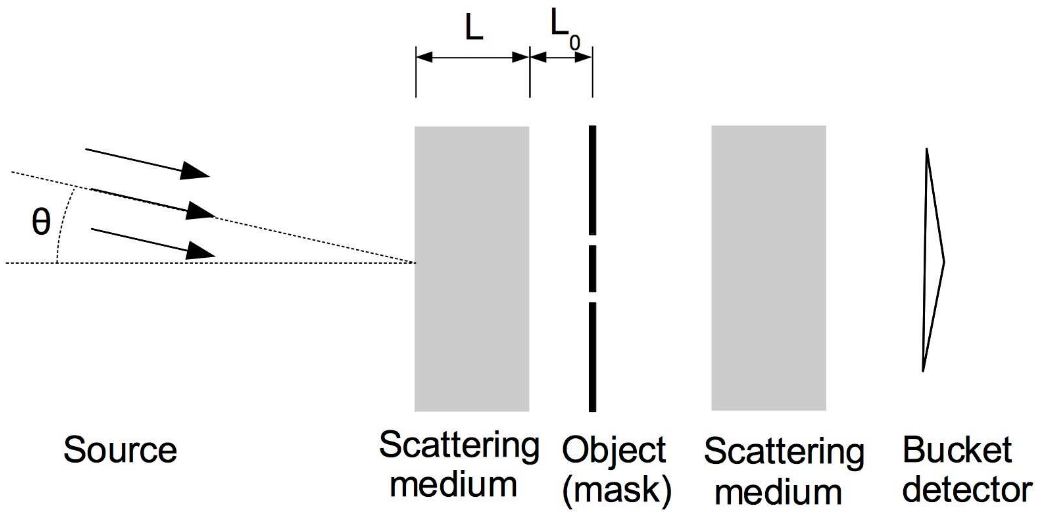
- We have

$$\mathcal{C}_{\boldsymbol{r},\boldsymbol{r}'} \simeq \mathbb{E}\big[\mathcal{C}_{\boldsymbol{r},\boldsymbol{r}'}\big] \approx \Big| \int |\hat{f}(\boldsymbol{\kappa})|^2 \exp\big(i\boldsymbol{\kappa}\cdot(\boldsymbol{r}'-\boldsymbol{r})\big) d\boldsymbol{\kappa}\Big|^2$$

$\hookrightarrow$ It is possible to reconstruct the incident field f by a two-step phase retrieval algorithm (Gerchberg-Saxon-type).

1) Given $C_{\boldsymbol{r},\boldsymbol{r}'}$, we know the modulus of the (I)FT of $|\hat{f}(\boldsymbol{\kappa})|^2$, and we know the phase of $|\hat{f}(\boldsymbol{\kappa})|^2$ (zero) $\to$ we can extract $|\hat{f}(\boldsymbol{\kappa})|^2$.
2) Given $|\hat{f}(\boldsymbol{\kappa})|^2$, we know the modulus of the FT of $f(\boldsymbol{x})$, and we know the phase of $f(\boldsymbol{x})$ (zero) $\to$ we can extract $f(\boldsymbol{x})$.

Speckle intensity correlation imaging through a scattering medium (II)

