Developments in Superstring Perturbation Theory

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Why superstring theory?

Gravity is undoubtedly one of the forces we see in nature

– needed for describing interaction between large objects, e.g. earth and us, planets and sun, stars, galaxies etc.

Quantum mechanics is a framework needed for describing interaction between small objects,

e.g. electrons and the nucleus of an atom, protons and neutrons inside a nucleus.

This makes it clear that a complete description of nature will have to incorporate gravity in the framework of quantum mechanics

During the last 50 years, quantum mechanics has been extremely successful in describing all other forces we see in nature

– strong, weak and electromagnetic forces

The framework we use for this is called quantum field theory

– a theory whose basic ingredients are fields – like the familiar electric and magnetic fields – but it also leads to the notion that the constituents of matter are particles

– a consequence of wave-particle duality

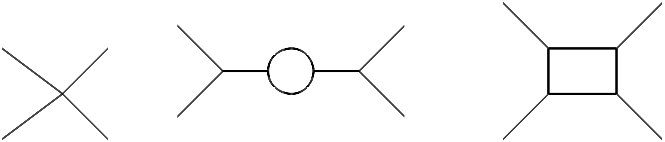
Most commonly used technique in quantum field theory is ‘perturbation theory’

– a Taylor series expansion in powers of interaction strength, assuming that the interaction strength is low

There is a systematic procedure for computing the coefficients of the series expansion

– known as Feynman diagram technique

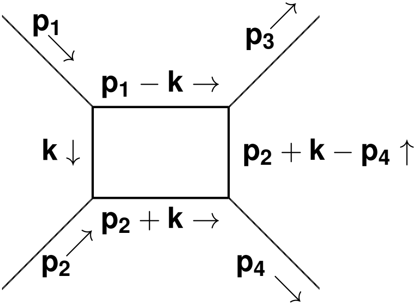
Examples of Feynman diagrams contributing to a 2 particle $\rightarrow$ 2 particle scattering amplitude:



Draw all possible graphs with four external lines

Quantum field theory gives definite rules for associating numbers to these diagrams

Diagrams with larger number of loops correspond to terms that are higher order in the perturbation expansion in powers of the interaction strength



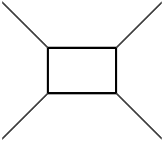
There are two equivalent ways of computing these diagrams, both involving integrals.

1. Integrate over the positions of the vertices

2. Integrate over momenta flowing in the loop

Occasionally we get infinite answer for these integrals

– known as ultraviolet (UV) divergences.



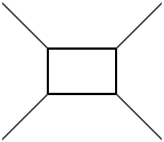
The UV divergences come from from the region of integration where

– the positions of the vertices come close to each other

– or momentum flowing in the loop become large.

Quantum field theory gives us a set of rules for removing these divergences and getting finite results

– known as renormalization

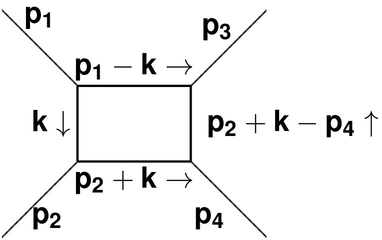


Quantum field theories also have ‘Infrared divergences’

– the integrand becomes infinite at some values of momenta

These represent physical phenomena and can be removed once we understand their origin

– will be discussed later



When we try to apply the quantum field theory methods to gravity, we run into difficulties.

The UV divergences are more severe than in the case of other forces

– related to the fact that the strength of gravitational force grows as the momenta carried by the particles increase.

The usual procedure for removing UV divergences fails.

String theory replaces the notion of point-like constituents of matter by string-like constituents of matter

– one dimensional objects

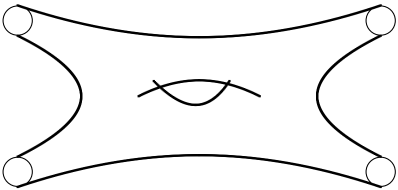
Like a normal musical string, a vibrating string can exist in many states of vibration, characterized by harmonics, amplitude and direction of vibration.

In quantum theory, each of these vibrational states appears as an elementary particle.

According to string theory the different elementary particles we see in nature are just different vibrational states of strings

One of them turns out to be the graviton – the mediator of gravitational force

Scattering amplitudes in string theory are described by different kinds of ‘Feynman diagrams’.



The ultraviolet divergences are absent due to the absence of definite interaction vertices.

This intuitive idea can be realized explicitly in a baby version of string theory known as the ‘bosonic string theory’.

In this theory a g-loop scattering amplitude with n-external states is given by

$$\int_{M_{g,n}} I(\{m\},\{Q\})$$

$M_{g,n}$: an abstract space labelled by 6g-6+2n coordinates

– known as moduli space of punctured Riemann surfaces.

6g-6 of these coordinates describe the shape of a Riemann surface with g handles

2n of the coordinates describe the locations of the n holes where the external strings attach (punctures)

$$\int_{\mathbf{M_{g,n}}} \mathbf{I}(\{\mathbf{m}\}, \{\mathbf{Q}\})$$

The integrand I depends on 6g-6+2n coordinates $\{\mathbf{m}\}$ of $\mathbf{M_{g,n}}$ and also on the momenta and other quantum numbers $\{\mathbf{Q}\}$ of the n external states.

