Speculative Trade and Market Newcomers

Bogdan Klishchuk

Humboldt-Universität zu Berlin

March 19, 2019

Motivation and Contribution

Motivation and Contribution

- Inexperienced, young or immigrant turned investors share markets with veteran traders

Motivation and Contribution

- Inexperienced, young or immigrant turned investors share markets with veteran traders
- Economists and finance scholars have two conflicting opinions about role of $\underbrace{\text{experienced investors}}$

Motivation and Contribution

- Inexperienced, young or immigrant turned investors share markets with veteran traders

- Economists and finance scholars have two conflicting opinions about role of $\underbrace{\text{experienced investors}}$

  (1) That they abate deviations of prices from fundamentals or even burst bubbles

Motivation and Contribution

- Inexperienced, young or immigrant turned investors share markets with veteran traders
- Economists and finance scholars have two conflicting opinions about role of $\underbrace{\text{experienced investors}}$
  (1) That they abate deviations of prices from fundamentals or even burst bubbles
  (2) That they systematically take advantage of newcomers and $\underbrace{\text{sell them assets above fundamental values}}$

Motivation and Contribution

- Inexperienced, young or immigrant turned investors share markets with veteran traders

- Economists and finance scholars have two conflicting opinions about role of $\underbrace{\text{experienced investors}}$
  (1) That they abate deviations of prices from fundamentals or even burst bubbles
  (2) That they systematically take advantage of newcomers and $\underbrace{\text{sell them assets above fundamental values}}$

- $\underbrace{\text{Sufficient conditions}}_{?}$ for perpetual bubble of this type?

Motivation and Contribution

- Inexperienced, young or immigrant turned investors share markets with veteran traders

- Economists and finance scholars have two conflicting opinions about role of $\underbrace{\text{experienced investors}}$

  (1) That they abate deviations of prices from fundamentals or even burst bubbles

  (2) That they systematically take advantage of newcomers and $\underbrace{\text{sell them assets above fundamental values}}$

- $\underbrace{\text{Sufficient conditions}}_{?}$ for perpetual bubble of this type?

- This is my contribution

When Do Such Bubbles Occur?

When Do Such Bubbles Occur?

- Generally speaking, my paper predicts such bubbles if

When Do Such Bubbles Occur?

- Generally speaking, my paper predicts such bubbles if
  (1) conditional on same information, most experienced cohort does not tend to be most optimistic about fundamentals

When Do Such Bubbles Occur?

- Generally speaking, my paper predicts such bubbles if
  1. (1) conditional on same information, most experienced cohort does not tend to be most optimistic about fundamentals
  2. (2) short-selling is limited

When Do Such Bubbles Occur?

- Generally speaking, my paper predicts such bubbles if
  (1) conditional on same information, most experienced cohort does not tend to be most optimistic about fundamentals
  (2) short-selling is limited

- Proved in deliberately simple, tractable model

When Do Such Bubbles Occur?

- Generally speaking, my paper predicts such bubbles if
  (1) conditional on same information, most experienced cohort does not tend to be most optimistic about fundamentals
  (2) short-selling is limited

- Proved in deliberately simple, tractable model

- But underlying intuition has real-world appeal

Intuition

Intuition

(1) Asset prices $\geq$ $\overbrace{\text{fundamental valuations}}^{\textit{subjective}}$ of $\underbrace{\text{most optimistic traders}}_{}$

Intuition

(1) Asset prices $\geq \overbrace{\text{fundamental valuations}}^{\textit{subjective}}$ of $\underbrace{\text{most optimistic traders}}$

(2) Anticipate to get $\underbrace{\text{premium}}$ at least at end of their lives

Intuition

(1) Asset prices $\geq \overbrace{\text{fundamental valuations}}^{\textit{subjective}}$ of $\underbrace{\text{most optimistic traders}}_{}$

(2) Anticipate to get $\underbrace{\text{premium}}_{}$ at least at end of their lives

- Eventually in public contingencies
  become relatively pessimistic (by assumption)

Intuition

(1) Asset prices $\geq$ $\overbrace{\text{fundamental valuations}}^{\textit{subjective}}$ of $\underbrace{\text{most optimistic traders}}$

