Abstracts

Automatic numbers and Mahler's method

Boris Adamczewski

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ABSTRACT

Some natural properties of real numbers, such as having a periodic expansion or having a computable expansion, do not depend on the integer base one chooses to represent them. In contrast, it is conjectured that the property of being generated by a finite automaton is strongly base-dependent. This problem remains completely open. In 1968, Cobham observed that generating functions of automatic sequences satisfy some linear difference equations associated with operators of the form $z \mapsto z^q$ for some natural number $q > 1$. Such functions are called Mahler functions after the pioneering work of Mahler. I will introduce a conjecture concerning the algebraic independence of values of Mahler functions at algebraic points that would solve, in a very strong form, the problem mentioned above. I will also explain how to attack this conjecture by using Mahler's method in several variables. This is a joint work with Colin Faverjon.

Spiral Delone set

Shigeki Akiyama

University of Tsukuba, Japan

ABSTRACT

We show that a constant angle progression on the Fermat spiral forms a Delone set if and only if its angle is badly approximable. More precisely, by using a classical three distance theorem on irrational rotation, we deduce quantitative statements between partial quotients of the angle and relative denseness and uniform discreteness of those points.

Normality equivalence and Pisot numbers

Shigeki Akiyama

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ABSTRACT

It is well-known that a-normality and b-normality is the same if and only if a and b are multiplicatively dependent. We discuss generalization of such results to ergodic piecewise linear maps sharing the same support of invariant measures. We find that it is very important to restrict ourselves to maps having Pisot number slope. We apply our result to give a one parameter family of generic point equivalent maps.

Normal numbers with digit dependencies

Verónica Becher

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ABSTRACT

We give metric theorems for the property of Borel normality for real numbers under the assumption of digit dependencies in their expansion in a given integer base. We quantify precisely how much digit dependence can be allowed such that, still, almost all real numbers are normal. Our theorem states that almost all real numbers are normal when at least slightly more than $\log\log n$ consecutive digits with indices starting at position n are independent. As the main application, we consider the Toeplitz set T_P, which is the set of all sequences $a_1a_2\dots$ of symbols from $\{0,\dots,b-1\}$ such that a_n is equal to a_{pn}, for every p in P and $n=1,2,\dots$. Here b is an integer base and P is a finite set of prime numbers. We show that almost every real number whose base b expansion is in T_P is normal to base b. In the case when P is the singleton set $\{2\}$ we prove that more is true: almost every real number whose base b expansion is in T_P is normal to all integer bases.

This is joint work between Christoph Aistleitner, Verónica Becher and Olivier Carton.

Bounded remainder sets and symbolic dynamics

Valérie Berthé

Université Paris-Diderot, France

ABSTRACT

We discuss dynamical approaches to the study of bounded remainder sets for Kronecker sequences inspired by symbolic dynamics where the notion of bounded remainder sets corresponds to bounded symbolic discrepancy. It provides particularly strong convergence properties of ergodic sums toward frequencies and is also closely related to the notion of balance in word combinatorics. We focus on the Pisot case for substitutive shifts and on the case of hypercubic billiards in the framework of aperiodic order.

On higher-dimensional three distance theorems

Valérie Berthé

Université Paris-Diderot, France

ABSTRACT

For a given real number alpha, let us place the points 0, alpha, 2 alpha,. . . , N alpha on the unit circle. These points partition the unit circle into intervals having at most three lengths, one being the sum of the other two. This is the three distance theorem. We discuss two dual two-dimensional versions of the three distance theorem. We present a strategy for having a small number of lengths based on Brun's continued fraction algorithm, at the cost that the corresponding shapes are fractal in nature. This a joint work with P. Arnoux, D. H. Kim, W. Steiner and J. Thuswaldner.

Equidistribution, negative bases and tiling

Anne Bertrand

Université de Poitiers, France

ABSTRACT

We define a Rauzy fractale for "negative Pisot Numbers" and look at the induced autosimilar multiple tiling. In some cases the fractale can be obtained as cut and project sets but the equidistribution of exponential sequences is the most fruitful argument.

