

Follow the money

Bayesian markets to aggregate expert opinions when the majority can be wrong

Aurelien Baillon, Benjamin Tereick, Tong V. Wang

Erasmus University Rotterdam, Netherlands

why

- political decisions, managerial decisions
- experts & wisdom of crowds
 - I. elicit / reward
 - II. aggregate / weight
- markets
 - in finance
 - prediction markets
 - I. elicit / reward
 - II. aggregate / weight

practical constraints

- prediction market: **contracts** on verifiable event
“Trump wins the election”
- **challenge I (reward)**: what if statement is unverifiable or fuzzy/vague?
 - unverifiable (in practice)
 - **vague/fuzzy** statements
“climate change is due to human activity”

majority

- prediction markets: mean beliefs*
 - *=under many assumptions, so most likely distorted
- Everyone same weight?
- challenge II (aggregate): should we trust majority?
- “do black swans exist?”



this paper

- solves both challenges at once
 - challenge I (reward): what if statement is unverifiable or fuzzy/vague?
 - challenge II (aggregate): should we trust majority?

I. Bayesian markets

- bet on what others say
- instead of event itself!

II. follow the money

- not the majority!

setting

based on Prelec et al. 2017

setting

- $S = \{Y, N\}$
 - we will never observe directly the state
 - prior $P(Y) = P(N) = \frac{1}{2}$
- agents $i \in \{1, \dots, n\}$
 - n infinite
- private signals $s_i \in \{0, 1\}$
- proportion ω of signal 1
- $\omega_Y \equiv P(s_i = 1|Y)$ and $\omega_N \equiv P(s_i = 1|N)$
 $\omega_Y > 0.5 > \omega_N$
 $\omega_Y > \omega_N$

consequences

- we still have: $P(Y|s_i = 1) > 0.5 > P(N|s_i = 0)$
 - link signals and beliefs about states
- ω (proportion of signals 1) can be ω_Y or ω_N
- $\omega_N < \bar{\omega}_0 \equiv E(\omega|s_i = 0) < \bar{\omega}_1 \equiv E(\omega|s_i = 1) < \omega_Y$



I. Bayesian market

Baillon (2017)

Bayesian markets

- **answer** a Y/N question (endorse Y or N)
- take or not a **bet** on proportion of Y answers (trade an asset)
 - e.g. “more than 42%?”
- payment
 - actual proportion – 42%
 - or 0

notation

- endorsements
 - $e_i = 1$ means agent endorses Y
 - $e_i = 0$ means agent endorses N

- asset value

$$v = \frac{\sum_{i=1}^n e_i}{n}$$

- truth-telling $e_i = s_i$
 - $P(Y|s_i = 1) > 0.5 > P(Y|s_i = 0)$

Bayesian market



agent i
endorses Y
 $e_i = 1$

agent i can buy the
asset from the
market maker at p

agent i pays p
to get v

market maker
randomly draw
price p

trades go through if
at least one agent
buy and one agent
sell at p

settlement value
 v is determined

agent j
endorses N
 $e_j = 0$

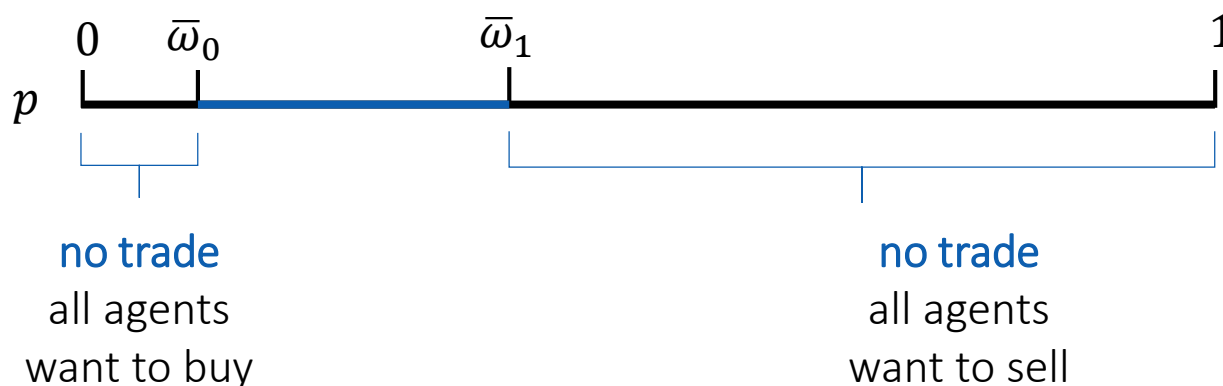
agent j can sell the
asset to the market
maker at p

agent j gets p
and pays v

theorem: truth-telling is a BNE

- intuition

- signal 1 agents expect v to be higher than signal 0 agents expect
- range of prices for which both signal 1 agents want to buy and signal 0 agents want to sell
- but no interest to take opposite position



conclusion of part I

- challenge I (reward): what if statement is unverifiable or fuzzy/vague?
 - solved!
 - agents still reveal their signals / beliefs
 - replaced bets on events by bets on what others believe