(2) Anticipate to get $\underbrace{\text{premium}}$ at least at end of their lives

- Eventually in public contingencies
  become relatively pessimistic (by assumption)
- Short-sales constraints
  $\Longrightarrow$
  possibility to sell assets
  above their updated fundamental valuations
  to more optimistic traders

Intuition

(1) Asset prices $\geq \overbrace{\text{fundamental valuations}}^{\textit{subjective}}$ of $\underbrace{\text{most optimistic traders}}$

(2) Anticipate to get $\underbrace{\text{premium}}$ at least at end of their lives

- Eventually in public contingencies
  become relatively pessimistic (by assumption)
- Short-sales constraints

  $\Longrightarrow$

  possibility to sell assets
  above their updated fundamental valuations
  to more optimistic traders

(3) Willing to pay above their fundamental valuations today

Intuition

(1) Asset prices $\geq$ $\overbrace{\text{fundamental valuations}}^{\textit{subjective}}$ of $\underbrace{\text{most optimistic traders}}$

(2) Anticipate to get $\underbrace{\text{premium}}$ at least at end of their lives

- Eventually in public contingencies
  become relatively pessimistic (by assumption)
- Short-sales constraints
  $\Longrightarrow$
  possibility to sell assets
  above their updated fundamental valuations
  to more optimistic traders

(3) Willing to pay above their fundamental valuations today

(4) Asset prices $\underbrace{>}$ most optimistic fundamental valuations

Intuition

(1) Asset prices $\geq$ $\overbrace{\text{fundamental valuations}}^{\textit{subjective}}$ of $\underbrace{\text{most optimistic traders}}$

(2) Anticipate to get $\underbrace{\text{premium}}$ at least at end of their lives

- Eventually in public contingencies
  become relatively pessimistic (by assumption)
- Short-sales constraints
  $\Longrightarrow$
  possibility to sell assets
  above their updated fundamental valuations
  to more optimistic traders

(3) Willing to pay above their fundamental valuations today

(4) Asset prices $\underbrace{>}$ most optimistic fundamental valuations

(5) Perpetual bubble

Intuition

(1) Asset prices $\geq$

$\overbrace{\text{fundamental valuations}}^{\textit{subjective}}$ of $\underbrace{\text{most optimistic traders}}$

(2) Anticipate to get $\underbrace{\text{premium}}$ at least at end of their lives

- Eventually in public contingencies
  become relatively pessimistic (by assumption)
- Short-sales constraints

  $\Longrightarrow$

  possibility to sell assets
  above their updated fundamental valuations
  to more optimistic traders

(3) Willing to pay above their fundamental valuations today

(4) Asset prices $\underbrace{>}$ most optimistic fundamental valuations

(5) Perpetual bubble

- Cf. Harrison and Kreps (1978) and Morris (1996)

Room for Such Disagreement of Opinion?

Room for Such Disagreement of Opinion?

- Claimed conditions for bubble
  require disagreement of opinion between veterans and novices
  conditional on public information

Room for Such Disagreement of Opinion?

- Claimed conditions for bubble
  require disagreement of opinion between veterans and novices
  conditional on public information

- Possible foundation in terms of newcomers' $\underbrace{\text{prior beliefs}}$

Room for Such Disagreement of Opinion?

- Claimed conditions for bubble
  require disagreement of opinion between veterans and novices conditional on public information
- Possible foundation in terms of newcomers' $\underbrace{\text{prior beliefs}}$
  - They are over future at time of market entry

Room for Such Disagreement of Opinion?

- Claimed conditions for bubble
  require disagreement of opinion between veterans and novices
  conditional on public information
- Possible foundation in terms of newcomers' $\underbrace{\text{prior beliefs}}$
  - They are over future at time of market entry
  - They are not over entire history

Room for Such Disagreement of Opinion?

- Claimed conditions for bubble
  require disagreement of opinion between veterans and novices
  conditional on public information
- Possible foundation in terms of newcomers' prior beliefs
  - They are over future at time of market entry
  - They are not over entire history
  - They incorporate information about past rather coarsely

Room for Such Disagreement of Opinion?