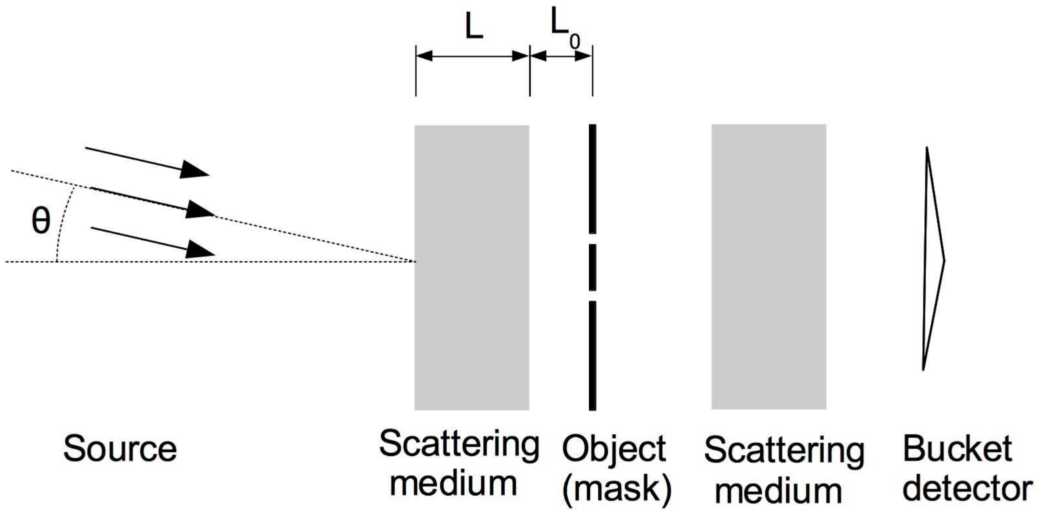


Experimental set-up

- A laser beam with incident angle $\boldsymbol{\theta}$ is shined on the scattering medium.
- The object to be imaged is a mask.
- The total intensity of the light that goes through the mask is collected by a bucket detector.

$\rightarrow$ For each incident angle $\boldsymbol{\theta}$ the total transmitted intensity $\mathcal{E}_{\boldsymbol{\theta}}$ is measured.

Speckle intensity correlation imaging through a scattering medium (II)



Consider:

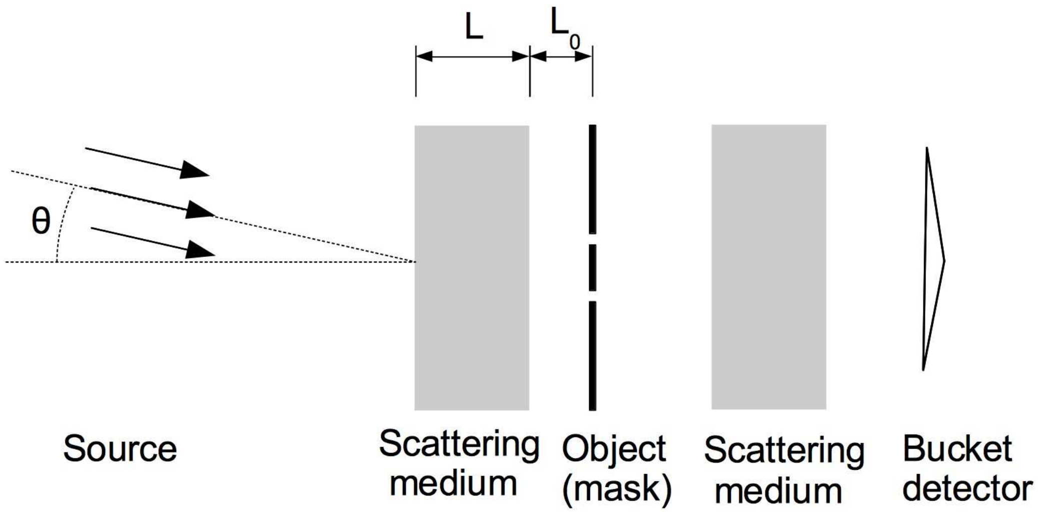
$$\mathcal{C}(\Delta\boldsymbol{\theta}) = \frac{1}{\Theta}\int_{\Theta} \mathcal{E}_{\boldsymbol{\theta}}\mathcal{E}_{\boldsymbol{\theta}+\Delta\boldsymbol{\theta}} d\boldsymbol{\theta} - \Big(\frac{1}{\Theta}\int_{\Theta} \mathcal{E}_{\boldsymbol{\theta}} d\boldsymbol{\theta}\Big)^2$$

- Result (in the paraxial regime):

$$\mathbb{E}[\mathcal{C}(\Delta\boldsymbol{\theta})] = \frac{1}{(2\pi)^2}\iint \exp\Big(\frac{\omega^2}{2c_o^2}\int_0^L \gamma\big(\boldsymbol{x}+\Delta\boldsymbol{\theta}(z+L_o)\big)dz\Big) e^{-i\boldsymbol{x}\cdot\boldsymbol{\kappa}} |\widehat{f^2}(\boldsymbol{\kappa})|^2 d\boldsymbol{\kappa} d\boldsymbol{x}$$
$$\times \exp\Big(-\frac{\omega^2\gamma(\mathbf{0})L}{2c_o^2}\Big) - |\widehat{f^2}(\mathbf{0})|^2 \exp\Big(-\frac{\omega^2\gamma_o(\mathbf{0})L}{2c_o^2}\Big),$$

with $\gamma(\boldsymbol{x}) = \int_{-\infty}^{\infty} \mathbb{E}[\mu(\mathbf{0},0)\mu(\boldsymbol{x},z)]dz$.

