Optimal auction duration: price formation point of view.

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Market models: from Limit Order Book...

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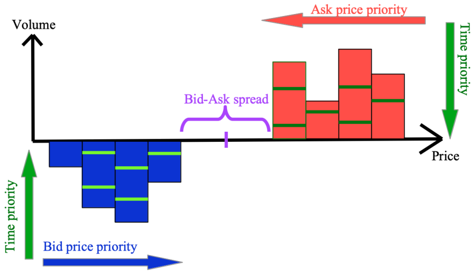

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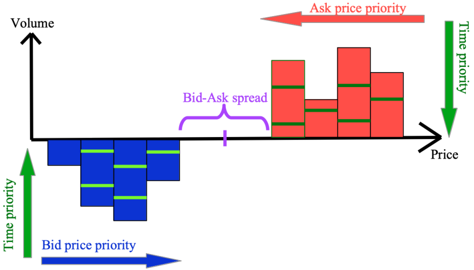

$\hookrightarrow$ Benefits by increasing the bid/ask spread and by being the faster.

- Market takers send market orders. Their arrival intensities are very sensitive w.r.t. the spread (see e.g. Madhavan, Richardson, and Roomans (1997), Wyart, Bouchaud, Kockelkoren, Potters, and Vettorazzo (2008), Dayri and Rosenbaum (2015)).

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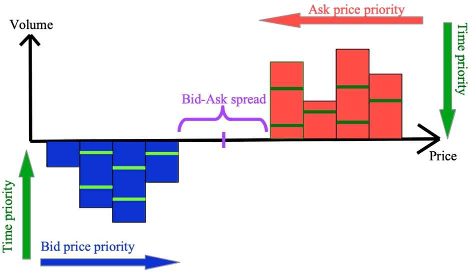

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- Exchanges aims at increasing the liquidity (make-take fees system).

... to batch auctions markets

$\hookrightarrow$ Concerning the electronic order book mechanism and to quote Farmer and Skouras, 1992:

"One consequence of this trading mechanism which we believe is underappreciated is that it endows a huge advantage to being faster than other traders, creating evolutionary pressures that drive an arms race for ever-more speed."

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- Suitable remedies: batch auctions.

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Some references

- Budish, Cramton, Shim. (2015): auction market reduces arbitrage opportunities systematically present in a CLOB market.

$\hookrightarrow$ No optimal duration investigation.

- Du and Zhu (2014): efficiency of an auction market with respect to the auction duration.

$\hookrightarrow$ Only for homogeneous agents.

- Garbade and Silber (1979): macroscopic model for price formation in auction markets.

  Fricke and Gerig (2018): extensions to multi assets or the presence of liquidity providers.

$\hookrightarrow$ No comparison with the CLOB.

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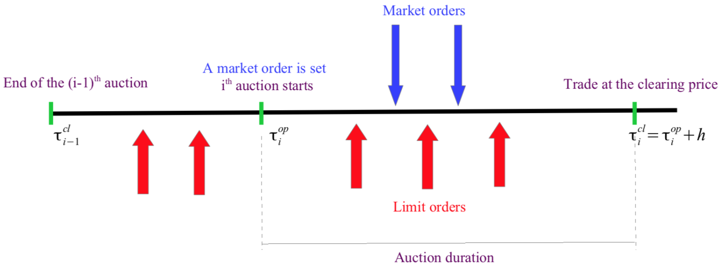
- After the clearing of the ith auction, the exchange opens the auction at time τ^{op} as soon as a market order is set during a duration h so that the clearing time is $\tau^{cl} = \tau^{op} + h$.

$\hookrightarrow$ It determines the clearing price ensuring to match the most orders.

In a nutshell



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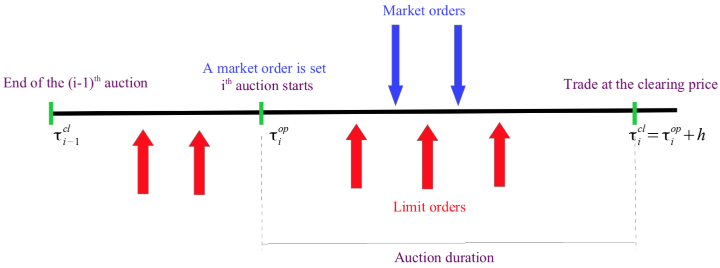

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- For each auction i: $(N^{i,a}, N^{i,b})$ and $N^{i,mm}$ are independent of P.
- After each auction clearing the market regenerates.