- Claimed conditions for bubble
  require disagreement of opinion between veterans and novices
  conditional on public information

- Possible foundation in terms of newcomers' $\underbrace{\text{prior beliefs}}$

  - They are over future at time of market entry
  - They are not over entire history
  - They incorporate information about past rather coarsely

- My model generates heterogeneous beliefs this way
  even though

Room for Such Disagreement of Opinion?

- Claimed conditions for bubble
  require disagreement of opinion between veterans and novices
  conditional on public information

- Possible foundation in terms of newcomers' prior beliefs
  - They are over future at time of market entry
  - They are not over entire history
  - They incorporate information about past rather coarsely

- My model generates heterogeneous beliefs this way
  even though
  - Traders are identical at time of market entry

Room for Such Disagreement of Opinion?

- Claimed conditions for bubble
  require disagreement of opinion between veterans and novices
  conditional on public information

- Possible foundation in terms of newcomers' prior beliefs
  - They are over future at time of market entry
  - They are not over entire history
  - They incorporate information about past rather coarsely

- My model generates heterogeneous beliefs this way
  even though
  - Traders are identical at time of market entry
  - Traders learn identically as time goes by

Room for Such Disagreement of Opinion?

- Claimed conditions for bubble
  require disagreement of opinion between veterans and novices
  conditional on public information
- Possible foundation in terms of newcomers' $\underbrace{\text{prior beliefs}}$
  - They are over future at time of market entry
  - They are not over entire history
  - They incorporate information about past rather coarsely
- My model generates heterogeneous beliefs this way
  even though
  - Traders are identical at time of market entry
  - Traders learn identically as time goes by
- Heterogeneity $\Longleftarrow$
  variation in duration of market participation and learning

Pessimistic Veteran Investors?

Pessimistic Veteran Investors?

- Claimed conditions for bubble
  require most experienced cohort to be relatively pessimistic

Pessimistic Veteran Investors?

- Claimed conditions for bubble
  require most experienced cohort to be relatively pessimistic

- How plausible?

Pessimistic Veteran Investors?

- Claimed conditions for bubble
  require most experienced cohort to be relatively pessimistic

- How plausible?
  - My model:
    for nonnegligible set of parameter values

Pessimistic Veteran Investors?

- Claimed conditions for bubble
  require most experienced cohort to be relatively pessimistic

- How plausible?
  - My model:
    for nonnegligible set of parameter values
  - Somewhat compatible empirical evidence

Pessimistic Veteran Investors?

- Claimed conditions for bubble
  require most experienced cohort to be relatively pessimistic

- How plausible?
  - My model:
    for nonnegligible set of parameter values
  - Somewhat compatible empirical evidence
    - Stock prices reaching two-year lows,
      last week of March 2001

Pessimistic Veteran Investors?

- Claimed conditions for bubble
  require most experienced cohort to be relatively pessimistic
- How plausible?
  - My model:
    for nonnegligible set of parameter values
  - Somewhat compatible empirical evidence
    - Stock prices reaching two-year lows,
      last week of March 2001
    - Investors above 60 were more likely to say
      stocks were overvalued (39% vs. 25%)

Pessimistic Veteran Investors?

- Claimed conditions for bubble
  require most experienced cohort to be relatively pessimistic
- How plausible?
  - My model:
    for nonnegligible set of parameter values
  - Somewhat compatible empirical evidence
    - Stock prices reaching two-year lows,
      last week of March 2001
    - Investors above 60 were more likely to say
      stocks were overvalued (39% vs. 25%)
    - No significant differences from bullish market in 1998

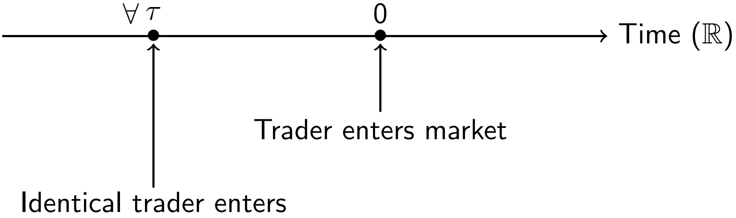
Pessimistic Veteran Investors?