Speckle intensity correlation imaging through a scattering medium (II)

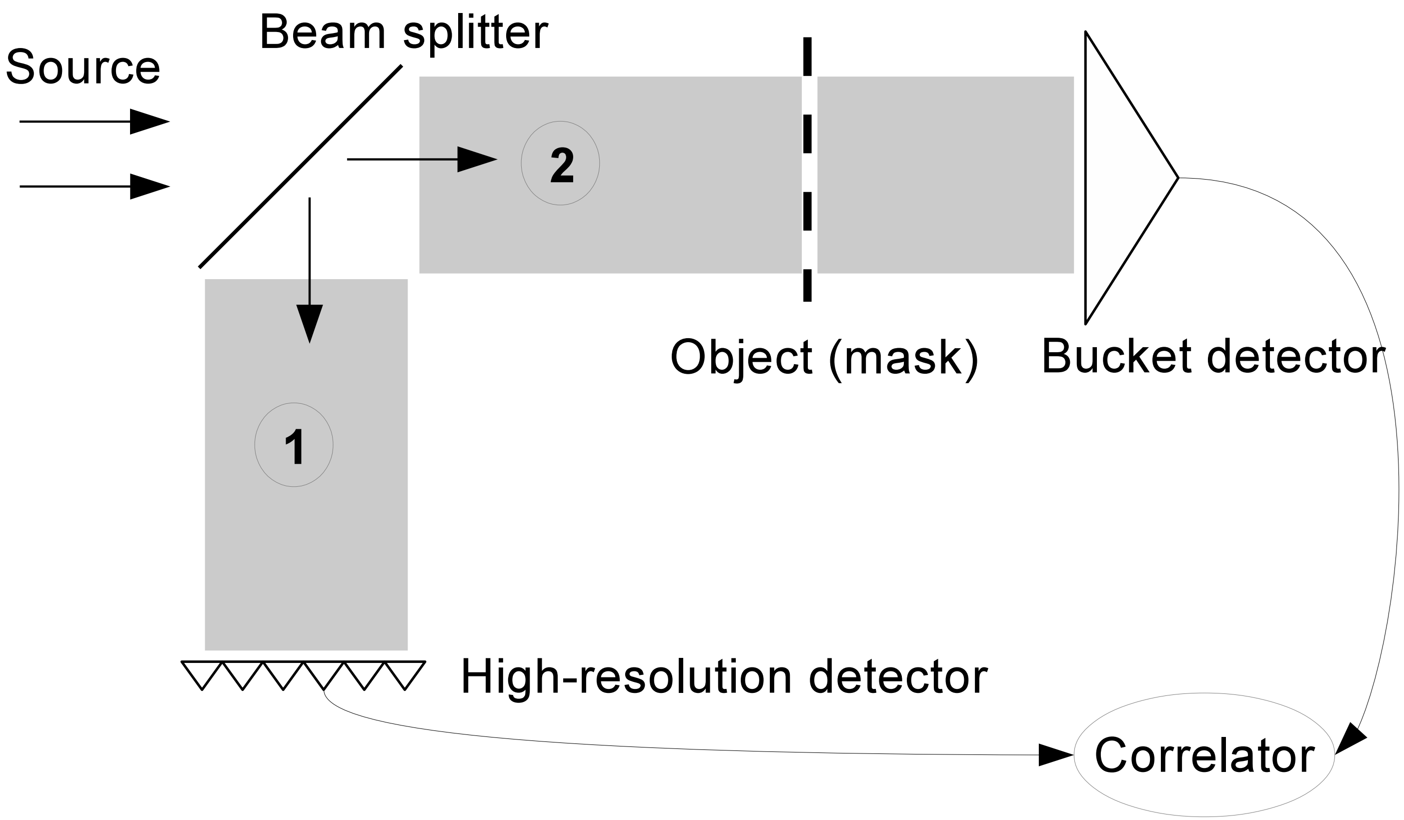


• Result: When $L \gg Z_{\rm sca} := \frac{8c_o^2}{\gamma(\mathbf{0})\omega^2}$, $\rho_L^2 = \frac{Z_{\rm sca}\ell_c^2}{L}$ is small enough and L_o is large enough, then

$$\mathcal{C}(\Delta\boldsymbol{\theta}) \simeq \mathbb{E}[\mathcal{C}(\Delta\boldsymbol{\theta})] \approx \int |\widehat{f^2}(\boldsymbol{\kappa})|^2 \exp\big(-i\boldsymbol{\kappa}\Delta\boldsymbol{\theta}L_o\big)d\boldsymbol{\kappa}$$

$\hookrightarrow$ One can extract $|\widehat{f^2}(\boldsymbol{\kappa})|^2$ (by Fourier transform or FFT) and then $f^2(\boldsymbol{x})$ by a (one-step) phase retrieval algorithm.

