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A Proof of the ABC Conjecture After Mochizuki

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ABSTRACT

I will explain Mochizuki's inter-universal Teichmüller theory and its Diophantine consequence.

Siegel series and intersection numbers

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POSTECH, Korea

ABSTRACT

The Siegel series is a local factor of the Siegel-Eisenstein series, which is expected to be a counter-part to the local deformation theory on (orthogonal or unitary) Shimura varieties.

In this talk, I will explain a conceptual reformulation of the Siegel series. This, combined with a recent work of Ikeda and Katsurada [2], gives a new interpretation on the intersection number studied by Gross and Keating in [1]. This is based on a joint work with T. Yamauchi.

References

- [1] B-H. Gross and K. Keating, *On the intersection of modular correspondences*, Invent math.112 (1993) 225-245.
- [2] T. Ikeda and H. Katsurada, *Explicit formula for the Siegel series of a half-integral matrix over the ring of integers in a non-archimedian local field*, arXiv:1602.06617.

On the cyclotomic p -adic triple product L -functions

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ABSTRACT

In this talk, we explain a construction of the four variable p -adic triple product L -functions attached to three Hida families of elliptic newforms and give the explicit interpolation formulae at all critical values in the balanced range. Our method uses a p -adic interpolation of Garrett's integral representation of triple product L -functions. We will also talk about a p -adic Gross-Zagier formula relating the central cyclotomic derivative of this p -adic triple product L -functions to the p -adic height of the diagonal cycles. This is a joint work in progress with Shunsuke Yamana.

A duality for Selmer group

SOMNATH JHA

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ABSTRACT

We will discuss a duality result for certain Selmer group over p -adic Lie extensions. This duality can be thought as an analogue of the ‘functional equation’ of the corresponding p -adic L -function. An important ingredient in this is a twisting Lemma.

This talk is based on joint works with T. Ochiai and G. Zabradi.

A non-vanishing theorem for the complex L -functions of Gross curves

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ABSTRACT

Let $K = \mathbb{Q}(\sqrt{-q})$, where q is any prime with $q \equiv 7 \pmod{8}$, and let H be the Hilbert class field of K . In his thesis [3], B. Gross constructed an elliptic curve A over H , with minimal discriminant $(-q^3)$, which is in some sense the smallest elliptic curve with complex multiplication by the full ring of integers of K . In the 1980's, David Rohrlich proved using complex analysis that $L(A/H, 1) \neq 0$, where $L(A/H, s)$ denotes the complex L -series of A . In my lecture, I will discuss some recent joint work with John Coates in which, when $q \equiv 7 \pmod{16}$, we give a completely different non-archimedean proof of Rohrlich's theorem using Iwasawa theory for the prime $p = 2$. In fact, the argument shows that $L(A/F, 1) \neq 0$ for F running over all finite extensions of H contained in $H(A_{\mathfrak{p}\infty})$, where \mathfrak{p} denotes either of the prime ideals dividing 2 in the ring of integers of K . Our method can also be used to prove a curious assertion about the complex L -series of essentially arbitrary quadratic twists of A , which was previously unknown even when $q = 7$, in which case A is then the modular curve $X_0(49)$.

References

- [1] J. Choi, J. Coates, *Iwasawa theory of quadratic twists of $X_0(49)$* , Acta Mathematica Sinica, English Series, 34 (2018), 19-28.
- [2] J. Coates, Y. Li, *A non-vanishing theorem for the complex L -functions of Gross curves*, preprint.
- [3] B. Gross, *Arithmetic on elliptic curves with complex multiplication*, Springer LN 776, 1980.

BSD Conjecture for Elliptic Curves and Modular Forms

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ABSTRACT

We report some recent progresses on BSD formulas for elliptic curves and their generalizations to modular forms in the rank 0 and 1 cases. In particular we now know that the full BSD conjecture holds for many infinite families of non-CM elliptic curves.