II. follow the money

(new)

majority

- on Bayesian markets, we observe

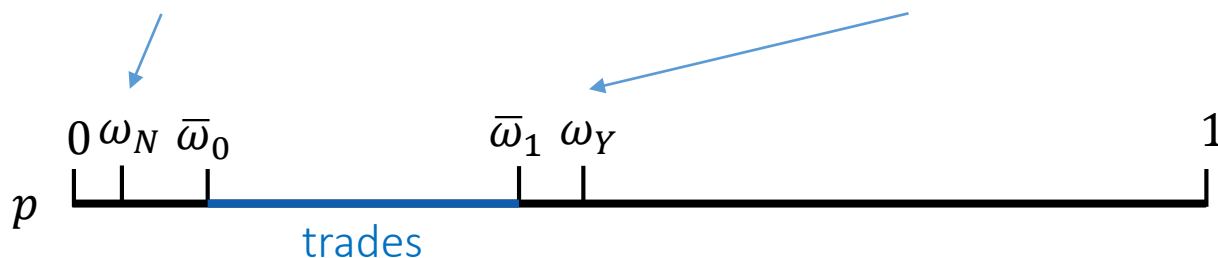
$$v = \frac{\sum_{i=1}^n e_i}{n} = \frac{\sum_{i=1}^n s_i}{n}$$

- should we conclude $v > 0.5$ means state Y ?
 - no!
 - imagine $\omega_Y = 0.2, \omega_N = 0.1$
 - then $v < 0.5$ no matter the state

follow the money

sellers (= endorsing N)
make money

buyers (= endorsing Y)
make money



- **theorem**: if market is at truth-telling equilibrium, only agents endorsing the actual state of nature makes a profit.

conclusion of part II

- challenge II (aggregate): should we trust majority?
 - no!
 - don't check if $v > 0.5$ but who makes money

Prelec et al. (2017)

- same model
- surprisingly popular algorithm
 - endorsement + predictions
 - select answer that is surprisingly popular
- differences
 - we handle challenge I (reward)
 - we use less information from agents

experiment

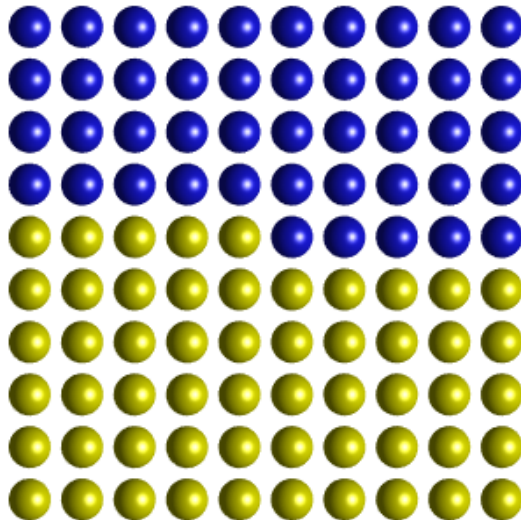
(fresh)

two states

$$Y, \omega_Y = 0.45$$

Urn Left

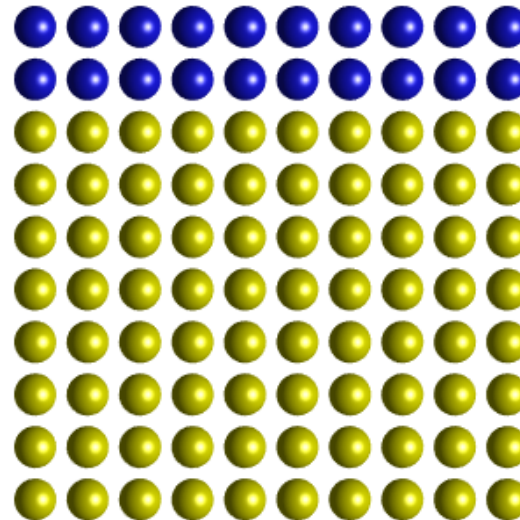
Number of Blue Balls: 45,
Number of Yellow Balls: 55.



$$N, \omega_N = 0.2$$

Urn Right

Number of Blue Balls: 20,
Number of Yellow Balls: 80.



signal and endorsement

Your draw:



Which urn do you guess was selected?