- Claimed conditions for bubble
  require most experienced cohort to be relatively pessimistic

- How plausible?
  - My model:
    for nonnegligible set of parameter values
  - Somewhat compatible empirical evidence
    - Stock prices reaching two-year lows,
      last week of March 2001
    - Investors above 60 were more likely to say
      stocks were overvalued (39% vs. 25%)
    - No significant differences from bullish market in 1998
    - This is from Dreman et al. (2001)

Timing in My Model

Timing in My Model



Time 0 Entrant

Time 0 Entrant

- Believes $\underbrace{\text{asset}}$ pays \$1 dividend at uncertain time $\theta \in (0, \infty)$

Time 0 Entrant

- Believes $\underbrace{\text{asset}}$ pays \$1 dividend at uncertain time $\theta \in (0, \infty)$
  - Single asset in my model

Time 0 Entrant

- Believes $\underbrace{\text{asset}}$ pays \$1 dividend at uncertain time $\theta \in (0, \infty)$
    - Single asset in my model
    - This asset actually does not pay dividends

Time 0 Entrant

- Believes $\underbrace{\text{asset}}$ pays \$1 dividend at uncertain time $\theta \in (0, \infty)$
  - Single asset in my model
  - This asset actually does not pay dividends
  - Finite and strictly positive number of units of asset

Time 0 Entrant

- Believes $\underbrace{\text{asset}}$ pays \$1 dividend at uncertain time $\theta \in (0, \infty)$
  - Single asset in my model
  - This asset actually does not pay dividends
  - Finite and strictly positive number of units of asset
- Beliefs about dividend time θ are Gamma (α, β)

Time 0 Entrant

- Believes $\underbrace{\text{asset}}$ pays \$1 dividend at uncertain time $\theta \in (0, \infty)$
  - Single asset in my model
  - This asset actually does not pay dividends
  - Finite and strictly positive number of units of asset
- Beliefs about dividend time θ are Gamma (α, β)
  - Density $f_{\alpha,\beta}$ defined by

$$f_{\alpha,\beta}(\theta) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)}\theta^{\alpha-1}e^{-\frac{\theta}{\beta}}$$

Time 0 Entrant

- Believes $\underbrace{\text{asset}}$ pays \$1 dividend at uncertain time $\theta \in (0, \infty)$
  - Single asset in my model
  - This asset actually does not pay dividends
  - Finite and strictly positive number of units of asset
- Beliefs about dividend time θ are Gamma(α, β)
  - Density $f_{\alpha,\beta}$ defined by

$$f_{\alpha,\beta}(\theta) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)}\theta^{\alpha-1}e^{-\frac{\theta}{\beta}}$$

  - Cumulative distribution $F_{\alpha,\beta}$ defined by

$$F_{\alpha,\beta}(t) = \int_0^t f_{\alpha,\beta}(\theta)\, d\theta$$

Time 0 Entrant's Valuation

Time 0 Entrant's Valuation

- Fundamental valuation measures
  willingness to pay for asset if obliged to hold it forever

Time 0 Entrant's Valuation

- Fundamental valuation measures
  willingness to pay for asset if obliged to hold it forever
- Beliefs+$\underbrace{\text{risk-free rate } r > 0}$ $\rightarrow$ fundamental valuation of asset

Time 0 Entrant's Valuation

- Fundamental valuation measures
  willingness to pay for asset if obliged to hold it forever
- Beliefs+$\underbrace{\text{risk-free rate } r > 0}$ $\rightarrow$ fundamental valuation of asset
  - Constant over time

Time 0 Entrant's Valuation

- Fundamental valuation measures
  willingness to pay for asset if obliged to hold it forever
- Beliefs$+\underbrace{\text{risk-free rate } r > 0}$ $\rightarrow$ fundamental valuation of asset
    - Constant over time
    - Same for all traders

