Qualitative Radar Imaging Under Randomized Field Illumination

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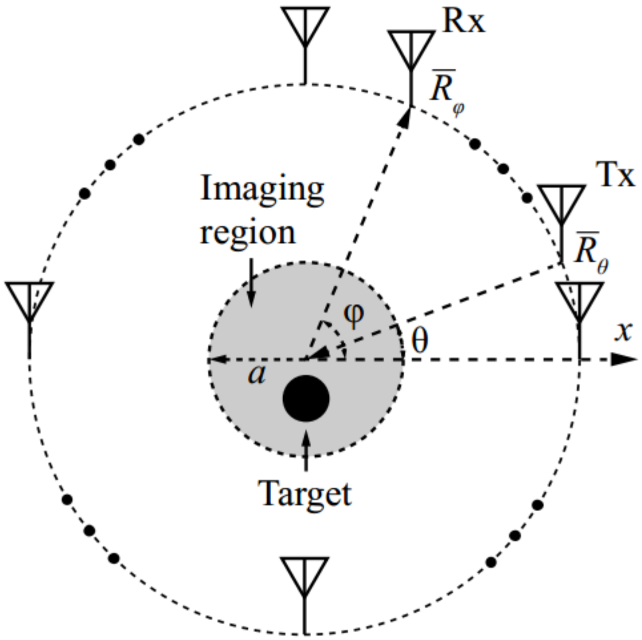
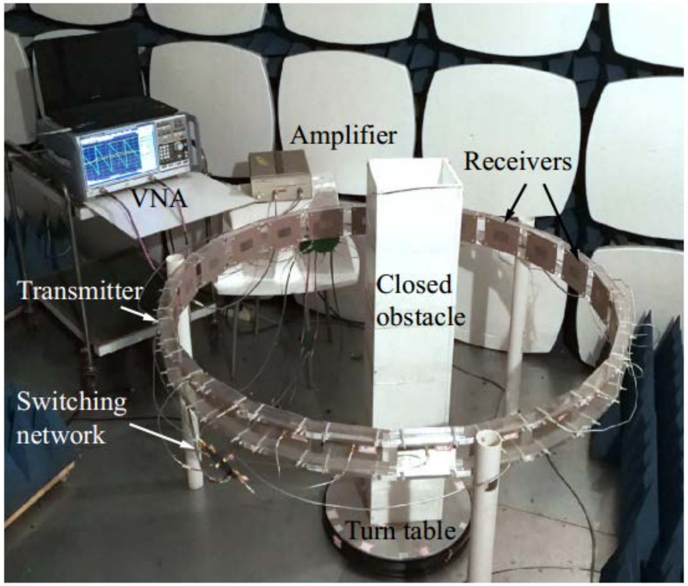
Topics

- Introduction
- Theory
- Simulation
- Experiment
- Discussion
- Summary

Introduction

Tomographic imaging

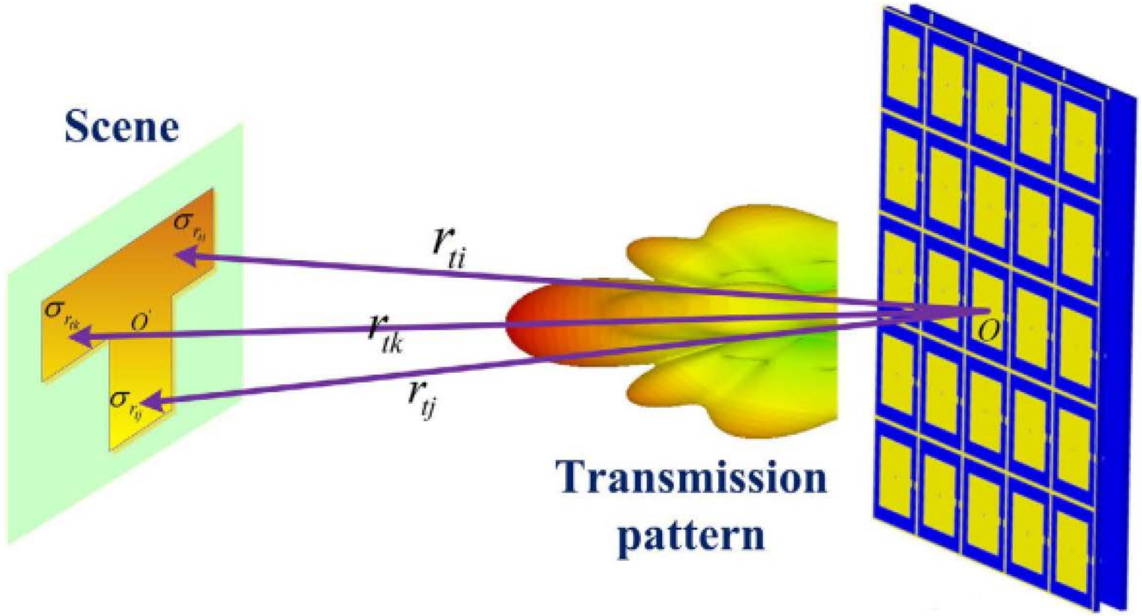




- MIMO structured
- **Quantitative:** reconstruct the spatial distribution of permittivity ($\boldsymbol{\varepsilon_r}$)
- Based on inverse scattering problem
- Essentially nonlinear, especially for larger ε_r

Introduction

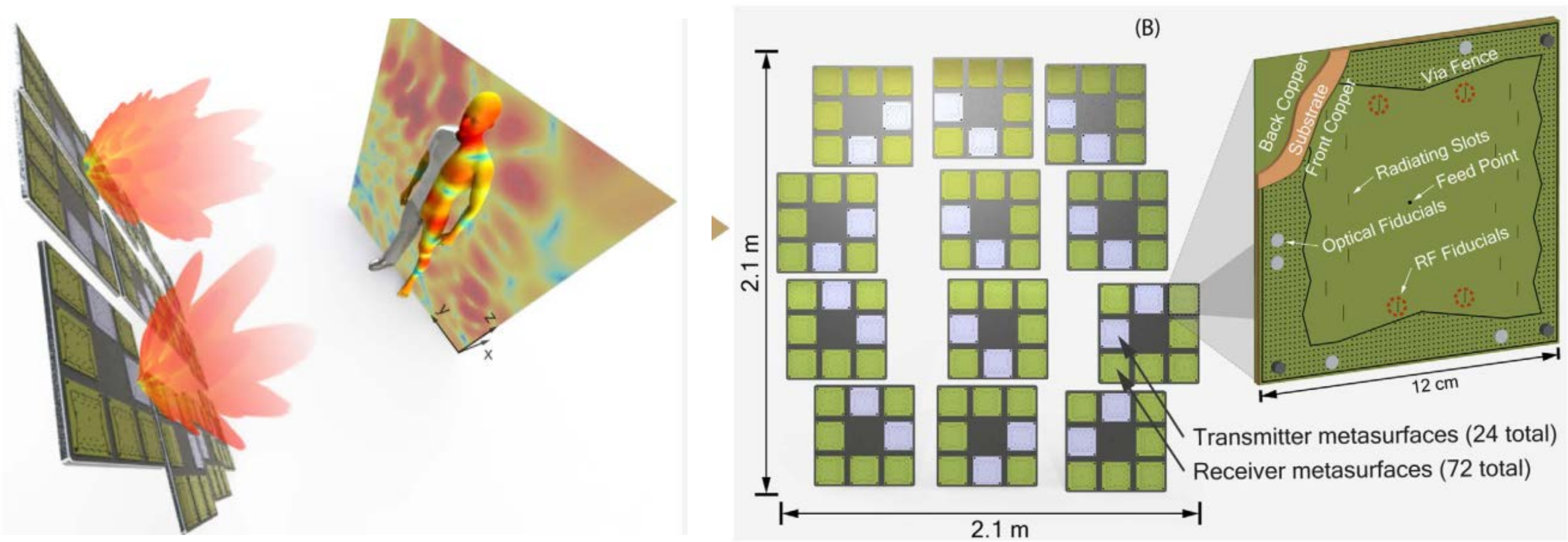
Radar imaging



- Phased array (MISO/SIMO) structured
- **Qualitative:** reconstruct the spatial distribution of reflectivity ($\sqrt{\varepsilon_r}$)
- Based on Born approximation
- Linearized, especially for larger ε_r

