

The Economics of Asset Securitization

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Introduction

- Asset-backed securitization is a major form of financial innovation in the last few decades
 - Over \$2 trillions issued each year in the US; last year, the issuance in China exceeded \$ 300 billion, with the YoY growth rate of 36%
 - Subprime mortgage-backed securities caused big financial institutions in the US to fail and led to the global financial crisis in 2008
- A number of theoretical studies to understand the tranche structure of asset-backed securities
- Empirical investigation uncovers some regularities that are hard to reconcile, some pointing to potential fraud

The Model

• The Set-up

- A discrete-time economy with $t = 0, 1, 2, \dots, T$, represented by $(\Omega, \mathcal{F}_t, t = 0, 1, \dots, T)$
- M non-tradable assets generating cash flow, $x_{i,t}$, $i = 1, \dots, M$
- A risk-neutral issuer attempting to issue securities based on the cash flow from the pool of M assets
- $N + 1$ types of mean-variance risk-averse investors interested in each of the $N + 1$ securities (tranches)

• Underlying Assets

- Each asset has a cash flow $x_t = f_t - y_t$, where f_t is the face value, y_t is the realized loss of value due to default, etc.
- The cash flow from the pool of M assets: $x_t \equiv \sum_i x_{i,t} = F_t - Y_t$, where $F_t \equiv \sum_i f_{i,t}$, and $Y_t \equiv \sum_i y_{i,t}$

Securitization Structure

- Total face value F and $N + 1$ tranches that are determined by N detachment points $0 < k_1 < k_2 < \dots < k_N < 1$
- Equity tranche payoff: $f_1(x) \equiv \max\{x - (1 - k_1)F, 0\}$
- Senior tranche: $f_{N+1}(x) \equiv \min\{x, k_N F\}$
- $f_i(x) = \max\{x - (1 - k_i)F, 0\} - \max\{x - (1 - k_{i-1})F, 0\}$ for all other mezzanine tranches for $i = 3, \dots, N$
- Each tranche has one unit, with issuance cost borne by the issuer:
 $C(n) = bn + b_0$ for each tranche

Investors

- $N + 1$ -types of investors, all are price-taker and mean-variance maximizers in each time period
- Each type of investors only invest in one particular tranche
- Risk aversion parameter for investors in tranche j is γ_j , with $\gamma_1 < \dots < \gamma_{N+1}$

Belief Structure

- The value of the underlying pool is written as $x = \mu + \eta$, μ is unobservable and η is the signal that is independently and identically normally distributed with zero mean. The variance of η is σ_η^2 , and each agent correctly models the distribution of η
- All are Bayesian, with prior $\mu \sim \mathcal{N}(\alpha_0, \sigma_0^2)$ for the issuer, and $\mu \sim \mathcal{N}(\alpha_{v,0}, \sigma_{v,0}^2)$ for investors
- We assume $\alpha_{v,0} > \alpha_0$, which states that the investor imposes a higher expected value on the underlying asset pool
- At each time $t = 1, \dots, T$, the information set \mathcal{F}_t is generated by $\{x_1, x_2, \dots, x_t\}$, and each agent has the same information set \mathcal{F}_t , but different belief on the distribution of the underlying pool value
- $\bar{x}_t = \frac{x_1 + \dots + x_t}{t}$ is used to represent the asset quality at time t
- Posterior beliefs for the issuer and the investors have conditional means α_t and $\alpha_{v,t}$, respectively, and conditional variances σ_t^2 and $\sigma_{v,t}^2$, respectively

Optimal Problems

- The optimal problem for investors in tranche j is

$$\max_{0 \leq \Phi_{t,j} \leq 1} \left\{ \mathbb{E}_{v,t} [W_{t+1,j}] - \frac{\gamma_j}{2} \text{Var}_{v,t} [W_{t+1,j}] \right\} \quad (1)$$

where $W_{t+1,j} = W_{t,j} + \Phi_{t,j} (f_j(x_{t+1}) - p_{t,j})$ is the terminal wealth of the investor at time $t+1$ and $W_{t,j}$ denotes the initial wealth at time t