In the unimodular case these tilings are probably simple (maybe they are some exceptions); We can prove the simplicity of the tiling if the degree is two or in the Tribonacci case.

Discrepancy bounds and related questions

Dmitriy Bilyk

University of Minnesota, USA

ABSTRACT

The talk will concentrate on open questions related to the optimal bounds for the discrepancy of an N-point set in the d-dimensional unit cube. We shall discuss the two main conjectures on the order of star-discrepancy, $(\log N)^{d-1}$ vs $(\log N)^{d/2}$, and present evidence in support of each one, stemming from various areas of mathematics. In addition, we shall talk about discrepancy in other geometrical settings (rotated rectangles, balls, points on the sphere etc).

On the expansions of a real number to two distinct bases

Yann Bugeaud

Université Strasbourg, France

ABSTRACT

Let r and s be multiplicatively independent positive integers. We show that an irrational number ξ cannot have simultaneously a simple expansion to base r and a simple expansion to base s. More precisely, if the expansion of ξ to base r is a Sturmian sequence over the alphabet $\{0, 1, \ldots, r-1\}$, then its expansion to base s cannot be a Sturmian sequence over the alphabet $\{0, 1, \ldots, s-1\}$. This is a joint work with Dong Han Kim.

Links between automata and normality

Olivier Carton

Université Paris Diderot, France

ABSTRACT

In this talk, we explain several links between normality and finite-state machines. This includes the characterization of normality by non-compressibility, preservation of normality by selection and computing frequencies with weighted automata.

Nested perfect necklaces and low discrepancy sequences

Olivier Carton

Université Paris Diderot, France

ABSTRACT

M. B. Levin constructed a real number x such that the first N terms of the sequence $b^n x$ mod 1 for n $\geq$ 1 have discrepancy $O((logN)^2/N)$. This is the lowest discrepancy known for this kind of sequences. In this talk, we present Levin's construction in terms of nested perfect necklaces, which are a variant of the classical de Bruijn sequences. For base 2 and the order being a power of 2, we give the exact number of nested perfect necklaces and an explicit method based on matrices to construct each of them.

Coupled randomness and a mutual divergence formula

ADAM CASE

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ABSTRACT

In this talk we investigate the notion of *coupled randomness*, that is, the algorithmic randomness of two sequences R_1 and R_2 with respect to probability measures that may be dependent on one another. For a restricted but interesting class of coupled probability measures we develop a formula for the *lower* and *upper mutual dimensions* $mdim(R_1 : R_2)$ and $Mdim(R_1 : R_2)$, which quantify lower and upper densities of algorithmic information that is shared by R_1 and R_2. We also show that the condition $Mdim(R_1 : R_2) = 0$ is necessary but not sufficient for R_1 and R_2 to be independently random. Finally, for sequences S and T, we identify conditions under which Billingsley generalizations of the mutual dimensions $mdim(S : T)$ and $Mdim(S : T)$ can be meaningfully defined and prove a divergence formula for the values of these generalized mutual dimensions. The research presented in this talk is joint work with Jack H. Lutz.

Perturbed metric discrepancy results for geometric progressions

Katusi Fukuyama

Kobe University, Japan

ABSTRACT

We investigate the discrepancies of a sequence $\{\theta^k x + \gamma_k\}$ where θ is a real number satisfying $|\theta| > 1$ and $\{\gamma_k\}$ is an arbitrary sequence of real numbers. We prove the law of the iterated logarithm for almost every x and try to determine the speed of decay of discrepancies.

Discrepancy estimates for rational points on varieties

Anish Ghosh

Tata Institute of Fundamental Research, India

ABSTRACT

I will discuss some quantitative equidistribution and discrepancy estimates for rational points on affine group varieties using ergodic methods.