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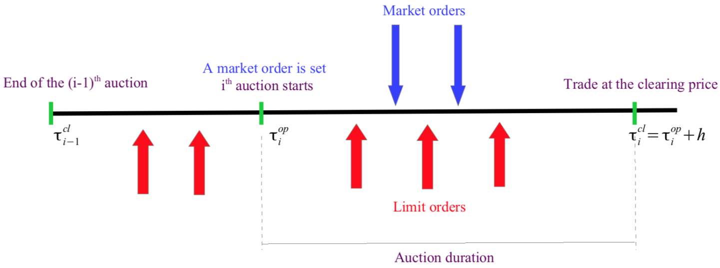

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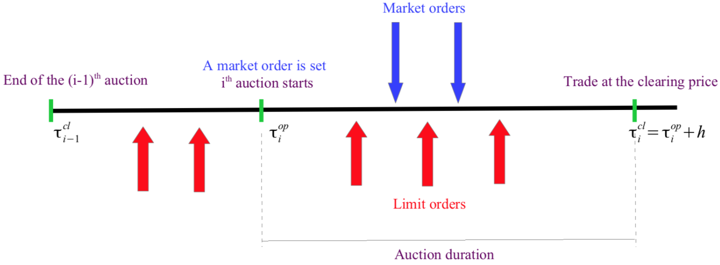

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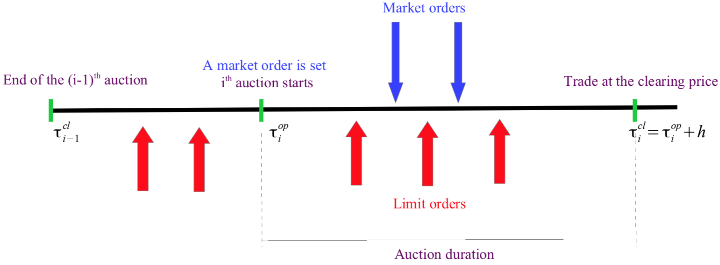

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- Clearing price sets to ensure the larger number of transactions.
- What is the optimal duration h? Comparison with the LOB ($h = 0$).

Clearing rule

The clearing price $P^{cl}_{\tau^{cl}_i}$ of the ith auction is determined as an equilibrium between supply and demand curves. Hence, P^{cl} is solution to

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Thus:

$$P^{cl}_{\tau^{cl}_i} = \underbrace{\frac{1}{N^{mm}_{\tau^{cl}_i} - N^{mm}_{\tau^{cl}_{i-1}}} \sum_{k=N^{mm}_{\tau^{cl}_{i-1}}+1}^{N^{mm}_{\tau^{cl}_i}} \tilde{P}_k}_{=:P^{mid}_{\tau^{cl}_i}} + \frac{1}{K} \frac{I_{\tau^{cl}_i} - I_{\tau^{op-}_i}}{N^{mm}_{\tau^{cl}_i} - N^{mm}_{\tau^{cl}_{i-1}}}.$$

Optimal duration

$$P^{cl}_{\tau_i^{cl}} = \frac{1}{N^{mm}_{\tau_i^{op}+h} - N^{mm}_{\tau^{cl}_{i-\mathbf{1}}}} \sum_{k=N^{mm}_{\tau^{cl}_{i-\mathbf{1}}}+1}^{N^{mm}_{\tau_i^{op}+h}} \tilde{P}_k + \frac{1}{K} \frac{I_{\tau_i^{op}+h} - I_{\tau_i^{op}-}}{N^{mm}_{\tau_i^{op}+h} - N^{mm}_{\tau^{cl}_{i-\mathbf{1}}}}.$$

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Criterion to measure the fairness of the transaction price:
Quadratic error between the clearing price and the fundamental price:

$$"Z_t^h = \int_0^t (P_s^{cl} - P_s)^2 \mathrm{d}s."$$

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Lemma

$$\lim_{t\to+\infty} \frac{Z^h_t}{t} = \mathbb{E}[(P^{cl}_{\tau^{op}_1+h} - P_{\tau^{op}_1+h})^2].$$

A duration $h^\star$ is optimal when it is a minimizer of the function

$$h \longmapsto E(h) = \mathbb{E}[(P^{cl}_{\tau^{op}_1+h} - P_{\tau^{op}_1+h})^2].$$

Optimal duration

We get

$$E(h) = E^{mid}(h) + \frac{\mathbb{E}[I^2_{\tau_1^{op}+h}]}{K^2} \frac{e^{\nu h}}{1 - e^{-\mu h} \frac{\nu}{\nu+\mu}} e^{\nu h} \int_h^{+\infty} \nu e^{-\nu t} e^{-\mu t} \int_0^{\mu t} \frac{1}{s} \int_0^s \frac{e^u - 1}{u} \mathrm{d}u \mathrm{d}s \mathrm{d}t.$$

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- $E(h)$ depends on the market takers' behaviours during the auction only through $\mathbb{E}[I^2_{\tau_1^{op}+h}]$.
- Until here, we have considers "unsophisticated" market takers having a fixed intensity ν of arrival between AND during the auction.