Urn Left

Urn Right

Bayesian market

You guessed that your draw came from **Urn Right**.

Do you bet that the number of participants guessing **Urn Right** is at least 55?

Yes

No

Bayesian market - payment

Your guess: **Urn Right**

You could bet on whether the number of participants guessing **Urn Right** would be at least 55.

This time **Urn Right** was selected.

Actual number of participants guessing **Urn Right**: 80

The earnings of the bet (in tokens) is this number minus 55:

Earnings = $80 - 55 = 25$

To make sure you don't lose money (if the earnings are negative), you will be endowed with 100 tokens, whether you take the bet or not.

You chose to take the bet.

Your bet goes through if someone took the opposite bet, which means, someone bet that less than 55 participants chose Urn Right (or in other words, more than 45 chose the other urn).

Your bet went through. Your payment for this task is $100 + \text{earnings} = 125$ tokens.

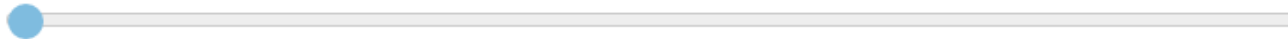
surprisingly popular algorithm

You guessed that your draw came from **Urn Right**.

How many participants do you predict to have guessed **Urn Right**?

0 10 20 30 40 50 60 70 80 90 100

Your prediction:



implementation

- last week on Prolific
- about 750 US students
- 30 situations (15 for each participants)
- follow the [money](#)...
 - use many prices
 - fit supply and demand curves
 - estimate theoretical profit

preliminary results

	average accuracy			
	majority rule		SPA	FTM
	data SPA	data FTM		
$\omega_N < 0.5 < \omega_Y$	96.0%	99.5%	92.3%	89.6%
$\omega_N < \omega_Y < 0.5$ or $0.5 < \omega_N < \omega_Y$	51.5%	53.2%	73.0%	77.0%

conclusion

this paper

- solves both challenges facing prediction markets at once
 - challenge I (reward): what if statement is unverifiable or fuzzy/vague?
 - challenge II (aggregate): should we trust majority?
- I. Bayesian markets
 - bet on what others say
 - instead of event itself!
- II. follow the money
 - not the majority!
- works in theory
- works in practice?

Bayesian markets to elicit private information

Aurélien Baillon^{a,1}

^aErasmus School of Economics, Erasmus University Rotterdam, 3000 DR Rotterdam, The Netherlands

Edited by Jose A. Scheinkman, Columbia University, New York, NY, and approved June 13, 2017 (received for review March 3, 2017)

Financial markets reveal what investors think about the future, and prediction markets are used to forecast election results. Could markets also encourage people to reveal private information, such as subjective judgments (e.g., “Are you satisfied with your life?”) or unverifiable facts? This paper shows how to design such markets, called Bayesian markets. People trade an asset whose value represents the proportion of affirmative answers to a question. Their trading position then reveals their own answer to the question. The results of this paper are based on a Bayesian setup in which people use their private information (their “type”) as a signal. Hence, beliefs about others’ types are correlated with one’s own type. Bayesian markets transform this correlation into a mechanism that rewards truth telling. These markets avoid two complications of alternative methods: they need no knowledge of prior information and no elicitation of metabeliefs regarding others’ signals.

prediction markets | economic incentives | truth telling | mechanism design | Bayesianism

When trading in a market or submitting a price to an auctioneer, people reveal the extent to which they value a good. In finance, option and future markets reveal investors’ beliefs. Although such revelations of tastes and beliefs originally were a by-product of regular markets, they have led to prediction markets whose primary goal is to reveal beliefs. In such markets, people can buy or sell a simple asset whose value is 1 unit (e.g., \$10) if a specified event occurs and is 0 otherwise. Buying a unit at a given price is a bet that the event is more likely than people think to be on average. The resulting market price reveals aggregate expectations. Prediction markets have been used by various public organizations and companies (1, 2); for example, social scientists use this type of market to predict the replicability of experiments (3, 4). Prediction markets also have been shown to outperform polls in predicting election results (5). Unfortunately, they can only be used for events whose occurrence can be objectively verified. When collecting personal data such as opinions and self-assessed measurements, objective verification often is conceptually or practically impossible.