Introduction

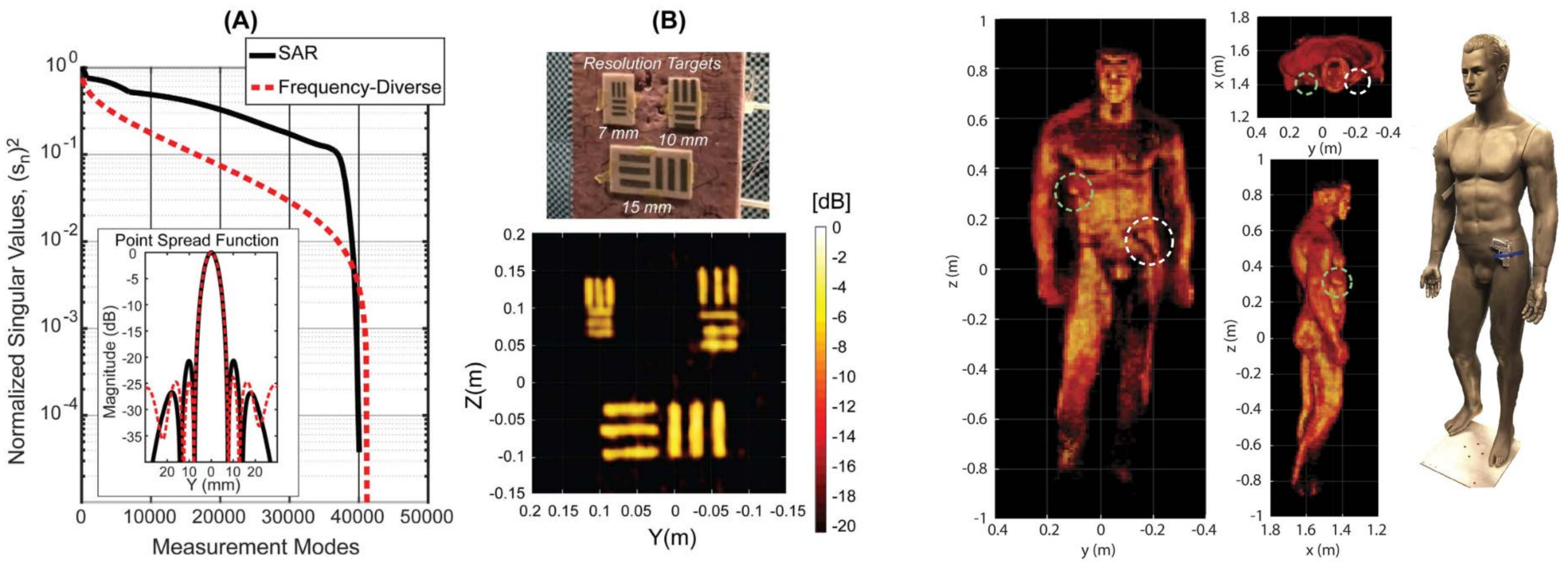
Radar imaging with random beams



- J. Gollub, et al, D. Smith, "*Large Metasurface Aperture for Millimeter Wave Computational Imaging at the Human-Scale*", Scientific Report, 2017
- Frequency-diverse metasurfaces, 17.5-26.5 GHz, spatially-diverse patterns
- 2.1m×2.1m, consisting of 24 transmitting and 72 receiving metasurfaces,

Introduction

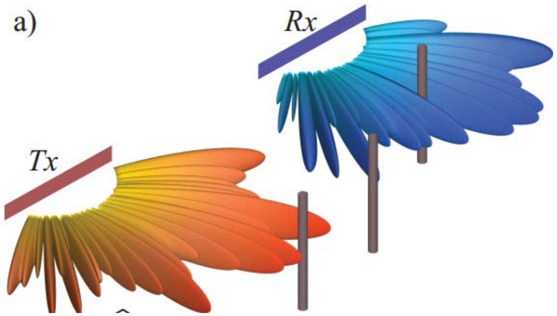
Latest result: Human-scale imaging

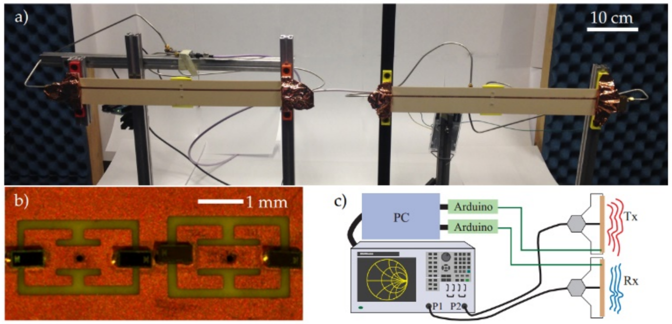


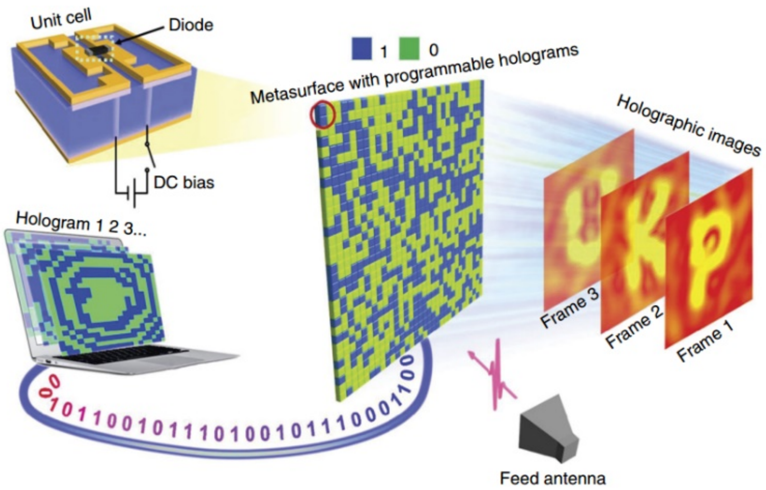
- Linear reconstruction matrix equation ***g = Hf +n***
- ***H***: linear transfer matrix should be established in advance
- Resolution researches 7 mm
- Identify gun and knife attached on a mannequin (painted by conductive layer)

Introduction

Other implementations



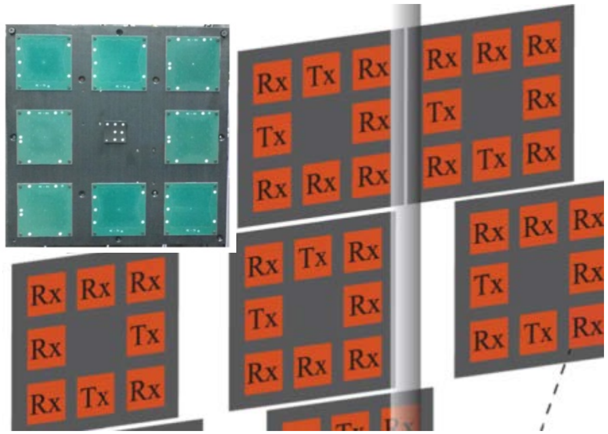


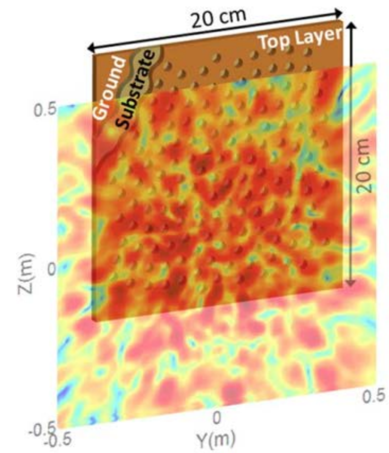


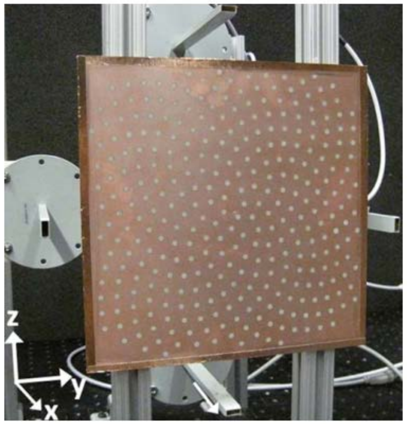
- Diode based dynamic 1-D and 2-D metasurfaces