Mock modular forms and modular traces of singular moduli

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ABSTRACT

For a fundamental discriminant $D < 0$, “singular moduli” means the value of a modular function at a CM point with discriminant D . Thanks to results of Zagier (for the case of level one), and Bruinier and Funke (for the case of general level), the generating function of modular traces of singular moduli for a fixed modular function is a mock forms of weight $3/2$. For a fundamental discriminant $D > 0$, Duke, Imamoglu, and Toth gave the definition of singular moduli of a modular function by using the integral of the modular function on a certain geodesic of a modular curve. They showed that modular traces of singular moduli ($D > 0$) for a fixed modular function is a mock forms of weight $1/2$.

Based on this progress on modular traces of singular moduli, in this talk I will talk about the following question: for a fixed modular function f , to find arithmetic connections between traces of singular moduli of $f(d > 0)$ and those of $f(d < 0)$. To introduce our results with Subong Lim on this question, first I will review basic notations of mock modular forms and results on modularity of traces of singular moduli. Next, I will announce our results and then give a brief sketch of the proof of the results.

Densities and stability via factorization homology

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ABSTRACT

Using factorization homology, we develop a uniform and conceptual approach for treating homological stability, homological densities, and arithmetic densities for configuration spaces (and generalizations thereof) in algebraic geometry. This categorifies and generalizes the coincidences appearing in the work of Farb-Wolfson-Wood, and in fact, provides a conceptual understanding of these coincidences. Our computation of the stable homological densities also yields rational homotopy types which answer a question posed by Vakil-Wood. Our approach hinges on the study of homological stability of cohomological Chevalley complexes, which is of independent interest.

Asymptotic distribution of Hecke eigenvalues on compact arithmetic quotients

SATOSHI WAKATSUKI

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ABSTRACT

In this talk, we describe the asymptotic distribution of Hecke eigenvalues in the Laplace eigenvalue aspect for a family of Hecke-Maass forms on compact arithmetic quotients. In particular, our family includes not only spherical, but also non-spherical Hecke-Maass forms. In preceding studies of distributions of Hecke eigenvalues, the trace formula played an important role. But since it is not available for our family at the present time, we worked out a new approach which is based on Fourier integral operator methods. This is a joint work with Pablo Ramacher.

Extended monstrous moonshine

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ABSTRACT

Monstrous Moonshine was extended in two complementary directions during the 1980s and 1990s, giving rise to Norton's Generalized Moonshine conjecture and Ryba's Modular Moonshine conjecture. Generalized Moonshine is now interpreted as a statement about orbifold conformal field theory, and Modular moonshine is interpreted as a statement about Tate cohomology of an integral form of the Moonshine module. Both conjectures have been unconditionally resolved in the last few years, so we outline the solutions, and consider some speculative conjectures that may extend and unify them.

2-Selmer groups of even hyperelliptic curves over function fields

DAO VAN THINH

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ABSTRACT

In this talk, I am going to compute the average size of 2-Selmer groups of the family of even hyperelliptic curves over function fields. The result will be obtained by a geometric method which is based on a Vinberg's representation of the group $G = \mathrm{PSO}(2n+2)$ and a Hitchin fibration. Their relation helps us to connect the size of 2-Selmer groups and the number of rational points on Hitchin fibers, where the latter are more accessible by using the canonical reduction theory of G-bundles.

On the diagonal cubic equation involving E_2 -numbers

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ABSTRACT

We consider the diagonal cubic equation $N = x_1^3 + \cdots + x_8^3$, where the variables $x_1, \dots, x_8 \in \mathbb{N}$ satisfy certain multiplicative restrictions. An open problem in this topic is to prove that for large even number N , the above equation is solvable in prime variables. We say the natural number x is a E_2 -number if x has exactly two prime factors. We shall solve the above equation, where all of $x_j (1 \leq j \leq 8)$ are E_2 -numbers. This is a joint work with Koichi Kawada.

Noncongruence subgroups and nonabelian level structures for elliptic curves

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ABSTRACT

The existence of noncongruence subgroups of $SL_2(\mathbb{Z})$ has been known since the 1890's, and yet compared to their congruence counterparts, their structure has remained relatively mysterious. In [1], it was shown that noncongruence modular curves arise as moduli spaces of nonabelian covers of elliptic curves branched at most above the origin, and that through this interpretation, one can deduce certain arithmetic properties satisfied by the associated noncongruence modular forms. In this talk we will explain these results and discuss some interesting questions and phenomena that arise in this setting.

