# Follow the money

## Bayesian markets to aggregate expert opinions when the majority can be wrong

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# why

- political decisions, managerial decisions
- experts & wisdom of crowds
  I. elicit / reward
  II. aggregate / weight

- markets
  - in finance
  - prediction markets
    I. elicit / reward
    II. aggregate / weight

# practical constraints

- prediction market: contracts on verifiable event
  *"Trump wins the election"*

- challenge I (reward): what if statement is unverifiable or fuzzy/vague?
  - unverifiable (in practice)
  - vague/fuzzy statements
    *"climate change is due to human activity"*

# majority

- prediction markets: mean beliefs*
  - *=under many assumptions, so most likely distorted
- Everyone same weight?

- challenge II (aggregate): should we trust majority?

- *"do black swans exist?"*

# this paper

- solves both challenges at once
  - challenge I (reward): what if statement is unverifiable or fuzzy/vague?
  - challenge II (aggregate): should we trust majority?

I. Bayesian markets
  - bet on what others say
  - instead of event itself!

II. follow the money
  - not the majority!

# setting

based on Prelec et al. 2017

# setting

- $S = \{Y, N\}$
  - we will never observe directly the state
  - prior $P(Y) = P(N) = \frac{1}{2}$
- agents $i \in \{1, \ldots, n\}$
  - $n$ infinite
- private signals $s_i \in \{0,1\}$
- proportion $\omega$ of signal 1
- $\omega_Y \equiv P(s_i = 1|Y)$ and $\omega_N \equiv P(s_i = 1|N)$

$$\omega_Y > 0.5 > \omega_N$$

$$\omega_Y > \omega_N$$

# consequences

- we still have: $P(Y|s_i = 1) > 0.5 > P(N|s_i = 0)$
  - link signals and beliefs about states

- $\omega$ (proportion of signals 1) can be $\omega_Y$ or $\omega_N$

- $\omega_N < \bar{\omega}_0 \equiv E(\omega|s_i = 0) < \bar{\omega}_1 \equiv E(\omega|s_i = 1) < \omega_Y$

# I. Bayesian market

Baillon (2017)

# Bayesian markets

- answer a Y/N question (endorse Y or N)

- take or not a bet on proportion of Y answers (trade an asset)
  - e.g. "more than 42%?"

- payment
  - actual proportion – 42%
  - or 0

# notation

- endorsements
  - $e_i = 1$ means agent endorses $Y$
  - $e_i = 0$ means agent endorses $N$

- asset value

$$v = \frac{\sum_{i=1}^{n} e_i}{n}$$

- truth-telling $e_i = s_i$
  - $P(Y|s_i = 1) > 0.5 > P(Y|s_i = 0)$

# Bayesian market

agent $i$ endorses $Y$
$e_i = 1$

agent $j$ endorses $N$
$e_j = 0$

market maker randomly draw price $p$

agent $i$ can buy the asset from the market maker at $p$

agent $j$ can sell the asset to the market maker at $p$

trades go through if at least one agent buy and one agent sell at $p$

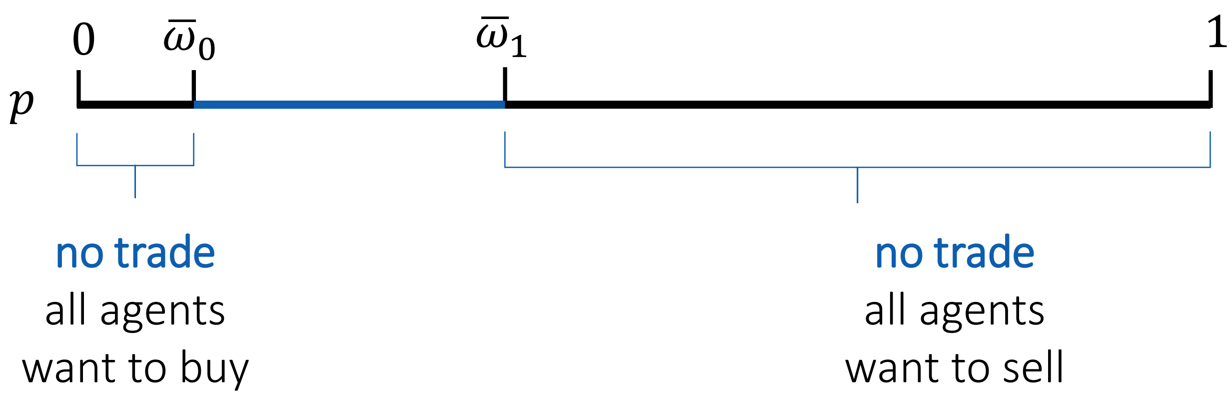
settlement value $v$ is determined

agent $i$ pays $p$ to get $v$

agent $j$ gets $p$ and pays $v$

# theorem: truth-telling is a BNE

- intuition
  - signal 1 agents expect $v$ to be higher than signal 0 agents expect
  - range of prices for which both signal 1 agents want to buy and signal 0 agents want to sell
  - but no interest to take opposite position