$\mathbf{I}(\{\mathbf{m}\}, \{\mathbf{Q}\})$ is finite in the interior of the moduli space

$\Rightarrow$ absence of ultraviolet divergences.

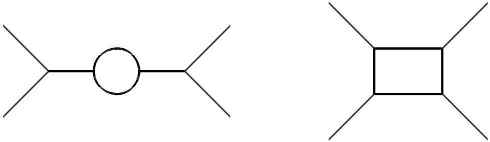
There are infrared divergences from the boundaries of $\mathbf{M_{g,n}}$ (to be discussed later)

Additional advantages in string theory

• String theory automatically contains gravity

– no need to add gravity as an additional force.

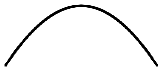
• In quantum field theories, there are many Feynman diagrams that we need to add for a given number of loops



In contrast string theory has only one term (for a given number of loops)

Unfortunately bosonic string theory is not fully consistent due to the existence of ‘tachyon’

The perturbation theory describes perturbation around the maximum of a potential



Some day we may find the minimum of the potential and develop perturbation theory around the minimum, but as of today, we do not know if there is a minimum.

This problem is overcome in superstring theory where we add more modes of vibration to the string

– does not have any instability

– shares the good properties with bosonic string theory, e.g. inclusion of gravity and ultraviolet finite perturbation theory

– gives a consistent ultraviolet finite quantum theory of gravity.

However there were some technical problems in the perturbation expansion which have been fully resolved only recently.

Spurious poles

g-loop scattering amplitude of m bosonic and 2n fermionic states has the form

$$\int_{M_{g,m+2n}} I(\{m\}, \{Q\}; y_1, \cdots y_{m+n+2g-2})$$

The integrand I depends on

- 6g-6+2m+4n coordinates $\{m\}$ of $M_{g,m+2n}$

- the momenta and other quantum numbers $\{Q\}$ of the external states

- the complex coordinates of $(m+n+2g-2)$ additional points $y_1, \cdots y_{m+n+2g-2}$ on the Riemann surface, known as 'locations of picture changing operators' Friedan, Martinec, Shenker

$$\int_{M_{g,m+2n}} I(\{m\},\{Q\};y_1,\cdots y_{m+n+2g-2})$$

The result seems to depend on the spurious data $y_1,\cdots y_{m+n+2g-2}$.

Under a change in the variables y_i, the integrand changes by a total derivative in the variables {m}

The change is zero if we can ignore boundary terms.

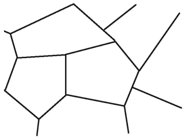
There is however a more serious problem.

The integrand has poles in $M_{g,m+2n}$ whose locations depend on the y_i's, making the integral ill-defined

Dijkgraaf, Verlinde, Verlinde

Solution

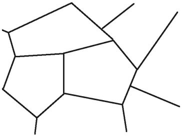
- Divide $M_{g,m+2n}$ into a collection of small cells



- In each cell choose the y_i's so that $I(\{m\};\{Q\};\{y\})$ does not have any pole in that cell.

- At the boundary between two cells y_i's jump

We add correction terms at the boundary to compensate for the jump.



We also need additional correction terms where the codimension one boundaries meet on a codimension two subspaces, and so on

– gives a systematic procedure for computing scattering amplitudes.

Final result is independent of the choice of the y_i's in each cell.

Infrared divergences

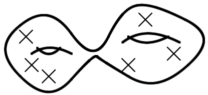
Scattering amplitude

$$\int_{\mathbf{M_{g,m+2n}}} \mathbf{I}(\{\mathbf{m}\},\{\mathbf{Q}\};\mathbf{y_1},\cdots\mathbf{y_{m+n+2g-2}})$$

$M_{g,N}$ has boundaries where the Riemann surface degenerates

– the surface develops a narrow neck

Example:



$$\int_{\mathbf{M_{g,m+2n}}} \mathbf{I}(\{\mathbf{m}\},\{\mathbf{Q}\};\mathbf{y_1},\cdots \mathbf{y_{m+n+2g-2}})$$

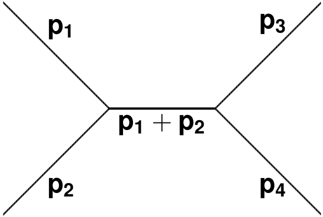
$\mathbf{I}(\{\mathbf{m}\},\{\mathbf{Q}\};\mathbf{y_1},\cdots \mathbf{y_{m+n+2g-2}})$ **diverges when** $\{\mathbf{m}\}$ **approaches one of these boundaries**

$\Rightarrow$ **often** $\int_{\mathbf{M_{g,m+2n}}} \mathbf{I}(\{\mathbf{m}\},\{\mathbf{Q}\};\mathbf{y_1},\cdots \mathbf{y_{m+n+2g-2}})$ **becomes ill-defined or divergent.**