Uniformity of the block distribution of planar words

Teturo Kamae

Osaka City University, Japan

ABSTRACT

Let $\mathbb{A} = \{0, 1, \cdots, d-1\}$ be an alphabet. We consider a configuration $x \in \mathbb{A}^{\mathbb{N}^2}$, where $\mathbb{N} = \{1, 2, \cdots\}$. We denote its restriction to a rectangle $C \times D \subset \mathbb{N}^2$ by $x[C \times D] \in \mathbb{A}^{C\times D}$. We denote its translation by $(i, j) \in (\mathbb{N} \cup \{0\})^2$ as $T^{(i,j)}x[C \times D] \in \mathbb{A}^{(C+i)\times(D+j)}$. That is,

$$T^{(i,j)}x[C \times D](u + i, v + j) = x[C \times D](u, v)$$

for any $(u, v) \in C \times D$. Let

$$\mathbb{A}^{+} = \bigcup_{n=1,2,\cdots;m=1,2,\cdots} \mathbb{A}^{[n]\times[m]},$$

where we denote $[n] = \{1, 2, \cdots, n\}$. For $\eta \in \mathbb{A}^{C\times D}$ and $\xi \in \mathbb{A}^{[n]\times[m]}$, we say that ξ is a (i, j)-prefix of η denoting $\xi \prec_{(i,j)} \eta$ if $([n] + i) \times ([m] + j) \subset C \times D$ and $\eta[([n] + i) \times ([m] + j)] = T^{(i,j)}\xi$. We denote by $|\eta|_\xi$ the number of occurrences of ξ in η. That is,

$$|\eta|_\xi = \#\{(i, j);\ \xi \prec_{(i,j)} \eta\}.$$

For a finite word $x \in \mathbb{A}^{[N]\times[M]}$, we define

$$\Sigma(x) = \sum_{\xi\in\mathbb{A}^{+}} |x|_\xi^2.$$

This value represents how uniform the block distribution of x is, so that if y is another finite word with the same domain $[N] \times [M]$, then $\Sigma(x) < \Sigma(y)$ implies x is more uniform (and random in some sense) than y.

Almost all realizations $x \in \mathbb{A}^{\mathbb{N}^2}$ of uniform and i.i.d. random variables $X_{i,j}$ $((i.j) \in \mathbb{N}^2)$ on $\mathbb{A}$ satisfy that

$$\lim_{N,M\to\infty} \frac{1}{N^2M^2}\Sigma(x[[N] \times [M]]) = \frac{1}{4} + \sum_{k=1}^{\infty} \frac{1}{d^k - 1}.$$

We call $x \in \mathbb{A}^{\mathbb{N}^2}$ as this Σ-random. We study the Σ-randomness in this setting.

Analogy with the Lagrange spectrum for the powers of quadratic Pisot unit

Hajime Kaneko

University of Tsukuba, Japan

ABSTRACT

In this talk, we will discuss uniform distribution theory related to arithmetic progressions and geometric progressions. Uniformity of arithmetic progressions modulo 1 is well-known. In fact, Weyl proved that an arithmetic progression $(n\alpha)_{n=0,1,\ldots}$ is uniformly distributed modulo 1 if and only if α is an irrational number.

The Lagrange spectrum is related to the Diophantine approximation property of an arithmetic progression $(n\alpha)_{n=0,1,\ldots}$ in the case where α is a badly approximable number. More precisely, let x be a real number. We denote by $\|x\|$ the distance between x and its nearest integer. For a badly approximable number α, let $l(\alpha) := 1/c_\alpha$, where $c_\alpha = \liminf_{n\to\infty} n\|n\alpha\|(<\infty)$. Then the Lagrange spectrum is defined by $\mathcal{L} = \{l(\alpha) \mid \alpha \text{ is a badly approximable number}\}$. Many mathematicians have studied the geometry of the set $\mathcal{L}$. For instance, Markov showed that $\mathcal{L} \cap (0,3)$ is a discrete set. Moreover, Hall proved that the half line $[6,\infty)$ is contained in $\mathcal{L}$.