Time 0 Entrant's Valuation

- Fundamental valuation measures
  willingness to pay for asset if obliged to hold it forever
- Beliefs+$\underbrace{\text{risk-free rate } r > 0}$ $\rightarrow$ fundamental valuation of asset
  - Constant over time
  - Same for all traders
- At every $t \in (0, \infty)$ fundamental valuation is

$$V_0(t) = \frac{1}{1 - F_{\alpha,\beta}(t)} \int_t^\infty e^{r(t-\theta)} f_{\alpha,\beta}(\theta)\, d\theta$$

Time 0 Entrant's Valuation

- Fundamental valuation measures
  willingness to pay for asset if obliged to hold it forever
- Beliefs+$\underbrace{\text{risk-free rate } r > 0}$ $\rightarrow$ fundamental valuation of asset
  - Constant over time
  - Same for all traders
- At every $t \in (0, \infty)$ fundamental valuation is

$$V_0(t) = \frac{1}{1 - F_{\alpha,\beta}(t)} \int_t^{\infty} e^{r(t-\theta)} f_{\alpha,\beta}(\theta)\, d\theta$$

- At 0 fundamental valuation is

$$V_0(0) = \lim_{t \to 0^+} V_0(t) = (\beta r + 1)^{-\alpha}$$

Time τ Entrant

Time τ Entrant

- Enters after past entrants observe that asset does not pay at τ

Time τ Entrant

- Enters after past entrants observe that asset does not pay at τ

- Believes that asset pays dividend at uncertain time $\theta \in (\tau, \infty)$

Time τ Entrant

- Enters after past entrants observe that asset does not pay at τ

- Believes that asset pays dividend at uncertain time $\theta \in (\tau, \infty)$

- Fundamental valuation at every $t \in [\tau, \infty)$ is

$$V_\tau(t) = V_0(t - \tau)$$

Market Exit

Market Exit

- All traders stay in market for fixed duration $T \in (0, \infty)$

Market Exit

- All traders stay in market for fixed duration $T \in (0, \infty)$

- Time τ entrant leaves at $\tau + T$ after new trader enters

Market Exit

- All traders stay in market for fixed duration $T \in (0, \infty)$
- Time τ entrant leaves at $\tau + T$ after new trader enters
- Traders do not know for how long they will live

Market Exit

- All traders stay in market for fixed duration $T \in (0, \infty)$
- Time τ entrant leaves at $\tau + T$ after new trader enters
- Traders do not know for how long they will live
- Traders can plan to hold asset forever

Trade and Prices

Trade and Prices

- Trade occurs with period $\Delta \in (0, T)$ at time points $\Delta\mathbb{Z}$

Trade and Prices

- Trade occurs with period $\Delta \in (0, T)$ at time points $\Delta\mathbb{Z}$
    - Strictly after new traders enter

Trade and Prices

- Trade occurs with period $\Delta \in (0, T)$ at time points $\Delta\mathbb{Z}$
    - Strictly after new traders enter
    - Strictly before old traders leave

Trade and Prices

- Trade occurs with period $\Delta \in (0, T)$ at time points $\Delta\mathbb{Z}$
  - Strictly after new traders enter
  - Strictly before old traders leave

- Steady-state price $p \in \mathbb{R}$

Trade and Prices

- Trade occurs with period $\Delta \in (0, T)$ at time points $\Delta\mathbb{Z}$
  - Strictly after new traders enter
  - Strictly before old traders leave

- Steady-state price $p \in \mathbb{R}$

- Limited short-selling

Trade and Prices

- Trade occurs with period $\Delta \in (0, T)$ at time points $\Delta\mathbb{Z}$
    - Strictly after new traders enter
    - Strictly before old traders leave

- Steady-state price $p \in \mathbb{R}$

- Limited short-selling

- Every trader's demand for asset
  when trader participates in market
  is simple function of trader's reservation prices

Trade and Prices

- Trade occurs with period $\Delta \in (0, T)$ at time points $\Delta\mathbb{Z}$
  - Strictly after new traders enter
  - Strictly before old traders leave

- Steady-state price $p \in \mathbb{R}$

- Limited short-selling

- Every trader's demand for asset when trader participates in market is simple function of trader's $\underbrace{\text{reservation prices}}$
  - Expected discounted returns of holding asset