These divergences are analogs of ‘infrared divergences’ in quantum field theory

– divergences associated with poles in the propagator

Example:

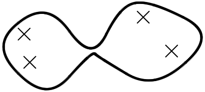


In this Feynman diagram the propagator carrying momentum $p_1 + p_2$ contributes

$$\{(p_1 + p_2)^2 + m^2\}^{-1}, \quad (p_1 + p_2)^2 \equiv -(p_1^0 + p_2^0)^2 + (\vec{p}_1 + \vec{p}_2)^2$$

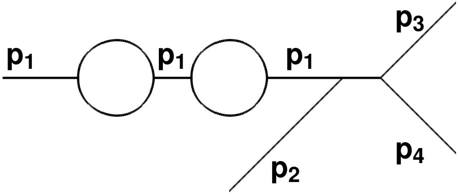
– has pole

In string theory this arises from the region of the moduli space where two punctures approach each other.



However understanding the physical origin of the divergence does not immediately offer a solution.

Example: Consider the following diagram in a quantum field theory.



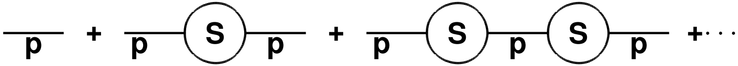
The intermediate propagators carrying momentum p_1 contributes $(p_1^2 + m^2)^{-1}$.

Usual relation between energy and momentum of the external state forces us to have

$$p_1^2 + m^2 = 0$$

causing this diagram to diverge.

In quantum field theory these divergences are addressed using mass renormalization.



is resummed as

$$\frac{1}{p^2+m^2}+\frac{1}{p^2+m^2}S(p)\frac{1}{p^2+m^2}+\cdots=\frac{1}{p^2+m^2-S(p)}$$

We then look for zeroes of $p^2+m^2-S(p)$ and identify that as 'renormalized' mass^2.

$\Rightarrow$ converts multiple poles into a single pole which can be handled by standard quantum field theory tricks.

In conventional string perturbation theory this option does not exist.

• Since there is only one term in every loop order, we cannot isolate divergent contributions at different loop orders and resum.

• For consistency of the formalism, the value of p^2 for external states is fixed from the beginning, and there is no option for changing it at the loop order.

Strategy: Combine the good features of string theory with the good features of quantum field theory

Use string field theory. Witten; Zwiebach; $\cdots$

Bosonic string field theory is a quantum field theory with infinite number of fields with the following properties:

Giddings, Martinec, Witten; Zwiebach; $\cdots$

• Scattering amplitudes are given by sum of Feynman diagrams as in a normal quantum field theory

• Contribution from each Feynman diagram may be expressed as integral over a cell of $\mathrm{M_{g,N}}$ with the correct integrand

• Sum of all Feynman diagrams gives us integration over the union of all the cells

– the whole moduli space.

If we have a superstring field theory, we can follow the correct rules, e.g. resum self-energy graphs and carry out mass renormalization to avoid the divergences

- would give a procedure for defining scattering amplitudes in superstring theory that are free from all divergences.

Main bottleneck: Writing down an action for the ‘Ramond sector’ fields.

Solved by doubling the number of string fields, with one copy remaining free even in the full quantum theory A.S.

This gives an action for all superstring field theories including type IIB string theory

– for this the action is not supposed to exist due to the existence of a self-dual 5-form field strength!

Corollary: This gives a method for writing down an action for a self dual field in 4n+2 dimensions by adding a free field that completely decouples from the theory

Field content:

1. A 2n-form field P

2. A (2n+1)-form self-dual field Q $\quad *\mathbf{Q} = \mathbf{Q}$

3. Other fields ϕ

Action

$$\int \left[\frac{1}{2} \mathbf{dP} \wedge * \mathbf{dP} - \mathbf{dP} \wedge \mathbf{Q} + \mathbf{L}(\mathbf{Q}, \phi) \right]$$

$$\int \left[\frac{1}{2} \mathbf{dP} \wedge * \mathbf{dP} - \mathbf{dP} \wedge \mathbf{Q} + \mathbf{L}(\mathbf{Q}, \phi) \right]$$

Equations of motion:

$$\mathbf{d}(*\mathbf{dP} + \mathbf{dP} - \mathbf{Q}) = \mathbf{0}, \quad \mathbf{dP} - *\mathbf{dP} + \cdots = \mathbf{0}, \quad \phi \text{ equations}$$

Degrees of freedom contain

– a free self-dual field $*\mathbf{dP} + \mathbf{dP} - \mathbf{Q}$

– an interacting self-dual field $\mathbf{dP} + \cdots$

Summary and Outlook

We now have a well-defined and unambiguous prescription for computing scattering amplitudes in superstring theory.

Goal: Make this into a practical tool

– develop codes for computing string scattering amplitudes to high order

Main bottleneck: A good parametrization of the moduli space of Riemann surfaces and a good choice of coordinate system on the Riemann surface.

Once we have the practical tools and get a few terms in the perturbation expansion, we can try resummation techniques to get information beyond perturbation theory