  $\hookrightarrow \mathbb{E}[I^2_{\tau_1^{op}+h}] = v^2(\nu h + \frac{1}{2})$.

  So that, we can compute $h^\star$ for "unsophisticated" market takers.

Comparison with "sophisticated" market takers

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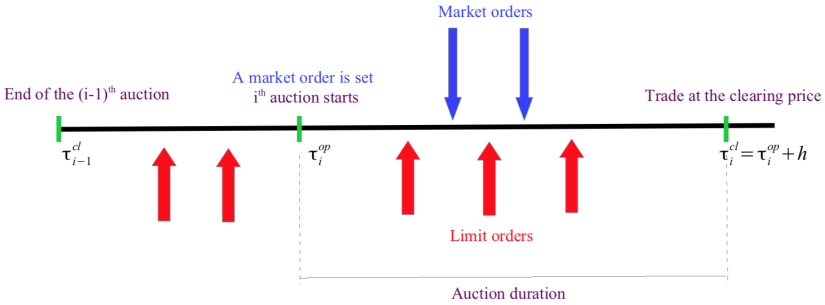

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During the auction: MTs optimize their trading costs w.r.t. their arrival intensities.

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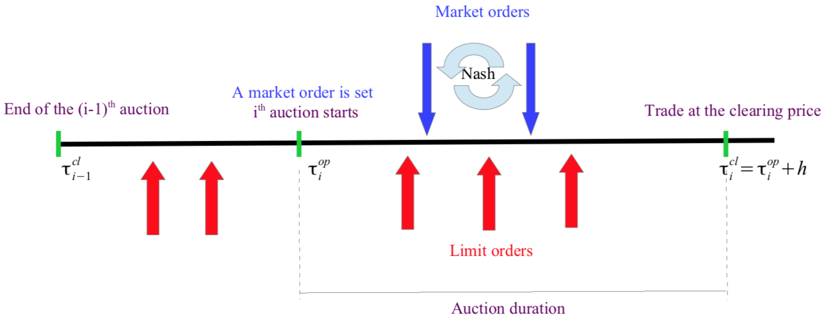

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Focus on the inbalance process

N_t^{cl} is the number of auctions cleared before t.
The aggregated total trading cost of buyers market takers at time t can be written:

$$C_t^a = \sum_{i=1}^{N_t^{cl}} (N^a_{\tau_i^{cl}} - N^a_{\tau_{i-1}^{cl}})(P^{cl}_{\tau_i^{cl}} - P_{\tau_i^{cl}}),$$

and similarly for sellers

$$C_t^b = \sum_{i=1}^{N_t^{cl}} (N^b_{\tau_i^{cl}} - N^b_{\tau_{i-1}^{cl}})(P_{\tau_i^{cl}} - P^{cl}_{\tau_i^{cl}}).$$

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Proposition

Long term error

$$\lim_{t\to+\infty} \frac{C_t^a}{t} \quad \propto \quad \mathbb{E}[N_h^a(N_h^a - N_h^b)].$$

$$\lim_{t\to+\infty} \frac{C_t^b}{t} \quad \propto \quad \mathbb{E}[N_h^b(N_h^b - N_h^a)].$$

Trading costs optimization of "sophisticated" market takers

Sophisticated market takers during the auction:

- Buyers market takers arrive with intensity λ^a
- Sellers market takers arrive with intensity λ^b

$\hookrightarrow$ induces a family of measures $\mathbb{P}^{a,b}$ with bounds $[\lambda_-, \lambda_+]$.