- The optimal problem for the Issuer is

$$\max_{\{p_{t,j}, j=1, \dots, N+1, 0 < k_1 < \dots < k_N < 1\}} \mathbb{E}_t \left[x_{t+1} + \sum_{j=1}^{N+1} (\Phi_{t,j} (p_{t,j} - f_j(x_{t+1})) - C(\Phi_{t,j})) \right], \quad (2)$$

where $p_{t,j} - f_j(x_{t+1})$ is the profit and loss derived from one unit of tranche j and $C(\Phi_{t,j})$ is the issuance cost on the tranche j for the issuer

Characterization of the Equilibrium

Proposition 1: For each tranche security $j = 1, \dots, N + 1$, there exists unique equilibrium as follows.

- (*Full issuance equilibrium*) When

$\mathbb{E}_{v,t}[f_j(x_{t+1})] \geq \mathbb{E}_t[f_j(x_{t+1})] + b + 2\gamma_j \text{Var}_{v,t}[f_j(x_{t+1})]$, the equilibrium volume of the tranche j is 1, and its price per unit is

$$p_{t,j}^* = \mathbb{E}_{v,t}[f_j(x_{t+1})] - \gamma_j \text{Var}_{v,t}[f_j(x_{t+1})]. \quad (4)$$

- (*Retention equilibrium*) When

$0 < \mathbb{E}_{v,t}[f_j(x_{t+1})] - \mathbb{E}_t[f_j(x_{t+1})] - b < 2\gamma_j \text{Var}_{v,t}[f_j(x_{t+1})]$, the equilibrium volume of the tranche j is

$$\Phi_{t,e}^* = \frac{1}{2\gamma_j} \frac{\mathbb{E}_{v,t}[f_j(x_{t+1})] - \mathbb{E}_t[f_j(x_{t+1})] - b}{\text{Var}_{v,t}[f_j(x_{t+1})]}, \quad (5)$$

and its price per unit is

$$p_{t,j}^* = \frac{\mathbb{E}_{v,t}[f_j(x_{t+1})] + \mathbb{E}_t[f_j(x_{t+1})] + b}{2}. \quad (6)$$

Characterization of the Equilibrium

Proposition 1 (cont.):

- (*No equilibrium*) When $\mathbb{E}_{v,t}[f_j(x_{t+1})] \leq \mathbb{E}_t[f_j(x_{t+1})] + b$, then the equilibrium volume of the tranche j is zero, that is, there is no trading volume on the tranche j .

In particular, the price per unit is positive in either retention or full issuance equilibrium.

Optimal Tranche Structure

- The expected utility of the issuer is a sum of α_t and

$$H(k_1, \dots, k_N) = \sum_{j=1}^{N+1} \Phi_{t,j}^* (p_{t,j}^* - \mathbb{E}_t[f_j(x_{t+1})] - b) \quad (7)$$

- The issuer's optimal tranche design problem is to solve

$$\max_{0 < k_1 < \dots < k_N < 1} H(k_1, \dots, k_N). \quad (8)$$

Two-tranche Problem: Equity Tranche

- Proposition 2:** $\Phi_{t,e}$ is positive if and only if the asset quality is high in the sense that \bar{x}_t is strictly *larger* than a specific number \bar{x} .
 Equivalently, the conditional default probability of the equity tranche for the issuer is strictly *smaller* than a specific probability θ if and only if $\Phi_{t,e}$ is positive.
- There exists market demand for the equity tranche in equilibrium if and only if the conditional default probability,
 $Prob\{x_{t+1} \leq (1 - k_1)F | \mathcal{F}_t\} < \theta$
- Or

$$\bar{x}_t > \frac{(1 - k_1)F - \sqrt{\sigma_t^2 + \sigma_\eta^2} N^{-1}(\theta)}{\alpha_0 g(\sigma_0)} \quad (9)$$

- If the asset quality \bar{x}_t is strong enough, there exists full issuance of the equity tranche