On the other hand, little is known on the uniformity of geometric progression $(\xi\alpha^n)_{n=0,1,\ldots}$ modulo 1. We discuss the value $\limsup_{n\to\infty} \|\xi\alpha^n\|$ in the case where α is a quadratic Pisot unit. The main purpose of this talk is to investigate the set $\mathcal{L}(\alpha) = \{\limsup_{n\to\infty} \|\xi\alpha^n\| \mid \xi \in \mathbb{R}\}$. Our main result implies that the geometry of the set $\mathcal{L}(\alpha)$ has similar properties to the Lagrange spectrum $\mathcal{L}$.

This is a joint work with Shigeki Akiyama and Teturo Kamae.

Lagrange spectrum of Romik's dynamical system

Dong Han Kim

Dongguk University, Korea

ABSTRACT

Let $L(S^1)$ be the Lagrange spectrum arising from the intrinsic Diophantine approximation of the unit circle S^1 by its rational points. In this talk, we give a complete description of the structure of $L(S^1)$ below its smallest accumulation point. We use certain digit expansions of points on S^1, which was initially introduced by Romik in 2008. This digit expansion is an analogue of simple continued fractions of real numbers. First, we prove that the smallest accumulation point of $L(S^1)$ is 2. Then we characterize the points on S^1 whose Lagrange numbers are less than 2 in terms of their Romik digit expansions. Our theorem is an analogue of a celebrated theorem of Markoff on badly approximable real numbers. We also adapt our method to the unit sphere S^2 and find a sharp Hurwitz bound, that is, the minimum of $L(S^2)$. This is joint work with Byungchul Cha.

Average case analysis of Euclidean algorithms

Loïck Lhôte

Université de Caen, France

ABSTRACT

This talk will be an introduction to the analysis of Euclidean algorithms or equivalently, to the average case analysis of "rational trajectories" in Euclidean dynamical systems.

Simultaneous Diophantine approximation of the orbits of the dynamical systems x2 and x3

LINGMIN LIAO

Université de Créteil, France

ABSTRACT

Consider the dynamical systems of multiplication by two modulo one and the multiplication by three modulo one. We study the sets of points whose orbits by these two dynamical systems are simultaneously approaching to a given point. With a given speed of approximation, the Hausdorff dimension of such a set is obtained. As a particular case, the Hausdorff dimension of the set of numbers which are simultaneously well approximated by dyadic and triadic rational numbers is calculated. This is a joint work with Bing LI.

Quantifying Equidistribution: a survey of finite-state dimensions

Jack H. Lutz

Iowa State University, USA

ABSTRACT

Finite-state dimensions are twenty-first century extensions of classical Hausdor and packing dimensions. For each infinite sequence S over a finite alphabet, the finite-state dimension of S is a real number in [0,1] that quantifies the equidistribution of S.For example, the sequence S is Borel normal if and only if its finite-state dimension is 1. This talk surveys finite-state dimensions and their connections with normality and equidistribution.

Borel complexity of sets of normal numbers via generic points in subshifts with specification

William Mance

Adam Mickiewicz University in Poznań, Poland

ABSTRACT

We study the Borel complexity of sets of normal numbers in several numeration systems. Taking a dynamical point of view, we offer a unified treatment for continued fraction expansions and base r expansions, and their various generalisations: generalised Lüroth series expansions and β-expansions. In fact, we consider subshifts over a countable alphabet generated by all possible expansions of numbers in $[0, 1)$. Then normal numbers correspond to generic points of shift-invariant measures. It turns out that for these subshifts the set of generic points for a shift-invariant probability measure is precisely at the third level of the Borel hierarchy (it is a Π^0_3-complete set, meaning that it is a countable intersection of F_σ-sets, but it is not possible to write it as a countable union of G_δ-sets). We also solve a problem of Sharkovsky–Sivak on the Borel complexity of the basin of statistical attraction. The crucial dynamical feature we need is a feeble form of specification. All expansions named above generate subshifts with this property. Hence the sets of normal numbers under consideration are Π^0_3-complete.

