Abstracts

Real numbers and Measures for which they are Random

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ABSTRACT

We will discuss the algorithmic formulation of randomness, and consider the question "For which real numbers x are there measures mu for which x is random?" The answer depends on the properties required of mu and the degree to which x is required to be mu-random. We will discuss the cases in which x is not a mu-atom, mu is continuous, mu is not supported by any set of small Hausdorff dimension, or the Fourier transform of mu vanishes at infinity at a preassigned rate. We will go into the details of the continuous case.

Lectures:

1. Martin-Lof randomness
    - formulations by measure and by prefix-free Kolmogorov complexity
    - Kucera-Gacs theorem
    - higher order randomness
    - Sacks Theorem
    - Franklin-Ng Theorem
2. Randomness for arbitrary measures
    - Every nonrecursive real number is ML-random for some measure for which it is not an atom.
    - discussion of Levin's neutral measure
    - Day-Miller theorem
3. Randomness for continuous measures
    - Discussion of equivalences for being random for some continuous measure
    - For each n, the set of real numbers that are not n-random for some continuous measure is countable.
4. Metamathematics
    - The main theorem of Day 3 cannot be proven using only finitely many iterates of the power set of N.
5. Randomness for more restricted families of measures.
    - Frostman's lemma for Hausdorff dimension and Reimann's theorem
    - Partial information for Fourier measures.

Model Theory of Fields and Local Rings

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ABSTRACT

Introductory Remarks: It is just over 50 years since work of Ax, Kochen and Ershov, and then Ax, transformed research in applied model theory. It brought logic much closer to number theory, algebraic geometry and ultimately motivic integration. In addition it has led to much deeper understanding of the algebraic structure of nonstandard models of arithmetic. In recent years the subject has connected with more abstract issues in modern model theory, for example from neostability. Using techniques of Feferman-Vaught type from their classic paper of 1959, one now has a deeper understanding of the rings of adeles over number fields, including measure-theoretic matters fundamental to the role of these rings in number theory. One has begun to understand much more deeply decidability and definability in infinite dimensional algebraic extensions of p-adic fields, and to confront big issues of ramification. Finally, model theory of Galois theory for p-adic fields plays an increasingly important role.

The plan is to cover, assuming only basic algebra and model theory, the general theory from the 1960's and 70's, and then to treat, in a less detailed, but hopefully illuminating way, the topics mentioned above. Provisionally the lectures will be arranged as follows:

Day 1. Valued fields and local fields. Hensel's Lemma.Decidability and Definability for p-adic fields.
Day 2. Ultraproducts of Finite Fields. Decidability and Definability. Uniformity in p. Application to rings of residues modulo varying prime powers.
Day 3. Adeles. Use of Feferman-Vaught. Decidability and Definability . Application to residue rings modulo all integers. Applications to residue rings of nonstandard models of Peano Arithmetic.
Day 4. Model theory of Galois Theory. Galois Stratification. Various works of Denef and collaborators, cell-decomposition. Application to p-adic Poincare series.
Day 5. Uniformity in p once more . Motivic Integration. Current research on model theory of ramification.

Ultimate L

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ABSTRACT

Gödel's consistency proof for the Axiom of Choice and the Continuum Hypothesis involves his discovery of the Constructible Universe of Sets nearly 80 years ago. The axiom "V = L" is the axiom which asserts that every set is constructible. This axiom settles the Continuum Hypothesis and more importantly, Cohen's method of forcing cannot be used in the context of the axiom "V = L".

However the axiom V = L is false since it limits the fundamental nature of infinity. In particular the axiom refutes (most) strong axioms of infinity.

A key question emerges. Is there an "ultimate" version of Gödel's constructible universe L yielding an axiom "V = Ultimate L" which retains the power of the axiom "V = L" for resolving questions like that of the Continuum Hypothesis, which is also immune against Cohen's method of forcing, and yet which does not refute strong axioms of infinity?

Until around 10 years ago there seemed to be a number of convincing arguments as to why no such ultimate L can possibly exist. But the situation is now changed and there is now a specific conjecture, the Ultimate L Conjecture, which is arguably the key problem.

We will cover the basic material for both the formulation of this conjecture and for the argument that this is really the key conjecture. Finally we will address the status of this conjecture.

Lecture 1: Inner models and Set Theory
Lecture 2: Stationary sets and the HOD Dichotomy
Lecture 3: Approximation and Covering I
Lecture 4: Approximation and Covering II
Lecture 5: The Ultimate L Conjecture

Generalizations of Hall's theorem in reverse mathematics

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ABSTRACT

Philip Hall's theorem gives necessary and sufficient conditions for the existence of a matching on a bipartite graph. In this talk, we briefly survey prior work on the reverse mathematics of this and related principles by Hirst (1990), and Hirst and Hughes (2015). We then present ongoing work concerning the reverse mathematics of several generalizations of Hall's theorem.

Posner-Robinson for hyperjumps of Turing degrees

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ABSTRACT

One version of the Posner-Robinson Theorem states that for any $A \geq_T 0'$ and non-recursive $Z \leq_T A$, there exists B such that $A \equiv_T B \oplus Z \equiv_T B'$. In this way, any non-recursive Z is (relative to some B) a Turing jump. Here we give an unpublished proof (due to Slaman, and also proven independently by Woodin) of the hyperjump version, namely that for any $A \geq_T \mathcal{O}$ and non-hyperarithmetical $Z \leq_T A$, there exists B such that $A \equiv_T B \oplus Z \equiv_T \mathcal{O}^B$, so that any hyperarithmetical Z is (relative to some B) a hyperjump. We also present some related results.