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Market takers aims at optimizing simultaneously

$$\textbf{(Nash)} \begin{cases} \sup_{\lambda^a \in [\lambda_-, \lambda_+]} \mathbb{E}^{\mathbb{P}^{\lambda_a, \lambda_b}}[N_h^a(N_h^a - N_h^b)] \\ \sup_{\lambda^b \in [\lambda_-, \lambda_+]} \mathbb{E}^{\mathbb{P}^{\lambda_a, \lambda_b}}[N_h^b(N_h^b - N_h^a)] \end{cases}$$

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Verification procedure: it reminds to solve a a bang-bang type integro-differential system of PDEs with Hamiltonian:

$$H^{a,\star}(z, \tilde{z}, \varepsilon) = z_1 \lambda_a^\star(z) + z_2 \lambda_b^\star(\tilde{z}, \varepsilon),\; H^{b,\star}(z, \tilde{z}, \varepsilon) = z_2 \lambda_b^\star(z) + z_1 \lambda_a^\star(\tilde{z}, \varepsilon),$$

where

$$\textbf{(foc)} \begin{cases} \lambda_a^\star(z, \varepsilon_a) & = \mathbf{1}_{z_1 > 0}\lambda_- + \mathbf{1}_{z_1 < 0}\lambda_+ + \varepsilon_a \mathbf{1}_{z_1 = 0} \\ \lambda_b^\star(z, \varepsilon_b) & = \mathbf{1}_{z_2 > 0}\lambda_- + \mathbf{1}_{z_2 < 0}\lambda_+ + \varepsilon_b \mathbf{1}_{z_2 = 0}. \end{cases}$$

Finding Nash equilibrium...

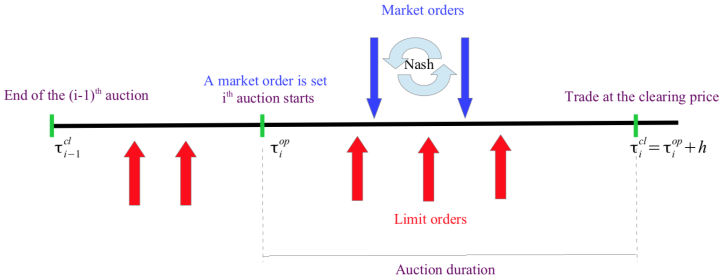
We mollify the functions $\lambda_a^\star$ and $\lambda_b^\star$ by introducing

$$\lambda^n(z) = \begin{cases} \lambda_+ & \text{if } z \leqslant -\frac{1}{n} \\ \lambda_- & \text{if } z \geqslant \frac{1}{n} \\ n\frac{\lambda_- - \lambda_+}{2} z + \frac{\lambda_+ + \lambda_-}{2} & \text{if } z \in (-\frac{1}{n}, \frac{1}{n}). \end{cases}$$

so that there exists a solution to a (Lipschitz) system of BSDE which gives a sequence $(\varepsilon_a^n, \varepsilon_b^n)$ of (Markovian) processes converging to a Nash equilibrium.

- We extend Hamadène and Mu (2014) to the Poisson case by using BSDE theory.
- The proof is constructive and gives a way to approach numerically this value.

Market takers and Nash equilibrium



Proposition

There exists a Nash equilibrium for the simultaneous optimization problem given by some Markovian controls $(\lambda_a^\star, \lambda_b^\star)$ *and so that*

$$\mathbb{E}[I^2_{\tau_1^{op}+h}] = V_h^a(\lambda_a^\star, \lambda_b^\star) + V_h^b(\lambda_a^\star, \lambda_b^\star),$$

where V^a *and* V^b *are the values of* (***Nash***).

Calibration model parameters on Euronext

Unknown variables

- MTs' arrival intensities ν between two auctions.
- MMs' arrivals given by μ,
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Euronext Data: details on the volumes of each stocks exchanged along the day.

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$\hookrightarrow$ $\nu = \frac{V}{T}$ where T: duration of the trading day, V: average daily volume of market orders.

$\hookrightarrow$ $K = (2e - \frac{\varsigma}{e})$ and $\mu = \nu(\frac{e}{K} - 1)$, where e and ς are the average and squared volumes (resp.) present in the first limit of the LOB.

For sophisticated MTs we take $\lambda_- = \nu/4$ and $\lambda_+ = \nu$.

Application to some stocks traded at Euronext exchange

	DiffNash	DiffPoiss	Poisson	Nash
Aperam	0.389	0.159	536	375.1
Sodexo	0.072	0.0	0.0	30
Air France - KLM	0.821	0.452	295	218
Hermes	0.296	0.02	295	202
ArcelorMittal	0.465	0.126	87	59
Credit Agricole	0.398	0.048	88	57
LVMH	0.497	0.155	121	87
Valeo	0.326	0.0	0	97
Air Liquide	0.916	0.647	627	459

Table: Optimal auction durations of a sample of stocks traded at Euronext (in seconds) and comparison with the LOB $E(0) - E(h^*)$.

Thank you.