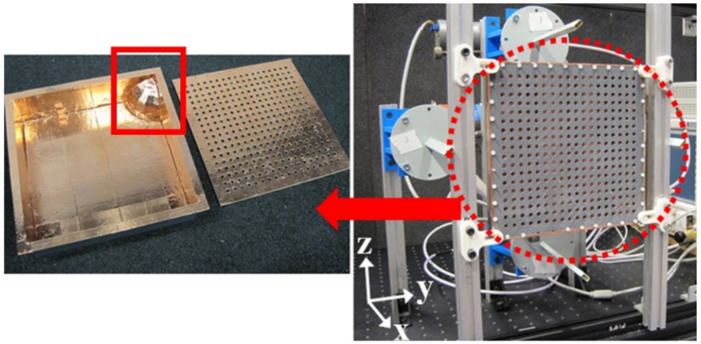
Introduction

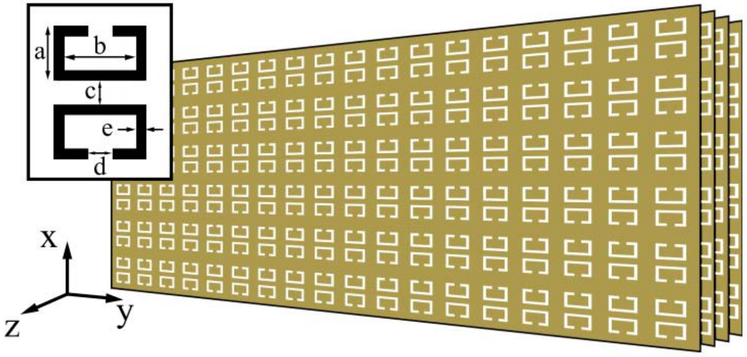
Other implementations











- Antenna array with randomized elements
- Frequency-diverse Resonant cavity with randomized slots
- Metasurface with complicated frequency dispersion

Introduction

Questions

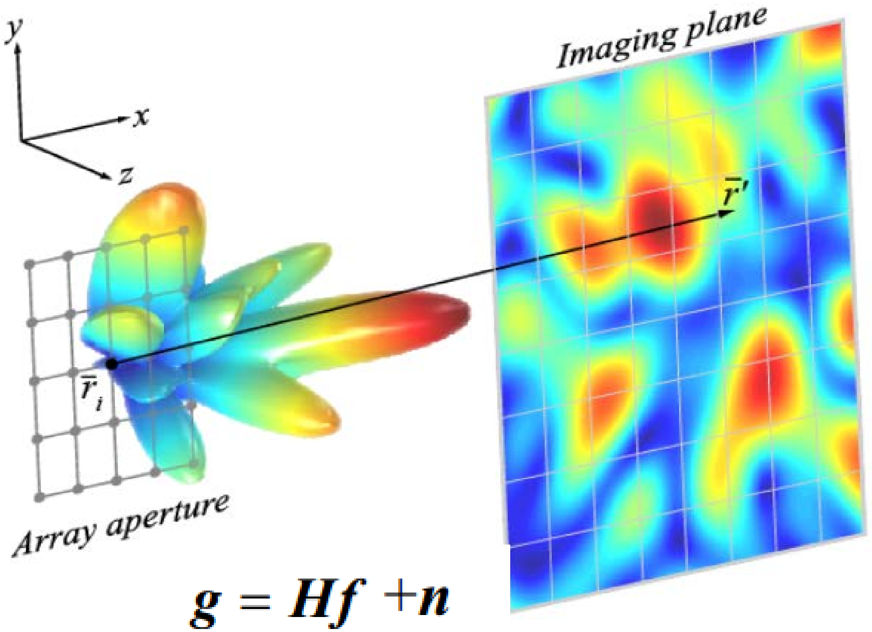
- Randomness of the Illuminations: random or pseudo random?
- Bandwidth: spectrum efficiency?
- Power consumption: energy efficiency?
- System scale: redundant?
- Imaging resolution: guaranteed?

Solution

- Making randomness definite
- Using conventional phased array
- Customization on demands

Theory

System description



$$g = Hf + n$$

- Array aperture: *N* element lies in *x*-*y* plane, *i*-th element located at $\bar{r}_i$
- Imaging plane: *N'* grids, *i'*-th grid located at $\bar{r}'_i$
- ***f***: spatial distribution of reflectivity, ***g***: measured scattered fields, ***n***: noises
- To solve ***f***, correlation between all of the row vectors of ***H*** should be zero

Theory

Derivations

- Incidence to the *i'*-th grid by the *i*-th element of the array

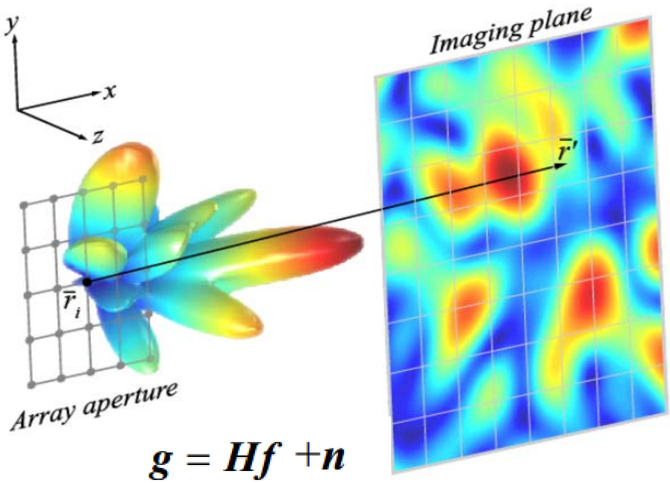
$$\bar{E}_i(\bar{r}') = jk\eta_0 I_0 e^{-j\varphi_i} G(\bar{r}',\bar{r}_i)$$

where $G(\bar{r}',\bar{r}_i) = e^{-jk|\bar{r}'-\bar{r}_i|} / 4\pi|\bar{r}'-\bar{r}_i|$

- Incidence to the *i'*-th grid by all the *N* elements of the array

$$\bar{E}_{inc}(\bar{r}') = \sum_{i=1}^{N} \bar{E}_i(\bar{r}') = jk\eta_0 \sum_{i=1}^{N} I_0 e^{-j\varphi_i} G(\bar{r}',\bar{r}_i) .$$

where $\boldsymbol{E}_{inc} = [\bar{E}_{inc}(\bar{r}'_1), \bar{E}_{inc}(\bar{r}'_2), \cdots, \bar{E}_{inc}(\bar{r}'_\infty)]$



$$\boldsymbol{g} = \boldsymbol{Hf} + \boldsymbol{n}$$

- The Green's functions of different elements are independent
- For *N* illuminations with completely randomized φ_i , the total electric field incident to the *i'*-th grid (and therefore all the grids) will be completely random
- Any additional illumination will be correlated with the previous *N* illuminations