Speckle intensity correlation imaging through a scattering medium (III)



- Noise source (laser light passed through a rotating glass diffuser).
- without object in path 1; a high-resolution detector measures the spatially-resolved intensity $I_1(t, \boldsymbol{x})$.
- with object (mask) in path 2; a single-pixel detector measures the spatially-integrated intensity $I_2(t)$.

Experiment: the correlation of $I_1(\cdot, \boldsymbol{x})$ and $I_2(\cdot)$ is an image of the mask [1,2].

[1] A. Valencia et al., *PRL* **94** (2005) 063601; [2] J. H. Shapiro et al., *Quantum Inf. Process* **1** (2012) 949.

Speckle intensity correlation imaging through a scattering medium (III)

- Wave equation in paths 1 and 2:

$$\frac{1}{c_j^2(\vec{\boldsymbol{x}})}\frac{\partial^2 u_j}{\partial t^2} - \Delta_{\vec{\boldsymbol{x}}} u_j = e^{-i\omega_o t} n(t,\boldsymbol{x})\delta(z) + c.c., \qquad \vec{\boldsymbol{x}} = (\boldsymbol{x}, z) \in \mathbb{R}^2 \times \mathbb{R}, \qquad j = 1,2$$

- Noise source (with mean zero):

$$\left\langle n(t,\boldsymbol{x})\overline{n(t,\boldsymbol{x}')}\right\rangle = F(t-t')\exp\Big(-\frac{|\boldsymbol{x}|^2}{r_o^2}\Big)\delta(\boldsymbol{x}-\boldsymbol{x}')$$

- Wave fields: $u_j(t,\vec{\boldsymbol{x}}) = v_j(t,\vec{\boldsymbol{x}})e^{-i\omega_o t} + c.c., \qquad j = 1,2$
- Intensity measurements:

$$\begin{aligned} I_1(t,\boldsymbol{x}) &= |v_1(t,(\boldsymbol{x},L))|^2 \text{ in the plane of the high-resolution detector} \\ I_2(t) &= \int_{\mathbb{R}^2} |v_2(t,(\boldsymbol{x}',L+L_0))|^2 d\boldsymbol{x}' \text{ in the plane of the bucket detector} \end{aligned}$$

- Correlation:

$$C_T(\boldsymbol{x}) = \frac{1}{T}\int_0^T I_1(t,\boldsymbol{x})I_2(t)dt - \Big(\frac{1}{T}\int_0^T I_1(t,\boldsymbol{x})dt\Big)\Big(\frac{1}{T}\int_0^T I_2(t)dt\Big)$$

Speckle intensity correlation imaging through a scattering medium (III)

- If the propagation distance is larger than the scattering mean free path, then

$$C_T(\boldsymbol{x}) \stackrel{T\to+\infty}{\longrightarrow} \int_{\mathbb{R}^2} \mathcal{H}(\boldsymbol{x}-\boldsymbol{y}) f(\boldsymbol{y})^4 d\boldsymbol{y},$$

where $f(\boldsymbol{x})$ is the mask indicator function and $\mathcal{H}(\boldsymbol{x})$ is a convolution kernel [1].

- If the medium is homogeneous:

$$\mathcal{H}(\boldsymbol{x}) = \frac{r_o^4}{2^8 \pi^2 L^4} \exp\Big(- \frac{|\boldsymbol{x}|^2}{4\rho_{\rm gi0}^2}\Big), \qquad \rho_{\rm gi0}^2 = \frac{c_o^2 L^2}{2\omega_o^2 r_o^2}.$$

- If the medium in path 1 and 2 is random (*independent* realizations):

$$\mathcal{H}(\boldsymbol{x}) = \frac{r_o^4 \rho_{\rm gi0}^2}{2^8 \pi^2 L^4 \rho_{\rm gi1}^2} \exp\Big(- \frac{|\boldsymbol{x}|^2}{4\rho_{\rm gi1}^2}\Big), \qquad \rho_{\rm gi1}^2 = \rho_{\rm gi0}^2 + \frac{4 c_o^2 L^3}{3\omega_o^2 Z_{\rm sca} \ell_{\rm c}^2}.$$

↪ Scattering (only slightly) reduces the resolution.