A model theoretic adic space

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ABSTRACT

(Ongoing joint work with Pablo Cubides-Kovacsics) Let K be a complete valued field of rank 1 and X be a variety over K. Hrushovski and Loeser showed how the space $\widehat{X}$ of generically stable types concentrating on X (over a large model of ACVF containing K) can be seen as a model-theoretic counterpart of the Berkovich analytification X^{an} of X. In this talk we will present an analogous construction which provides a model-theoretic counterpart $\widetilde{X}$ of the Huber's analytification of X. We show that, the same as for $\widehat{X}$, the space $\widetilde{X}$ is strict pro-definable in the geometric language of valued fields. Furthermore, there will be canonical liftings of the deformation retraction developed by Hrushovski and Loeser.

The remarkable expressivity of first-order logic in profinite groups

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ABSTRACT

Profinite groups are the inverse limits of finite groups, or equivalently, the compact totally disconnected groups. First-order logic in the signature of groups can directly talk only about their algebraic structure. We address the question whether a profinite group G can be determined by a single first-order sentence: is there a sentence ϕ such that $H \models \phi$ if and only if H is topologically isomorphic to G, for each profinite group H?

Let $p \geq 3$ be a prime. We show that this property holds for the groups $\mathrm{SL}_2(\mathbb{Z}_p)$ and $\mathrm{PSL}_2(\mathbb{Z}_p)$ where $\mathbb{Z}_p$ is the ring of p-adic integers. If we restrict the reference class to the inverse limits of p-groups, we obtain many further examples, e.g. all groups with a bound on the dimension of the closed subgroups (such as the abelian group $\mathbb{Z}_p$).

This is joint work with Dan Segal and Katrin Tent.

Computable quotient presentations of nonstandard models of arithmetic

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ABSTRACT

A computable quotient presentation of a mathematical structure $\mathcal{A}$ consists of a computable structure on the natural numbers $\langle \mathbb{N}, \star, *, \dots \rangle$, meaning that the operations and relations of the structure are computable, and an equivalence relation E on $\mathbb{N}$, not necessarily computable but which is a congruence with respect to this structure, such that the quotient $\langle \mathbb{N}, \star, *, \dots \rangle$ is isomorphic to the given structure $\mathcal{A}$. Thus, one may consider computable quotient presentations of graphs, groups, orders, rings and so on.

A natural question asked by B. Khoussainov in 2016, is if the Tennenbaum Theorem extends to the context of computable presentations of nonstandard models of arithmetic. By using the Low Basis Theorem, one can show that any c.e. theory in a purely functional language has a model that admits a computable quotient presentation w.r.t. an equivalence relation of low degree, so in particular of Δ^0_2-degree.. A further natural question then is: can we have nonstandard models of arithmetic that admit computable quotient presentations w.r.t. relations that are Σ^0_1 or Π^0_1.

In a joint work with J.D. Hamkins we have proved that no nonstandard model of arithmetic admits a computable quotient presentation by a computably enumerable equivalence relation on the natural numbers. What is more, no ?1-sound nonstandard model of arithmetic has a computable quotient presentation by a co-c.e. equivalence relation. No nonstandard model of arithmetic in the language $+$, $\cdot$, ? has a computably enumerable quotient presentation by any equivalence relation of any complexity. No model of ZFC or even much weaker set theories has a computable quotient presentation by any equivalence relation of any complexity. And similarly no nonstandard model of finite set theory has a computable quotient presentation.

However, as it happens, there exists a nonstandard model of arithmetic admitting a computable quotient presentation by a co-c.e. equivalence relation. Actually, there are infinitely many of those. The idea of the proof consists in simulating the Henkin construction via finite injury priority argument. What is quite surprising, the construction works (i.e. injury lemma holds) by Hilbert's Basis Theorem.

Descending sequences of hyperdegrees and the second incompleteness theorem

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ABSTRACT

It follows from classical results due to Spector that there is no sequence of reals $A_0, A_1, A_2, \ldots$ such that for each n, $A_n \geq_H \mathcal{O}^{A_{n+1}}$. We will give a new proof of this result using the second incompleteness theorem. We will then mention how this fact can be used to give an alternative proof of a result of Simpson and Mummert on a semantic version of the second incompleteness theorem for β_n models. Both of these results seem to suggest a more general connection between well-foundedness of certain partial orders and the second incompleteness theorem. We will mention several other examples of this connection. This talk is based on joint work with James Walsh.

Random sequences of quantum bits

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ABSTRACT

Martin-Loef (1966) formalised the intuitive notion of randomness for infinite sequences of bits via algorithmic tests. What if we replace classical bits by quantum bits?

We first provide a framework to formalise infinite sequences of qubits as states of a suitable C* algebra. Thereafter we introduce an analog of Martin-Loef's notion. We show that for classical bit sequences the two notions coincide. We also discuss quantum Kolmogorov complexity for finite sequences of qubits and its relationship to quantum Martin-Loef randomness. Finally we consider an effective version of the Shannon-McMillan-Breiman theorem in the quantum setting.

This is joint work with Volkher Scholz. Paper available at arxiv.org/abs/1709.08422.