Theory

Derivations

- For all the grids on the imaging plane

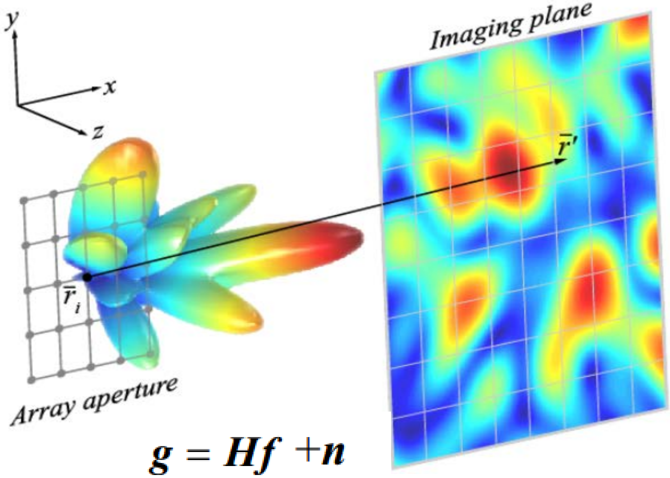
$$\bar{\boldsymbol{E}}_{inc} = jk\eta_0 I_0 \boldsymbol{\varphi} \boldsymbol{G}$$

where

$$\bar{\boldsymbol{E}}_{inc} = [\bar{E}_{inc}(\bar{r}'_1), \bar{E}_{inc}(\bar{r}'_2), \cdots, \bar{E}_{inc}(\bar{r}'_\infty)]$$

$$\boldsymbol{\varphi} = [e^{-j\varphi_1} \cdots e^{-j\varphi_i} \cdots e^{-j\varphi_N}]$$

$$\boldsymbol{G} = \begin{bmatrix} G(\bar{r}'_1,\bar{r}_1) & G(\bar{r}'_2,\bar{r}_1) & \cdots & G(\bar{r}'_\infty,\bar{r}_1) \\ \vdots & \vdots & & \vdots \\ G(\bar{r}'_1,\bar{r}_i) & G(\bar{r}'_2,\bar{r}_i) & & G(\bar{r}'_\infty,\bar{r}_i) \\ \vdots & & \ddots & \vdots \\ G(\bar{r}'_1,\bar{r}_N) & G(\bar{r}'_2,\bar{r}_N) & \cdots & G(\bar{r}'_\infty,\bar{r}_N) \end{bmatrix}$$



Theory

Derivations

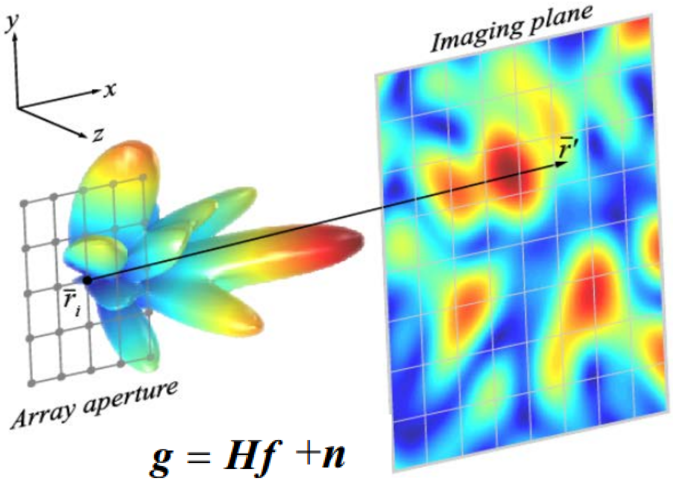
- For M measurements to $\bar{\boldsymbol{E}}_{inc}$

$$\bar{\boldsymbol{E}} = jk\eta_0 I_0 \boldsymbol{\Phi}\,\boldsymbol{G}$$

where

$$\bar{\boldsymbol{E}} = [\bar{\boldsymbol{E}}_{inc}^{(1)}, \bar{\boldsymbol{E}}_{inc}^{(2)} \cdots \bar{\boldsymbol{E}}_{inc}^{(M)}]^T$$

$$\boldsymbol{\Phi} = [\boldsymbol{\varphi}^{(1)}, \boldsymbol{\varphi}^{(2)} \cdots \boldsymbol{\varphi}^{(M)}]^T$$



$$\boldsymbol{g} = \boldsymbol{H}\boldsymbol{f} + \boldsymbol{n}$$

- According to matrix theory, the rank of ***E*** will satisfy

$$R(\bar{\boldsymbol{E}}) \le \min\{R(\boldsymbol{\Phi}), R(\boldsymbol{G})\}$$

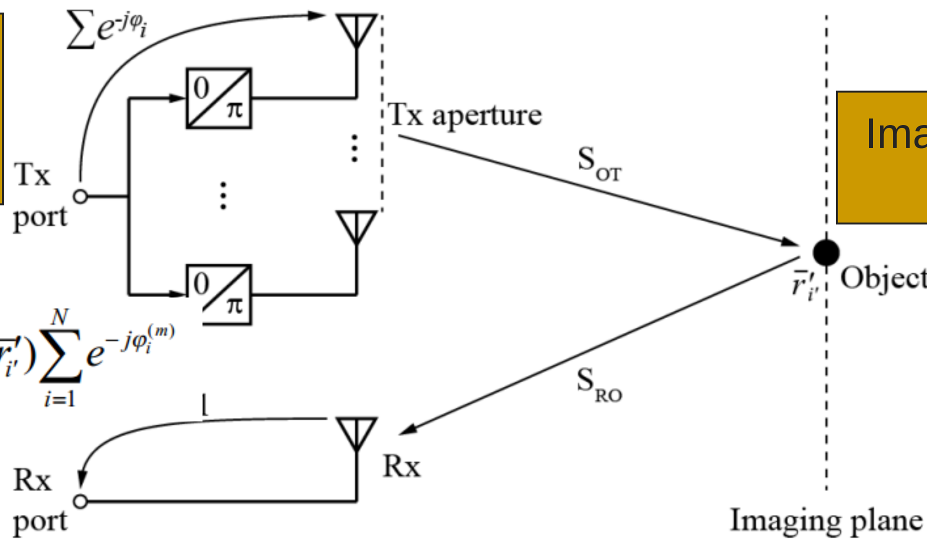
- Since the maximum rank of ***R***(*G*) is ***N***, and the maximum rank of ***R(Φ)*** is the smaller one in *M* and *N,* when *M* is increased to *N*, the rank of ***E*** can be maximized to *N*.
- In this case, the *N* measurements will be independent, leading to the most independent equations for the reconstruction of the image.
- Any additional measurement over *N* will be redundant.

Theory

Imaging equation

N-way divider
N 1-bit modulator
N-element

Imaging plane meshed to *N'* grids



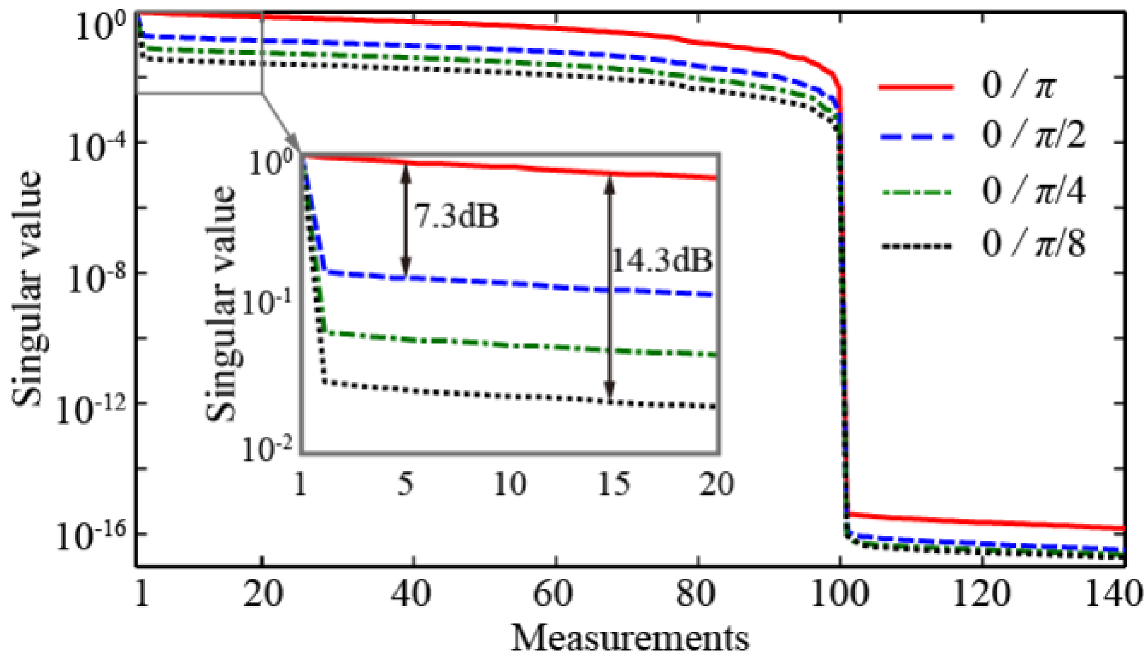
$$H^{(m)}(\bar{r}'_{i'}) = S_{RO}(\bar{r}'_{i'})S_{OT}(\bar{r}'_{i'})\sum_{i=1}^{N} e^{-j\varphi_i^{(m)}}$$