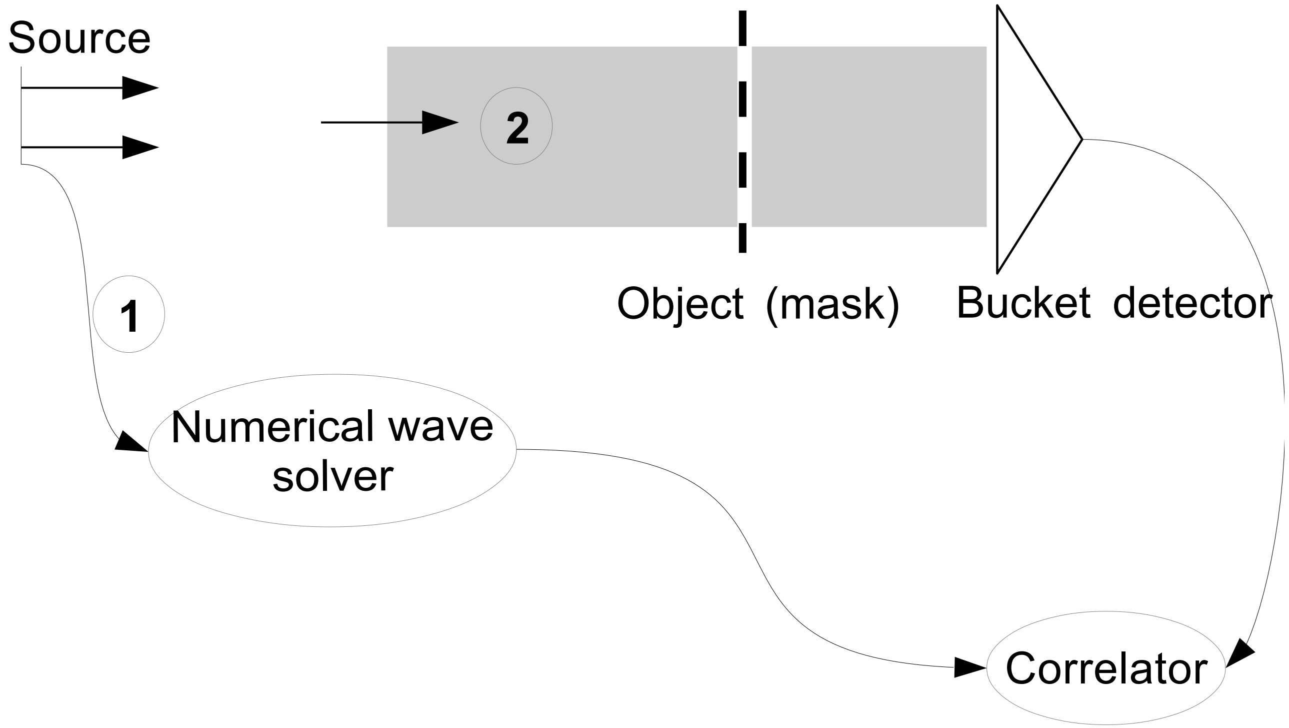
- If the medium in path 1 and 2 is random (*same* realization):

$$\mathcal{H}(\boldsymbol{x}) = \frac{r_o^4}{2^8 \pi^2 L^4} \exp\Big(- \frac{|\boldsymbol{x}|^2}{4\rho_{\rm gi2}^2}\Big), \qquad \frac{1}{\rho_{\rm gi2}^2} = \frac{1}{\rho_{\rm gi0}^2} + \frac{16 L}{Z_{\rm sca} \ell_{\rm c}^2}.$$

↪ Scattering enhances the resolution !

[1] J. Garnier, *Inverse Problems and Imaging* **10** (2016) 409.