Essentially non-normal numbers

William Mance

Adam Mickiewicz University in Poznań, Poland

ABSTRACT

A real number in base b is *essentially non-normal* if each of the digits $0, 1, \cdots, b-1$ do not occur with any frequency in its b-ary expansion. The set of essentially non-normal numbers in base b is known to have full Hausdorff dimension and is comeagre. It is possible to make similar definitions for other expansions and similar results are known.

We strongly generalize previous results by considering essentially non-normal numbers for Q-Cantor series expansions, where Q is a generic point for a weakly mixing measure preserving system with fully supported measure. We take a stronger definition that an essentially non-normal number must have all blocks of digits of all lengths occur without a frequency. We prove for these classes of basic sequences Q that not only is the set of essentially non-normal numbers of full Hausdorff dimension and is comeagre, but so is the set of numbers where no blocks have a frequency when sampled along any arithmetic progression. Interesting examples of this result include b-ary expansions and letting Q be given by a translation of the partial quotients of a real number that is normal for the regular continued fraction expansion.

Additionally, we prove even stronger results for Q that are fixed points of primitive substitution. In particular, if Q is a translation of the Thue-Morse sequence, we can prove a similar theorem not just for real numbers sampled along arithmetic progressions, but also along sequences of the form $\lfloor n^c \rfloor$ for $c \in (1, 1.4) \cup \{2\}$.

Lastly, under some conditions we may provide examples of "explicit" subsets of these sets of essentially non-normal numbers of arbitrarily large Hausdorff dimension. More precisely these sets are Π_1^0 sets in the Borel lightface hierarchy. From these sets, we may extract computable examples of essentially non-normal numbers.

Asymptotic divergences and strong dichotomy

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ABSTRACT

The *Schnorr-Stimm dichotomy theorem* [1] concerns finite-state gamblers that bet on infinite sequences of symbols taken from a finite alphabet Σ. The theorem asserts that, for any such sequence S, the asymptotic behavior of finite-state gamblers is radically (exponentially) different depending on whether the sequence S is (Borel)-normal or not.

In this paper we use the Kullback-Leibler divergence to formulate the *(upper)lower asymptotic divergence* of a probability measure α on Σ from a sequence S over Σ and in such a way that a sequence S is α-normal (meaning that every string w has asymptotic frequency $\alpha(w)$ in S) if and only if the upper asymptotic divergence of α from S is zero. We also use the Kullback-Leibler divergence to quantify the *total risk* that a finite-state gambler G takes when betting along a prefix w of S.

Our main theorem is a *strong dichotomy theorem* that uses the above notions to *quantify* the exponential rates of winning and losing on the two sides of the Schnorr-Stimm dichotomy theorem extended from normality to α-normality.

We also use our main Theorem to show upper bounds on the finite-state α-dimension and finite-state strong α-dimension of a sequence S.

References

[1] Claus-Peter Schnorr and Hermann Stimm. Endliche Automaten und Zufallsfolgen. *Acta Informatica*, 1(4):345–359, 1972.

Low discrepancy sequences failing Poissonian pair correlations

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ABSTRACT

M. Levin defined a real number x that satisfies that the sequence of the fractional parts of $(2^n x)_{n\geq 1}$ are such that the first N terms have discrepancy $O((\log N)^2/N)$, which is the smallest discrepancy known for this kind of parametric sequences. In this work we show that the fractional parts of the sequence $(2^n x)_{n\geq 1}$ fail to have Poissonian pair correlations. Moreover, we show that all the real numbers x that are variants of Levin's number using Pascal triangle matrices are such that the fractional parts of the sequence $(2^n x)_{n\geq 1}$ fail to have Poissonian pair correlations. This is joint work with Verónica becher and Olivier Carton.

Continued fractions and subsequence selections

Satyadev Nandakumar

Institute of Technology Kanpur, India

ABSTRACT

We study the stability of frequency properties of continued fractions under subsequence selections, following the work of Heersink and Vandehey.