$$\begin{bmatrix} g^{(1)} \\ g^{(2)} \\ \vdots \\ g^{(M)} \end{bmatrix} = \begin{bmatrix} H^{(1)}(\bar{r}'_1) & H^{(1)}(\bar{r}'_2) & \cdots & H^{(1)}(\bar{r}'_{N'}) \\ H^{(2)}(\bar{r}'_1) & H^{(2)}(\bar{r}'_2) & & H^{(2)}(\bar{r}'_{N'}) \\ \vdots & & \ddots & \\ H^{(M)}(\bar{r}'_1) & H^{(M)}(\bar{r}'_2) & & H^{(M)}(\bar{r}'_{N'}) \end{bmatrix} \begin{bmatrix} f(\bar{r}'_1) \\ f(\bar{r}'_2) \\ \vdots \\ f(\bar{r}'_{N'}) \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_M \end{bmatrix}$$

- If the imaging region was meshed so that *N'* = *N*, $\boldsymbol{f} = \boldsymbol{H}^{-1}\boldsymbol{g}$
- If *N'* > *N*, ill-posed.

Simulations

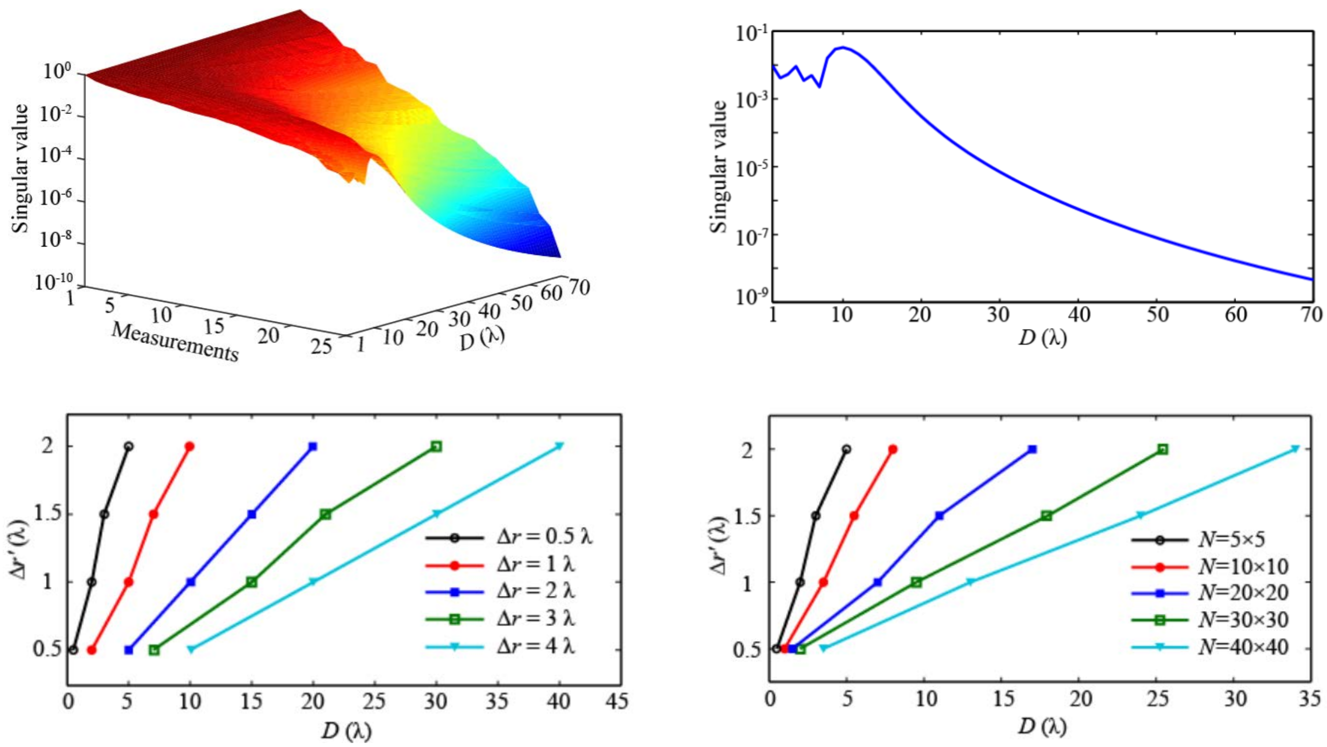
Randomness of ***H***



- 10×10-element array, 20×20 0.5-*λ* grids, periodicity 1*λ*
- Normalized singular values suddenly drop at $M = N' = N$ =100 measurement
- 1-bit phase toggling between 0 and *π* phase works best.
- When $M = N' = N$, ***H*** will be full-rank, $\boldsymbol{f} = \boldsymbol{H}^{-1}\boldsymbol{g}$, implying an efficient imaging