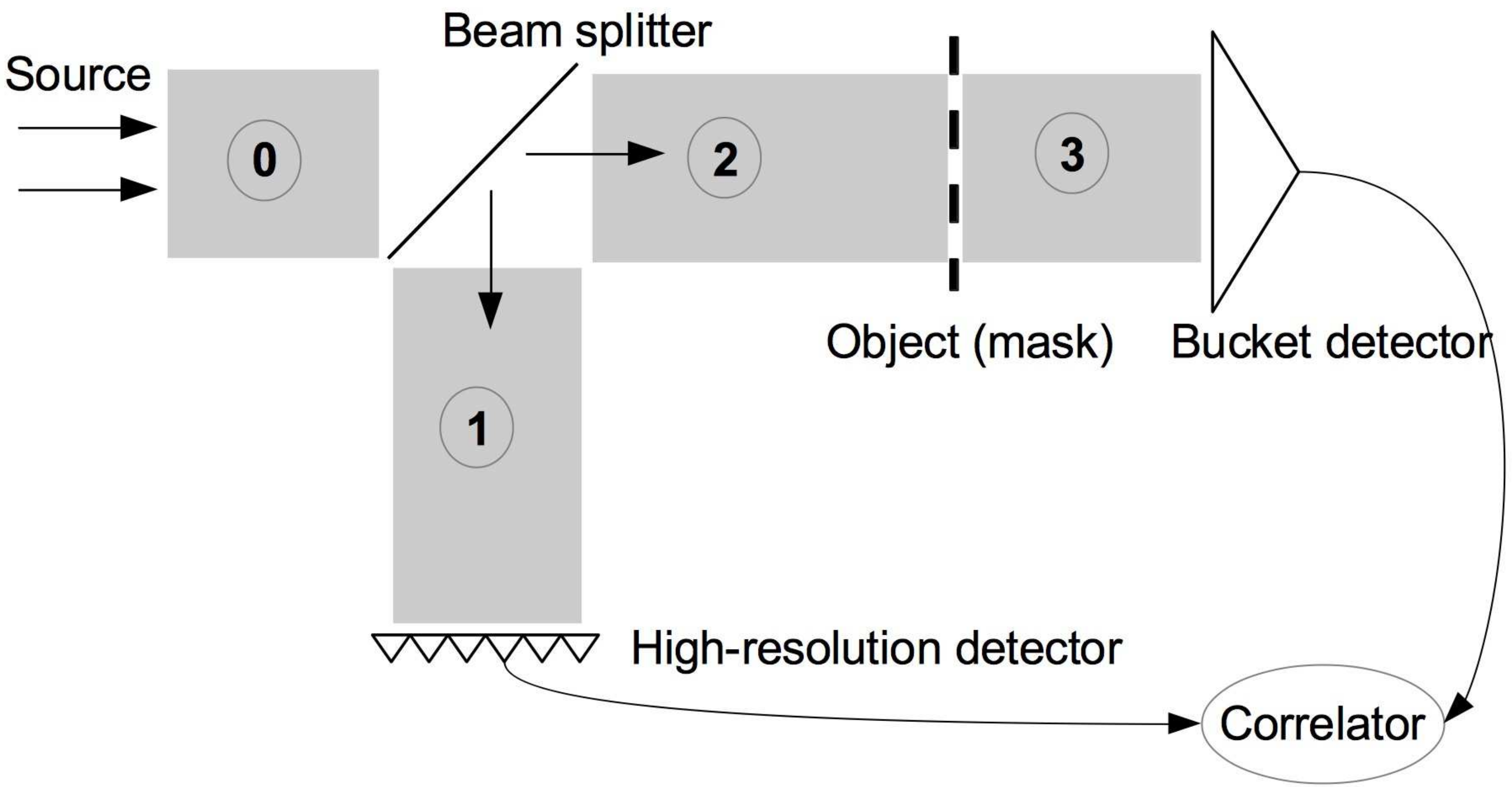
Speckle intensity correlation imaging through a scattering medium (IV)



- The medium in path 2 is randomly heterogeneous.
- There is no other measurement than $I_2(t)$.
- The realization of the source is known (use of a Spatial Light Modulator) and the medium is taken to be homogeneous in the "virtual path 1" $\rightarrow$ one can *compute* the field (and therefore its intensity $I_1(t, \boldsymbol{x})$) in the "virtual" output plane of path 1.

$\hookrightarrow$ a *one-pixel camera* can give a high-resolution image of the mask!

Conclusion: On the role of the random medium



Is random medium good or bad for imaging ?

Random medium in region 0 is *good*.

Random medium in regions 1 and 2 is *bad*.

Random medium in region 3 plays *no role*.