# conclusion of part I

- challenge I (reward): what if statement is unverifiable or fuzzy/vague?
  - solved!
  - agents still reveal their signals / beliefs
  - replaced bets on events by bets on what others believe

# II. follow the money

(new)

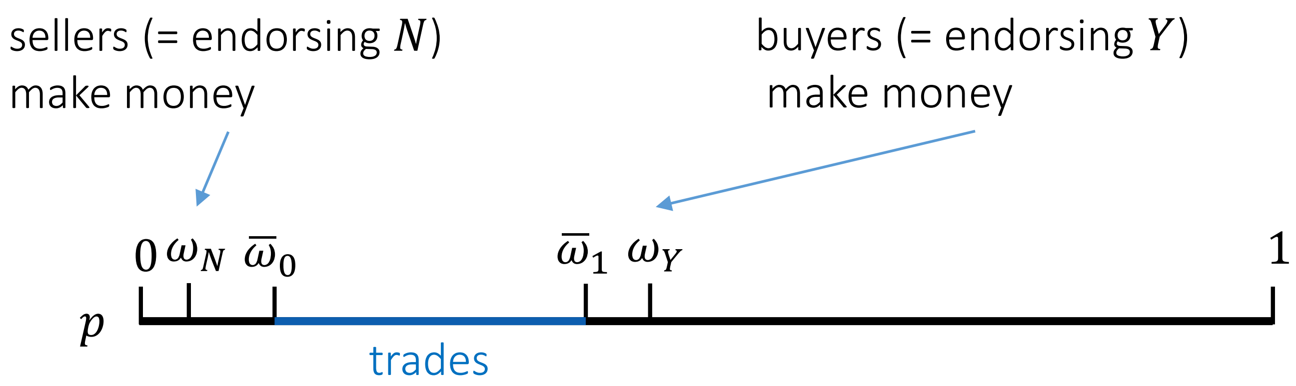
# majority

- on Bayesian markets, we observe

$$v = \frac{\sum_{i=1}^{n} e_i}{n} = \frac{\sum_{i=1}^{n} s_i}{n}$$

- should we conclude $v > 0.5$ means state $Y$?
  - no!
  - imagine $\omega_Y = 0.2$, $\omega_N = 0.1$
  - then $v < 0.5$ no matter the state

# follow the money




- theorem: if market is at truth-telling equilibrium, only agents endorsing the actual state of nature makes a profit.

# conclusion of part II

- challenge II (aggregate): should we trust majority?
  - no!
  - don't check if $v > \mathbf{0.5}$ but who makes money

# Prelec et al. (2017)

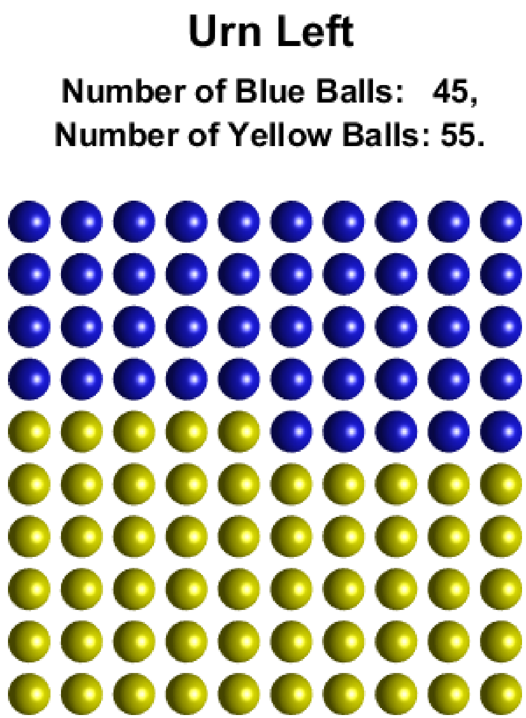
- same model
- surprisingly popular algorithm
  - endorsement + predictions
  - select answer that is surprisingly popular
- differences
  - we handle challenge I (reward)
  - we use less information from agents