Simulations

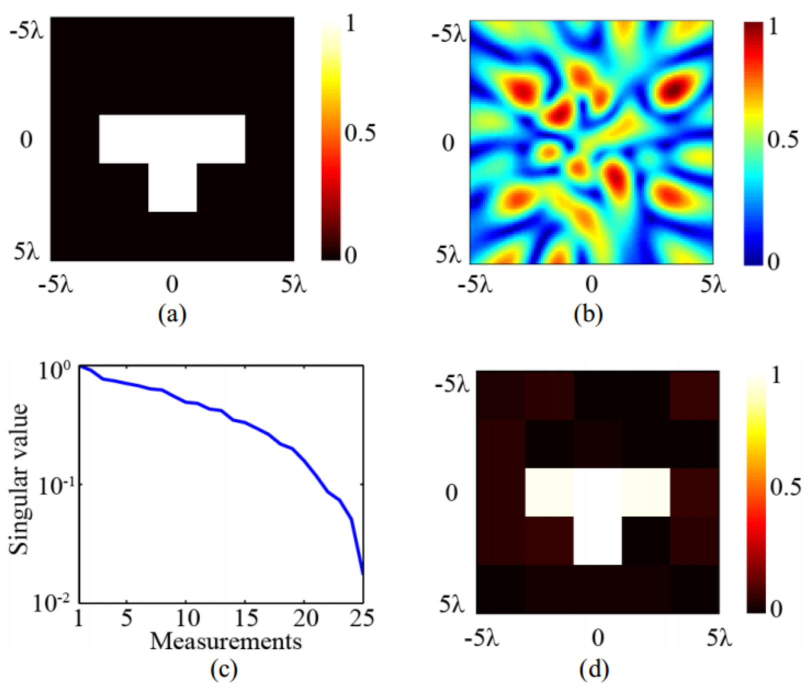
Customization on demand

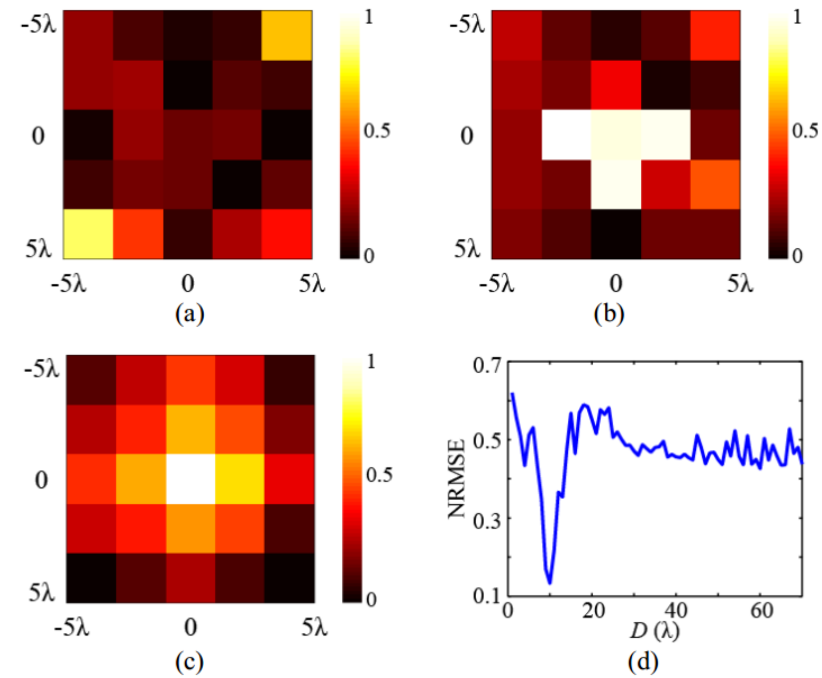


- **Input**: approximate size of the object and desired imaging resolution *Δr'*
- **Output**: the size of the imaging region, total grids number *N*' , antenna element number *N*, element periodicity *Δr*, measurement number *M*, imaging distance *D*

Simulations

Verification

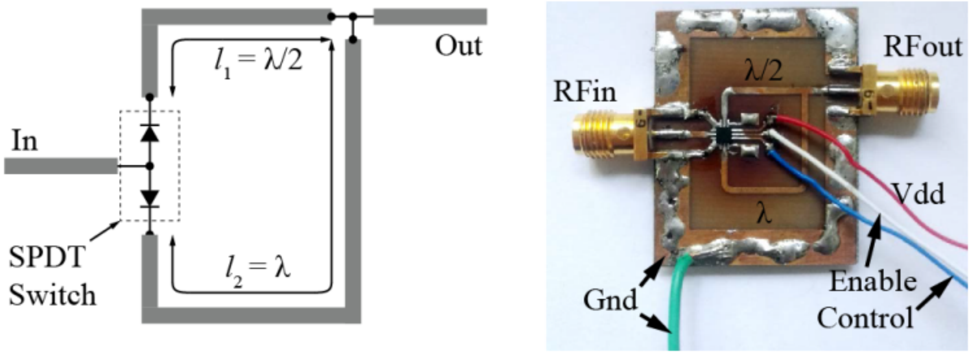




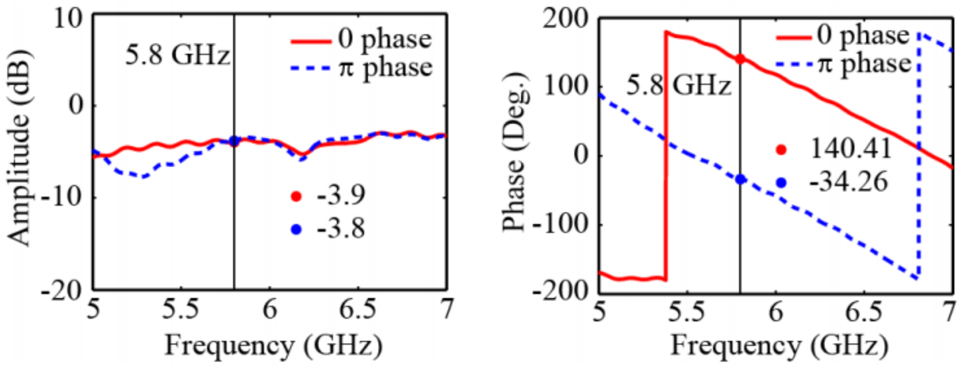
- **Left:** With a 25-dB SNR: (a) T-shaped object, (b) random illumination, (c) singular values of ***H***, (d) reconstructed imaging
- **Right**: Reconstructed images with a 15-dB SNR: (a) 5-λ, (b)10-*λ and* (c) 15-*λ* imaging distances. (d) NRMS errors for different imaging distances.

Experiments

1-bit phase modulator



(a)

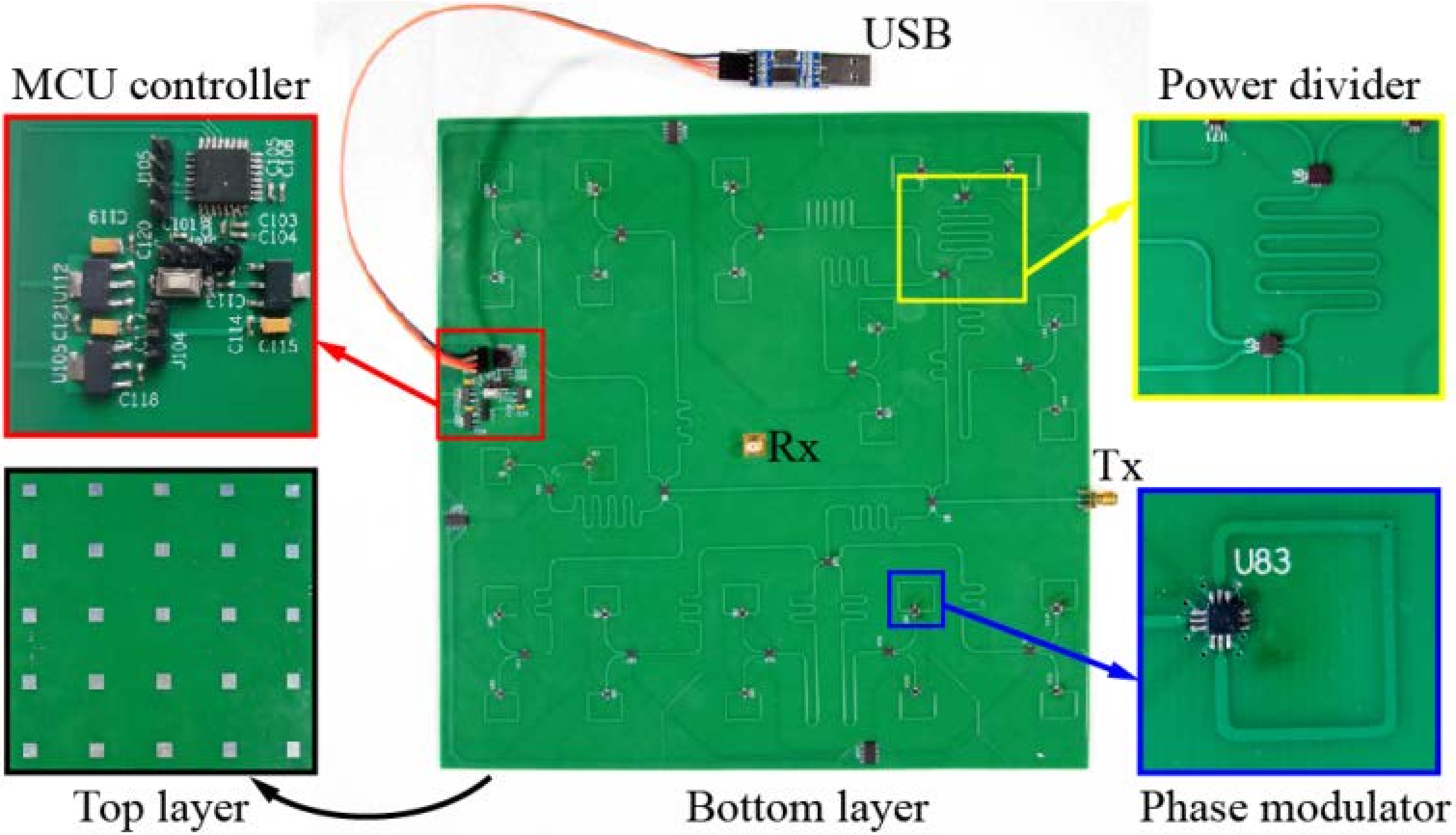


(b)

- Designed at 5.8 GHz
- (a) Circuit and test board; (b) Measured amplitude and phase difference.