References

- [1] Chen, William Yun. “Moduli Interpretations for Noncongruence Modular Curves”, *Mathematische Annalen* (2015): 1-86.

The kernel of an Eisenstein prime at composite level

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ABSTRACT

In this talk, we study the kernel of an Eisenstein ideal on the Jacobian variety of the modular curve of composite level. This work is completely known by Mazur in 1977 if the level is prime. After introducing some notations, we will propose a conjecture on the dimension of this kernel under certain assumptions based on SAGE computations. Then, we present a sketch of a proof of known cases. When the level is squarefree the result is based on joint work with Ken Ribet.

Mixed Hodge structures with modulus

TAKAO YAMAZAKI

Tohoku University, Japan

ABSTRACT

We define a category of mixed Hodge structures with modulus that generalizes classical mixed Hodge structures. It contains as a full subcategory the category of Laumon 1-motives. To a smooth proper variety equipped with two effective divisors we attach a mixed Hodge structure with modulus. As an application we generalize Kato-Russell's Albanese varieties with modulus to 1-motives.

This is joint work with F. Ivorra.

Length-Preserving Factorization of Affine Weyl Groups

MING-HSUAN KANG

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ABSTRACT

In this talk, we will introduce a new length preserving factorization of affine Weyl groups which involves the alternating product of all parabolic subgroups. Such factorization indeed arise from the study of Iwahori-Hecke algebra and zeta functions of high dimensional complexes. Moreover, it also gives us some new integer invariants of affine Weyl groups.

Ramanujan and π

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ABSTRACT

In 1914, S. Ramanujan published a paper titled “Modular equations and approximations to π ” (Quarterly Journal of Mathematics (45) (1914), 350-372). This paper did not receive wide attention from mathematicians until around 1985, when B. Gosper computed around 17 million digits of π using one of Ramanujan’s series for $1/\pi$ recorded in the paper. Since Gosper’s work, many research articles inspired by Ramanujan’s series have appeared. In this talk, I will present some of these works.

Hasse principles for multinorm equations

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ABSTRACT

A classical result of Hasse states that the norm principle holds for finite cyclic extensions of global fields, in other words local norms are global norms. We investigate the norm principle for finite dimensional commutative étale algebras over global fields; since such an algebra is a product of separable extensions, this is often called the multinorm principle. Under the assumption that the étale algebra contains a cyclic factor, we give a necessary and sufficient condition for the Hasse principle to hold, in terms of an explicitly constructed element of a finite abelian group. This can be seen as an explicit description of the Brauer-Manin obstruction to the Hasse principle.

Logarithmic capacity and equidistribution of algebraic integers

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ABSTRACT

I shall discuss several questions involving the distribution of algebraic integers, and in particular the eigenvalues of Frobenius endomorphisms, and of integer matrices. The natural tool for such topics is logarithmic capacity theory.

Goldfeld's conjecture and congruences between Heegner points

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ABSTRACT

Given an elliptic curve E over Q , a celebrated conjecture of Goldfeld asserts that a positive proportion of its quadratic twists should have analytic rank 0 (resp. 1). We show this conjecture holds whenever E has a rational 3-isogeny. We also prove the analogous result for the sextic twists of j -invariant 0 curves. For a more general elliptic curve E , we show that the number of quadratic twists of E up to twisting discriminant X of analytic rank 0 (resp. 1) is $\gg X/\log^{5/6}X$, improving the current best general bound towards Goldfeld's conjecture due to Ono–Skinner (resp. Perelli–Pomykala). We prove these results by establishing a congruence formula between p -adic logarithms of Heegner points based on Coleman's integration. This is joint work with Daniel Kriz.

Reductions of crystalline representations and hypergeometric polynomials

GO YAMASHITA

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ABSTRACT

We calculate the reduction modulo p of two dimensional crystalline representations of the absolute Galois group of \mathbb{Q}_p with Hodge-Tate weights $\{0, k-1\}$, where k is greater than or equal to 2 and less than or equal to $(p^2 + 1)/2$, by using integral p -adic Hodge theory. In the calculations, the hypergeometric polynomials mysteriously appear.

This is a joint work with Seidai Yasuda.