Expected Discounted Returns

Expected Discounted Returns

- Time 0 entrant

Expected Discounted Returns

- Time 0 entrant
  - At every $t \in (0, \infty)$,
    for holding asset to arbitrary $s \in (t, \infty)$:

$$
\begin{aligned}
&R_0(t,s) \\
&= \frac{1}{1 - F_{\alpha,\beta}(t)} \int_t^s e^{r(t-\theta)} f_{\alpha,\beta}(\theta)\, d\theta + \frac{1 - F_{\alpha,\beta}(s)}{1 - F_{\alpha,\beta}(t)} e^{r(t-s)} p \\
&= V_0(t) + \frac{1 - F_{\alpha,\beta}(s)}{1 - F_{\alpha,\beta}(t)} e^{r(t-s)} (p - V_0(s))
\end{aligned}
$$

Expected Discounted Returns

- Time 0 entrant
  - At every $t \in (0, \infty)$,
    for holding asset to arbitrary $s \in (t, \infty)$:

    $$\begin{aligned} & R_0(t,s) \\ & = \frac{1}{1 - F_{\alpha,\beta}(t)} \int_t^s e^{r(t-\theta)} f_{\alpha,\beta}(\theta)\, d\theta + \frac{1 - F_{\alpha,\beta}(s)}{1 - F_{\alpha,\beta}(t)} e^{r(t-s)} p \\ & = V_0(t) + \frac{1 - F_{\alpha,\beta}(s)}{1 - F_{\alpha,\beta}(t)} e^{r(t-s)} (p - V_0(s)) \end{aligned}$$

  - At 0, for holding asset to arbitrary $s \in (0, \infty)$:

    $$R_0(0,s) = V_0(0) + (1 - F_{\alpha,\beta}(s))\, e^{-rs} (p - V_0(s))$$

Expected Discounted Returns

- Time 0 entrant
  - At every $t \in (0, \infty)$,
    for holding asset to arbitrary $s \in (t, \infty)$:

$$
\begin{aligned}
& R_0(t, s) \\
& = \frac{1}{1 - F_{\alpha,\beta}(t)} \int_t^s e^{r(t-\theta)} f_{\alpha,\beta}(\theta)\, d\theta + \frac{1 - F_{\alpha,\beta}(s)}{1 - F_{\alpha,\beta}(t)} e^{r(t-s)} p \\
& = V_0(t) + \frac{1 - F_{\alpha,\beta}(s)}{1 - F_{\alpha,\beta}(t)} e^{r(t-s)} (p - V_0(s))
\end{aligned}
$$

  - At 0, for holding asset to arbitrary $s \in (0, \infty)$:

$$
R_0(0, s) = V_0(0) + (1 - F_{\alpha,\beta}(s))\, e^{-rs} (p - V_0(s))
$$

- Time τ entrant

Expected Discounted Returns

- Time 0 entrant
  - At every $t \in (0, \infty)$,
    for holding asset to arbitrary $s \in (t, \infty)$:

    $$\begin{aligned} & R_0(t, s) \\ & = \frac{1}{1 - F_{\alpha,\beta}(t)} \int_t^s e^{r(t-\theta)} f_{\alpha,\beta}(\theta)\, d\theta + \frac{1 - F_{\alpha,\beta}(s)}{1 - F_{\alpha,\beta}(t)} e^{r(t-s)} p \\ & = V_0(t) + \frac{1 - F_{\alpha,\beta}(s)}{1 - F_{\alpha,\beta}(t)} e^{r(t-s)} (p - V_0(s)) \end{aligned}$$

  - At 0, for holding asset to arbitrary $s \in (0, \infty)$:

    $$R_0(0, s) = V_0(0) + (1 - F_{\alpha,\beta}(s))\, e^{-rs} (p - V_0(s))$$

- Time τ entrant
  - At every $t \in [\tau, \infty)$, for holding to arbitrary $s \in (t, \infty)$:

    $$R_\tau(t, s) = R_0(t - \tau, s - \tau)$$

Reservation Prices

Reservation Prices

- Reservation prices,
  for every time τ entrant, at every $t \in [\tau, \tau + T) \cap (\Delta\mathbb{Z})$