Experiments

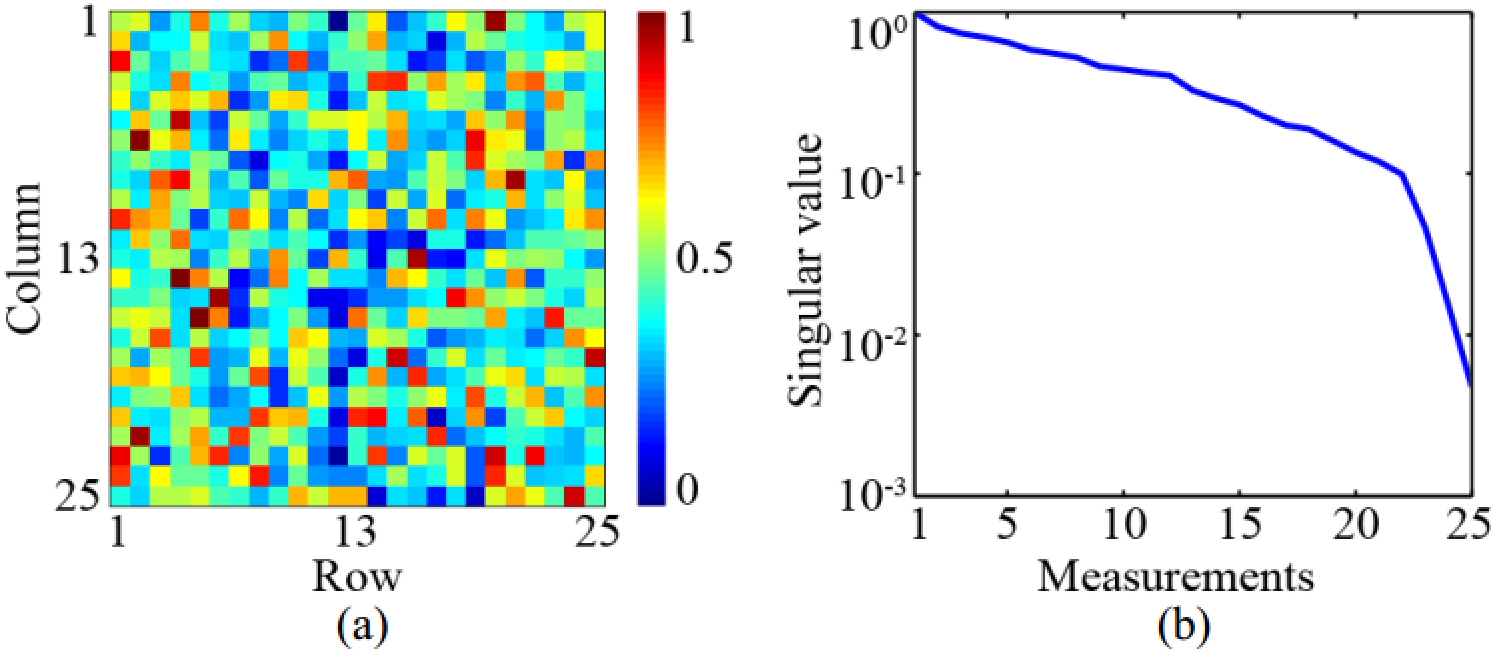
Implementation



- Board-integrated imaging system
- 5×5 elements, 25 measurement, each finished in microseconds