# experiment

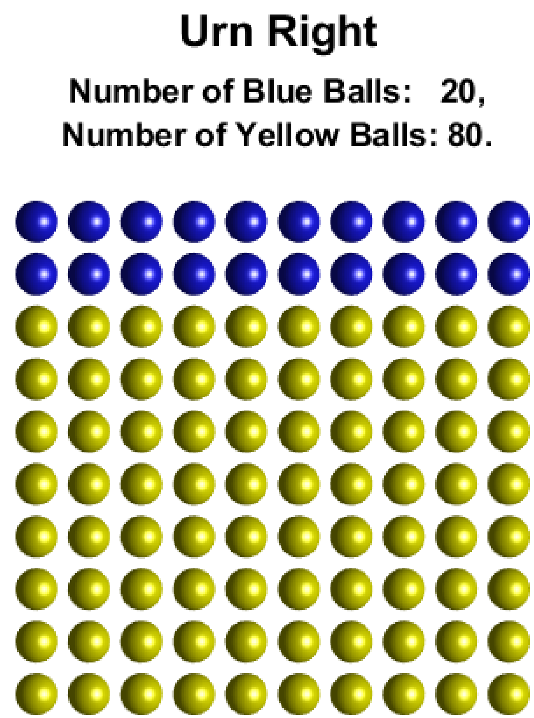
(fresh)

# two states

$Y, \omega_Y = 0.45$

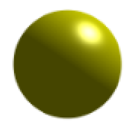



$N, \omega_N = 0.2$



# signal and endorsement

Your draw:



Which urn do you guess was selected?

Urn Left

Urn Right

# Bayesian market

You guessed that your draw came from **Urn Right**.

Do you bet that the number of participants guessing **Urn Right** is at least 55?

Yes

No

# Bayesian market - payment

Your guess: **Urn Right**

You could bet on whether the number of participants guessing **Urn Right** would be at least **55**.

This time **Urn Right** was selected.
Actual number of participants guessing **Urn Right**: **80**

**The earnings of the bet (in tokens) is this number minus 55:**
**Earnings = 80 - 55 = 25**

**To make sure you don't lose money (if the earnings are negative), you will be endowed with 100 tokens, whether you take the bet or not.**

You chose to take the bet.

Your bet goes through if someone took the opposite bet, which means, someone bet that less than 55 participants chose Urn Right (or in other words, more than 45 chose the other urn).

**Your bet went through. Your payment for this task is 100 + earnings = 125 tokens.**

# surprisingly popular algorithm

You guessed that your draw came from **Urn Right**.

How many participants do you predict to have guessed **Urn Right**?

0 10 20 30 40 50 60 70 80 90 100

Your prediction:

# implementation

- last week on Prolific

- about 750 US students
- 30 situations (15 for each participants)

- follow the money…
  - use many prices
  - fit supply and demand curves
  - estimate theoretical profit

# preliminary results

| | average accuracy | | | |
|---|---|---|---|---|
| | majority rule | | SPA | FTM |
| | data SPA | data FTM | | |
| $\omega_N < 0.5 < \omega_Y$ | 96.0% | 99.5% | 92.3% | 89.6% |
| $\omega_N < \omega_Y < 0.5$ or $0.5 < \omega_N < \omega_Y$ | 51.5% | 53.2% | 73.0% | 77.0% |

# conclusion

# this paper

- solves both challenges facing prediction markets at once
  - challenge I (reward): what if statement is unverifiable or fuzzy/vague?
  - challenge II (aggregate): should we trust majority?

I. Bayesian markets
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II. follow the money
  - not the majority!

- works in theory
- works in practice?



Baillon, Aurélien (2017) [Bayesian markets to elicit private information](). ***Proceedings of the National Academy of Sciences*** vol. 114:30, pp. 7958–7962.

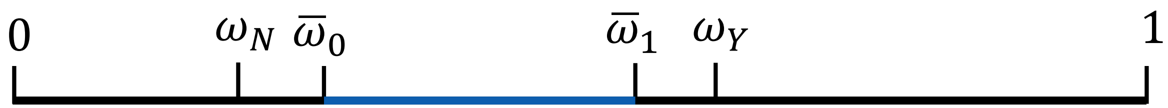
# thank you

# average profit

- average profit in state $Y$ over all possible $p$

$$\pi_1^Y = -\pi_0^Y = \int_{\bar{\omega}_0}^{\bar{\omega}_1} (\omega_Y - p)dp$$

$$= \frac{1}{2}[(\omega_Y - \bar{\omega}_0)^2 - (\omega_Y - \bar{\omega}_1)^2] > 0$$




- symmetric in state N

# supply and demand