Reservation Prices

- Reservation prices,
  for every time τ entrant, at every $t \in [\tau, \tau + T) \cap (\Delta\mathbb{Z})$
  - For holding asset to arbitrary $s \in (t, \infty) \cap (\Delta\mathbb{Z})$:

$$P_\tau(t, s) = R_\tau(t, s)$$

Reservation Prices

- Reservation prices,
  for every time τ entrant, at every $t \in [\tau, \tau + T) \cap (\Delta\mathbb{Z})$
  - For holding asset to arbitrary $s \in (t, \infty) \cap (\Delta\mathbb{Z})$:

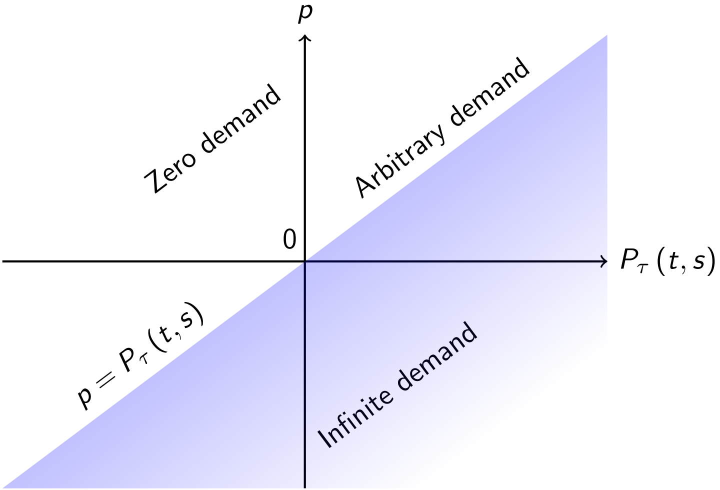
$$P_\tau(t, s) = R_\tau(t, s)$$

  - For holding asset forever:

$$P_\tau(t, \infty) = V_\tau(t)$$

Demand

Demand



- Demand by time τ entrant at t for holding asset to s
- $s \in ((t,\infty) \cap (\Delta\mathbb{Z})) \cup \{\infty\}$

Harrison-Kreps Equilibrium Price

Harrison-Kreps Equilibrium Price

- p clears market at $t \in \Delta\mathbb{Z} \iff p$ is max of

$$\overbrace{\left\{P_\tau\left(t,s\right) : \tau \in \left[t-T,t\right], s \in \left(\left(t,\infty\right) \cap \left(\Delta\mathbb{Z}\right)\right) \cup \left\{\infty\right\}\right\}}^{\text{reservation prices}}$$
$$= \left\{P_\tau\left(0,s\right) : \tau \in \left[-T,0\right], s \in \left(\left(0,\infty\right) \cap \left(\Delta\mathbb{Z}\right)\right) \cup \left\{\infty\right\}\right\}$$

Harrison-Kreps Equilibrium Price

- p clears market at $t \in \Delta\mathbb{Z} \iff p$ is max of

$$\overbrace{\{P_\tau(t,s) : \tau \in [t-T, t], s \in ((t,\infty) \cap (\Delta\mathbb{Z})) \cup \{\infty\}\}}^{\text{reservation prices}}$$
$$= \{P_\tau(0,s) : \tau \in [-T, 0], s \in ((0,\infty) \cap (\Delta\mathbb{Z})) \cup \{\infty\}\}$$

- Due to above identity,
  p clears market at $0 \iff p$ clears market at all $t \in \Delta\mathbb{Z}$

Harrison-Kreps Equilibrium Price

- p clears market at $t \in \Delta\mathbb{Z} \iff p$ is max of

$$\overbrace{\{P_\tau(t,s) : \tau \in [t-T,t], s \in ((t,\infty) \cap (\Delta\mathbb{Z})) \cup \{\infty\}\}}^{\text{reservation prices}}$$
$$= \{P_\tau(0,s) : \tau \in [-T,0], s \in ((0,\infty) \cap (\Delta\mathbb{Z})) \cup \{\infty\}\}$$

- Due to above identity,
  p clears market at $0 \iff p$ clears market at all $t \in \Delta\mathbb{Z}$