Experiments

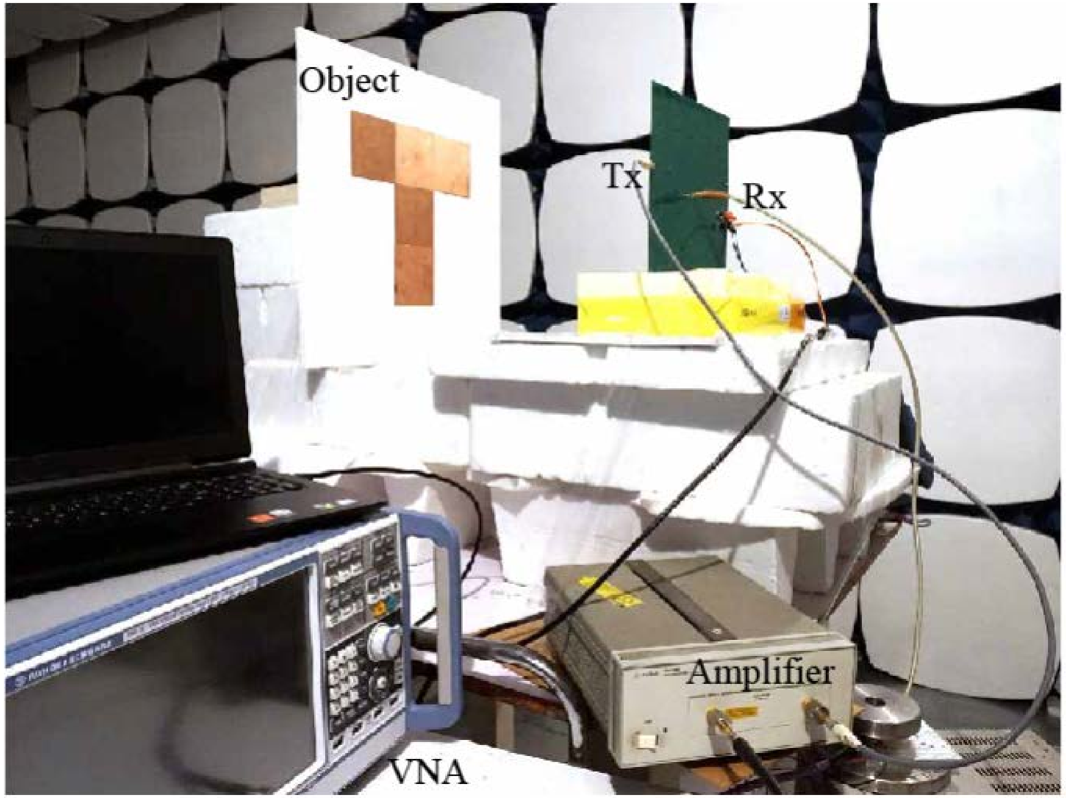
Calibration



- Randomness of the experimental ***H*** matrix

Experiments

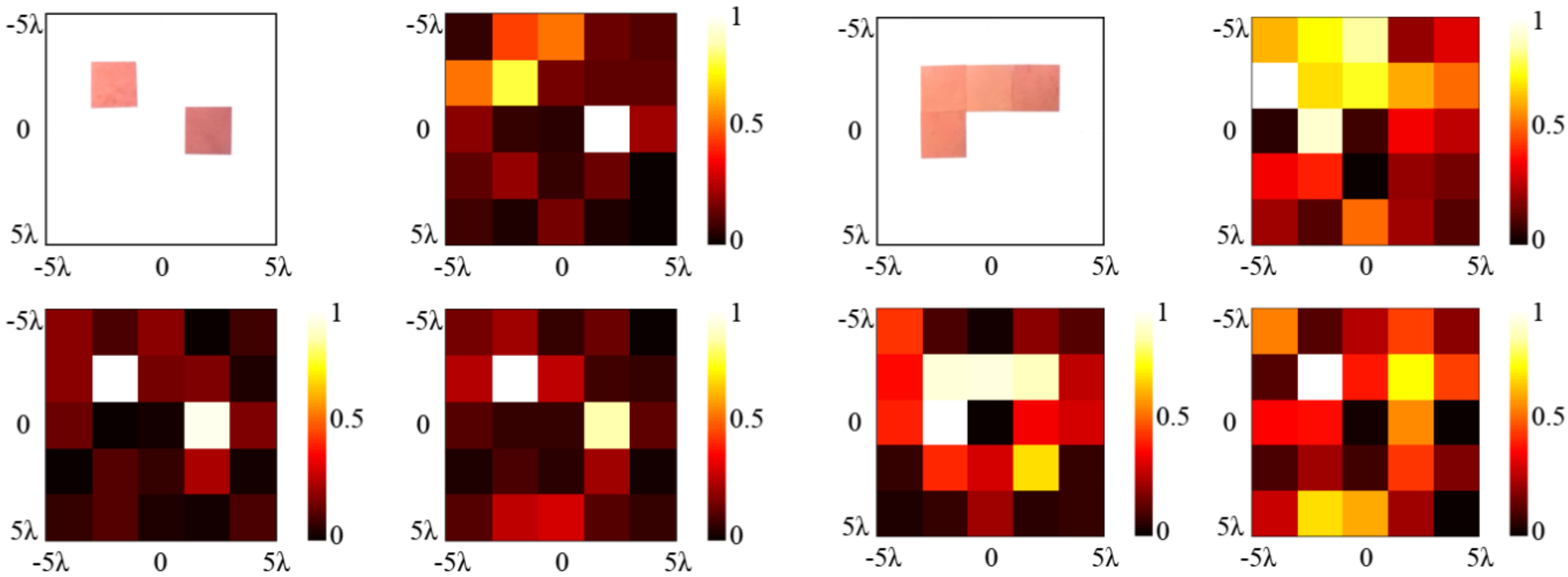
Setup



- Measurement inside a chamber

Experiments

Results

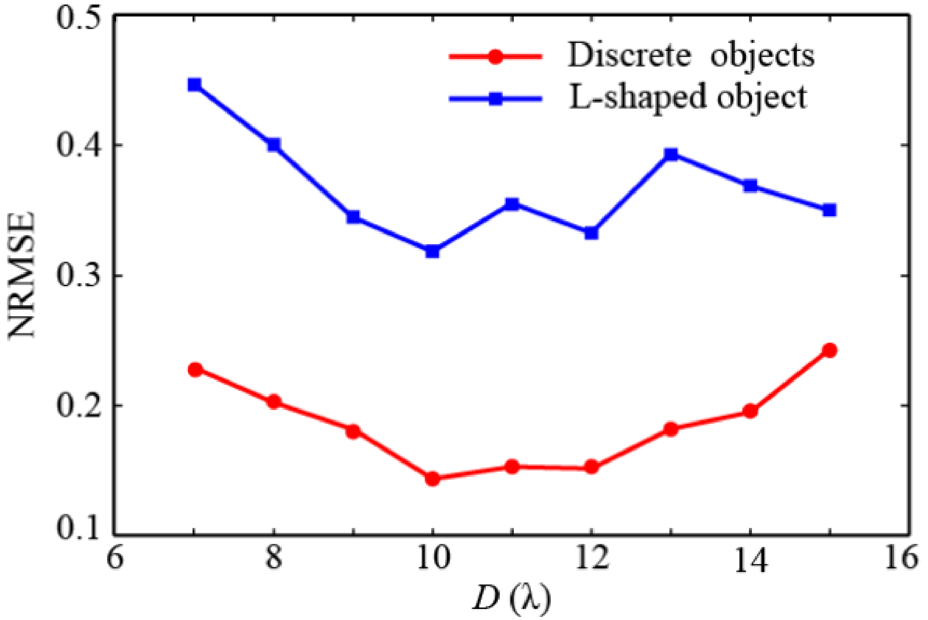


(a) (b)

- (a) Imaging to two discrete copper sheets at 7-, 10- and 13-λ distances
- (b) Imaging to an L-shaped copper sheet at the same distances.

Experiments

Error analysis

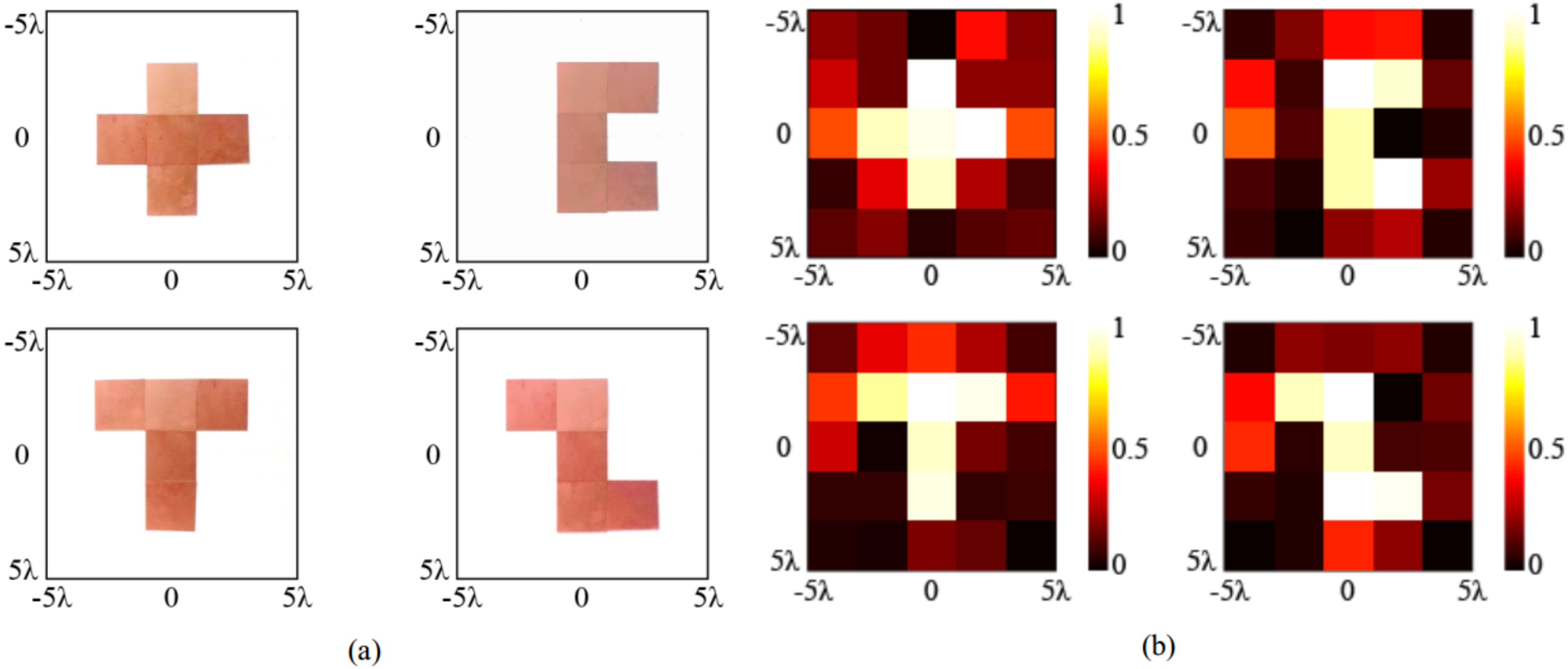


$$NRMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\hat{f}_i - f_i)^2} / (f_{\max} - f_{\min})$$

- At the optimal distance, the imaging quality is the best
- Reconstructed background also contributes to the error

Experiments

Results



- Other shapes also work
- The above results verified the proposed imaging system and algorithms.

Discussions

- Based on the ensured, complete randomness, imaging is highly efficient
- The imaging system can be customized on demand, to realize various minimal imaging system, costing minimum resources
- Including: minimum number of antenna elements, full-rank ***H*** matrix with a minimum dimension, minimum measurements
- Imaging can be finished in milliseconds
- Best spectrum efficiency and power efficiency
- Solutions for larger objects or higher resolutions
  - Increasing the scale of the illumination array
  - Using higher frequencies: switches are available at sub-THz band
  - Using step-frequency, multicarrier or wideband signals
  - Adopting inverse problem based imaging

Thank you for your attention!