Beyond uniform distribution: level spacing and minimal gaps

Zeev Rudnick

Tel Aviv University, Israel

ABSTRACT

The study of uniform distribution of sequences is a century old. However, the property of a sequence to be uniformly distributed is a very crude one, and there are much finer quantities that can be studied. I will discuss what is known about nearest-neighbour gaps of sequences, both the typical gaps, whose probability distribution is called the level spacing distribution in the physics literature, and the extremal gaps (minimal and maximal). Examples of interesting sequences for which one can study these quantities include classical sequences studied in uniform distribution theory, the zeros of the Riemann zeta function, energy levels of quantum systems, and more. These examples offer a huge source of problems for probabilists, physicists and number theorists.

Joint distribution of the base-q and Ostrowski digital sums

Divyum Sharma

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ABSTRACT

In 1922, A. Ostrowski introduced a numeration system based on the denominators of the convergents in the continued fraction expansion of a fixed irrational number α. Coquet, Rhin and Toffin studied the joint distribution in residue classes of the usual base-q sum-of-digits function S_q and the Ostrowski sum-of-digits function S_α. They gave certain sufficient conditions for the set

$$\{n \in \mathbb{N} : S_q(n) \equiv a_1 \pmod{m_1},\ S_\alpha(n) \equiv a_2 \pmod{m_2}\}$$

to have asymptotic density $1/m_1 m_2$. In this talk, we present a quantitative version of their result when

$$\alpha = [0; \overline{1, m}],\ m \geq 2.$$

New bounds on the discrepancy of fractional parts of real polynomials

Igor Shparlinski

University of New South Wales, Australia

ABSTRACT

We also improve the bound of Wooley (2015) on the discrepancy of polynomial sequences

$$u_1 n + \ldots + u_d n^d, \qquad n = 1, 2, \ldots,$$

with $(u_1, \ldots, u_d) \in [0,1)^d$, which hold for almost all $(u_i)_{i\in I}$ and all $(u_j)_{j\in J}$, where $I \cup J$ is a partition of $\{1, \ldots, d\}$.

We also extend these results to very general settings of arbitrary orthogonal projections of the vectors of the coefficients $(u_1, \ldots, u_d)$ onto a lower dimensional subspace, which seems to be a new point of view.

In the opposite direction, we show that the set of vectors for which the fractional parts of $u_1 n + \ldots + u_d n^d$, $n = 1, 2, \ldots$, are poorly distributed is quite massive in the sense of Hausdorff dimension.

This is a joint work with Changhao Chen.

Uniform distribution of generalized polynomials and applications

Younghwan Son

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ABSTRACT

Generalized polynomials are real-valued functions which are obtained from conventional polynomials by the use of the operations of addition, multiplication, and taking the integer part. They form a natural extension of conventional polynomials, and appear, under different names, in a variety of mathematical contexts, from dynamics on nilmanifolds to number theory and mathematical games. Unlike the conventional polynomials, generalized polynomials may have quite intricate distributional properties. In this talk we will present recent results on uniform distribution of a large class of generalized polynomials and discuss some ergodic theoretical applications. This is a joint work with Vitaly Bergelson and Inger Håland Knutson.

In the proximity of dimension 1

Linda Brown Westrick

Pennsylvania State University, USA

ABSTRACT

The effective dimension of an infinite sequence measures its information density. A random sequence has effective dimension 1. Starting from a sequence of effective dimension 1, one can make a sequence of effective dimension 1/2 by changing every even bit to zero (changing one in every four bits on average, since half the even bits were zero already). Is this optimal, or can we get to a sequence of dimension 1/2 with less than density 1/4 of changes? In the other direction, starting from a sequence of effective dimension 1/2, what density of changes will allow us to obtain a sequence of effective dimension 1? We answer these questions for the general case of effective dimensions s and t, where $0 < s < t < 1$, and raise some related ones in the context of normality and metric spaces. Joint work with N. Greenberg, J. Miller, and A. Shen.