- Any such price is called *Harrison-Kreps equilibrim price*

Optimism as Function of Experience

Optimism as Function of Experience

Proposition

Shape of valuation function $V_0 : [0, \infty) \to \mathbb{R}$ depends on parameter α as follows:

- *If $\alpha < 1$, then V_0 is strictly decreasing*
- *If $\alpha = 1$, then V_0 is constant*
- *If $\alpha > 1$, then V_0 is strictly increasing*

Optimism as Function of Experience

Proposition

Shape of valuation function $V_0 : [0, \infty) \to \mathbb{R}$ *depends on parameter* α *as follows:*

- *If* $\alpha < 1$, *then* V_0 *is strictly decreasing*
- *If* $\alpha = 1$, *then* V_0 *is constant*
- *If* $\alpha > 1$, *then* V_0 *is strictly increasing*

Table: Relative optimistm of most experienced trader as function of α

α	Most experienced trader is	Claim that bubble occurs
< 1	least optimistic	yes
$= 1$	as optimistic as everyone else	no
> 1	most optimistic	no

Claimed conditions for bubble hold in my model $\iff \alpha < 1$

Sufficiency Formally

Proposition (BUBBLE)

If $\overbrace{\alpha < 1}^{\textit{sufficient condition}}$ *and p is Harrison-Kreps equilibrium price, then* $\underbrace{p > V_0(t), \forall t \geq 0}_{\textit{BUBBLE}}$

Existence and Properties of Equilibrium

Existence and Properties of Equilibrium

Proposition

If $\alpha < 1$*,*
then unique Harrison-Kreps equilibrium price is

$$P(0,\Delta) = \frac{V_0(0) - (1 - F_{\alpha,\beta}(\Delta))\, e^{-r\Delta} V_0(\Delta)}{1 - (1 - F_{\alpha,\beta}(\Delta))\, e^{-r\Delta}}$$

In this equilibrium,
at every trading time point $t \in \Delta\mathbb{Z}$ *only time* t *entrant holds asset*

Existence and Properties of Equilibrium

Proposition

If $\alpha < 1$,
then unique Harrison-Kreps equilibrium price is

$$P(0,\Delta) = \frac{V_0(0) - (1 - F_{\alpha,\beta}(\Delta))\, e^{-r\Delta} V_0(\Delta)}{1 - (1 - F_{\alpha,\beta}(\Delta))\, e^{-r\Delta}}$$

In this equilibrium,
at every trading time point $t \in \Delta\mathbb{Z}$ only time t entrant holds asset

Proposition

Mapping $\Delta \mapsto P(0,\Delta)$ on $(0,\infty)$ into $\mathbb{R}$ is strictly decreasing
—i.e., bubble is increasing in trading frequency

Case without Bubble

Case without Bubble

Proposition

If $\alpha = 1$*,*
then unique Harrison-Kreps equilibrium price is $V_0(0)$
In this equilibrium,
at every trading time point $t \in \Delta\mathbb{Z}$ *any trader can hold asset*

Case without Bubble

Proposition

If $\alpha = 1$,
then unique Harrison-Kreps equilibrium price is $V_0(0)$
In this equilibrium,
at every trading time point $t \in \Delta\mathbb{Z}$ *any trader can hold asset*

Proposition

If $\alpha > 1$,
then unique Harrison-Kreps equilibrium price is $V_0(T)$
In this equilibrium,
at every trading time $t \in \Delta\mathbb{Z}$ *only time* $t - T$ *entrant holds asset*

Take-away

Take-away

- Relatively optimistic inexperienced investors are prey for relatively pessimistic veteran traders

Take-away

- Relatively optimistic inexperienced investors are prey for relatively pessimistic veteran traders
- Limited short-selling enables success of this perpetual scheme

Take-away

- Relatively optimistic inexperienced investors are prey for relatively pessimistic veteran traders

- Limited short-selling enables success of this perpetual scheme

- Result is perpetual bubble

Take-away

- Relatively optimistic inexperienced investors are prey for relatively pessimistic veteran traders
- Limited short-selling enables success of this perpetual scheme
- Result is perpetual bubble
- My paper gives formal proofs of this intuitive conjecture in simple model