This paper introduces Bayesian markets, which are designed to elicit private information in binary settings (yes-or-no questions) when objective verification is impossible. Bayesian markets rely on the assumption that the private information that people possess influences their belief about others. Such inference is justified by Bayesian reasoning, a widely used theory of rational reasoning (6). Answering yes (Y) provides information (a signal) that can be used to update one’s prior expectation about the proportion of Y answers. In Bayesian markets, the assets traded have a value determined by the proportion of Y answers to a given question, for example, “Are you satisfied with your life?” Using Bayesian reasoning, we can predict that people who actually answer Y will expect a higher asset value than those who answer no (N). Hence, there exists a range of prices for which Y people want to buy the asset and N people want to sell it. In other words, for any price in this range, Y people bet that the asset will be worth more than the price, and N people bet it will be worth less. Bayesian markets use the difference in betting behavior to provide incentives for people to tell the truth about unverifiable information.

In prediction markets, betting on an event reveals one’s beliefs about that event. In Bayesian markets, betting on how many people are satisfied with their lives, for instance, reveals the bettor’s beliefs about others’ life satisfaction, which in turn reveals the bettor’s own life satisfaction. Bayesian markets can also be used to elicit people’s opinions about an event far in the future, such as the very long-term consequences of climate change, for which prediction markets are not adequate.

Bayesian markets complement alternative methods proposed to elicit private information, which are the Bayesian truth serum (7), the peer prediction method (8), and their refinements (9–11). Bayesian markets share the same Bayesian setting with these methods, notably the assumption that agents have a common prior about the population. However, by using simple betting decisions instead of eliciting a probability or estimating metabeliefs, Bayesian markets are simpler and more transparent than these alternatives and are robust to certain deviations from the common prior assumption (unlike refs. 7–10). They are restricted to binary questions, however. After presenting the setup, the main result, and an extension to small samples, I discuss related literature and situations in which the required assumptions are satisfied.

The Agents and Their Information

There are n agents (referred to as “he” in the singular). Consider a question Q about the agents’ private information, with two possible answers $\{0, 1\}$ to choose from. The type $t_i \in \{0, 1\}$ of agent $i \in \{1, \dots, n\}$ corresponds to his truth, which is private information. The proportion of type 1 agents is denoted $\omega = (\sum_{i=1}^n t_i/n) \in [0, 1]$. Following the literature (7–13), I assume that it is common knowledge that all agents share a prior belief $f(\omega)$ describing how likely they would consider various proportions to be, had they not (yet) known their own type. Harsanyi (14) provided justifications of this common prior assumption.

It is also common knowledge that types are impersonally informative, as defined by Prelec (7): $f(\omega|t_i) = f(\omega|t_j)$ is equivalent to $t_i = t_j$. This property includes two aspects. First, types are impersonal. That is, all agents i with $t_i = 0$ have the same updated belief $f(\omega|t_i = 0)$, with expectations denoted ω_0 , and all agents j

Significance

People’s private information can be revealed by the way in which they trade specifically designed assets in a new type of market. People trade an asset whose value is the proportion of affirmative answers to a question. Their trading position then reveals their own answer to the question. In Bayesian markets, people can be rewarded for telling the truth even when the truth is not verifiable. Bayesian markets are simpler and more transparent than alternative methods, avoiding the measurements of metabeliefs about others and prior beliefs.

Author contributions: A.B. wrote the paper.

The author declares no conflict of interest.

This article is a PNAS Direct Submission.

Freely available online through the PNAS open access option.

¹E-mail: baillon@ese.eur.nl.

This article contains supporting information online at www.pnas.org/lookup/suppl/doi:10.1073/pnas.1703486114/-/DCSupplemental.



Baillon, Aurélien (2017)
[Bayesian markets to elicit private information.](#)

Proceedings of the National Academy of Sciences

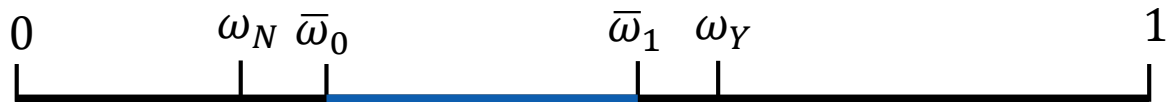
vol. 114:30, pp. 7958–7962.

thank you

average profit

- average profit in state Y over all possible p

$$\begin{aligned}\pi_1^Y &= -\pi_0^Y = \int_{\bar{\omega}_0}^{\bar{\omega}_1} (\omega_Y - p) dp \\ &= \frac{1}{2} [(\omega_Y - \bar{\omega}_0)^2 - (\omega_Y - \bar{\omega}_1)^2] > 0\end{aligned}$$



- symmetric in state N

supply and demand