Two-tranche Problem: Senior Tranche

- Proposition 3:** There exists market demand for the senior tranche if and only if the conditional survival probability of the senior tranche for the issuer is strictly *smaller* than a specific probability ψ .
 Equivalently, $\Phi_{s,t}$ is positive if and only if the asset quality is low in the sense that \bar{x}_t is strictly smaller than a specific number \hat{x} .
- This is equivalent to

$$\bar{x}_t < \frac{F - \alpha_0(1 - g(\alpha_0)) + \sqrt{\sigma_t^2 + \sigma_\eta^2} N^{-1}(\psi)}{g(\sigma_0)} \quad (10)$$

- A rationale for the high yield on senior tranches
- Retention puzzle for senior tranches: Issuers tend to retain senior tranches when underlying asset quality is high

Two-tranche Problem: Optimal Detachment Point

- **Proposition 4:** In a two-tranche asset securitization, \mathcal{K} is a range of the detachment point k_1 such that both the equity tranche and the senior tranche are fully issued in the market. Then the optimal detachment point $k_1(\bar{x}_t) \in \mathcal{K}$ is *negatively* related to the asset quality.
- The optimizing problem is equivalent to

$$\max_k Cov_t((x_{t+1} - (1 - k)F)^+, x_{t+1} - (x_{t+1} - (1 - k)F)^+) \quad (11)$$

- The main insight of Proposition 4 is the *decreasing* equity size with respect to the asset quality in an optimal tranche structure

Three-tranche Problem: The Mezzanine Tranche

- **Proposition 5:** If either $\bar{x}_t > \frac{\alpha^* - \alpha_0(1-g(\sigma_0))}{g(\sigma_0)}$ or $\bar{x}_t < \frac{\alpha^* - \alpha_{v,0}(1-g(\sigma_0))}{g(\sigma_0)}$, then there exists no market demand for the mezzanine tranche in equilibrium, where $\alpha^* = (1 - \frac{k+l}{2})F$, and $g(\sigma)$ is defined as

$$g(\sigma) \equiv \frac{\frac{t}{\sigma_\eta^2}}{\frac{1}{\sigma^2} + \frac{t}{\sigma_\eta^2}}, \forall \sigma > 0$$

- The Mezzanine tranche may only be sold or desired when the asset quality is moderate
- The retention of the mezzanine tranche is not only likely, but persistent

Three-tranche Problem: The Effect of Asset Quality

- When the quality of the underlying asset improves, both the equity tranche price and the senior tranche price increases. Moreover, the mezzanine tranche price also increases with respect to the asset quality in its retention equilibrium.
- Both the equity tranche spread and the senior tranche spread decrease when the quality of the underlying pool is improved. The mezzanine tranche spread has the same monotonic property in the retention equilibrium.

Three-tranche Problem: Optimal Detachment Point

- Define $k(\bar{x}_t) = \inf \{k : \mathbb{E}_{v,t}[f_e(x_{t+1})] - \mathbb{E}_t[f_e(x_{t+1})] - b > 0\}$
- Define $l(\bar{x}_t) = \sup \{l : \mathbb{E}_{v,t}[f_s(x_{t+1})] - \mathbb{E}_t[f_s(x_{t+1})] - b > 0\}$
- $\frac{\partial k(\bar{x}_t)}{\partial x_t} < 0, \frac{\partial l(\bar{x}_t)}{\partial x_t} < 0$
- Roughly speaking, the quality of the underlying pool has a negative relation to the equity tranche size, and a positive relation to the senior tranche size

Summary

- An equilibrium model to characterize the instances when issuance of each tranche of ABS could go fully sold, partially retained or completely unsold
- The model offers a way to obtain optimal detachment points in the tranche structure, and allows characterization of the effect of asset quality on optimal tranche design
- The model shows that retaining the senior tranche is optimal for the issuer when asset quality is high, while retaining the equity tranche is optimal when asset quality is low, consistent with empirical evidence
- The model also shows that asset quality is negatively related to the size of the equity tranche, positively related to the size of the senior tranche
- The results offer a different perspective for the high-yield and retention puzzles for the senior tranche as observed in the data