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On the preservation of very large cardinals under class forcing

JOAN BAGARIA

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ABSTRACT

We will present a very general result, obtained in collaboration work with A. Poveda, on the preservation of extendible cardinals, and beyond, under class forcing iterations. We will also give several applications.

Uniform Martin’s conjecture, locally

VITTORIO BARD

Università Degli Studi Di Torino, Italy

ABSTRACT

We present a proof of the following fact: given Turing degrees x and y , there exists a non-constant UTI function $f : x \rightarrow y$ if and only if $x \leq_T y$. We show that Slaman and Steel’s theorem that part I of Martin’s conjecture holds for UTI functions can be derived (using just Turing determinacy) from this fact. Hence, part I of uniform Martin’s conjecture follows from a “local” phenomenon. We thus raise the question whether part II of uniform Martin’s conjecture follows from local phenomena, too. We remark that a positive answer to this question would probably be of metamathematical interest.

Complexity of maximal objects

JOERG BRENDLE

Kobe University, Japan

ABSTRACT

Definability of families of sets of natural numbers that are maximal in some sense, like ultrafilters, towers, maximal almost disjoint families (mad families) or maximal independent families, has been investigated intensively in the past few years. We give a survey on results in the area, with focus on mad families and maximal independent families, sketch proofs of some recent results and discuss a couple of open problems.

Algebraic properties of elementary embeddings

SCOTT CRAMER

California State University, San Bernardino , USA

ABSTRACT

We will investigate the algebraic structures created by rank-into-rank elementary embeddings. Our starting point will be R. Laver's theorem that any rank-into-rank embedding generates a free left-distributive algebra on one generator. We will consider extensions of this and related results. One surprising result is the existence of a 'purely algebraic' rank-into-rank elementary embedding. Our results will also lead to some surprisingly coherent conjectures on the algebraic structure of rank-into-rank embeddings in general.

A completeness criterion for Borel sets

ADAM DAY

Victoria University of Wellington, New Zealand

ABSTRACT

I will present a criterion for establishing whether a set is complete for various levels of the Borel hierarchy. I will also present an application which makes use of a theorem of Harrington about enumerations of Borel sets of a fixed complexity. (Joint work with Andrew Marks).

Foundations of online model theory

ROD DOWNEY

Victoria University of Wellington, New Zealand

ABSTRACT

I will talk about work seeking to give a foundation to the idea of online algorithms. This involves either lack of delay, or uniformity.

Joint work with Melnikov and Ng

Decision problems in Borel combinatorics

SU GAO

University of North Texas, USA

ABSTRACT

Let $F(2^{\mathbb{Z}^2})$ be the Cayley graph on the free part of the Beroulli shift on $2^{\mathbb{Z}^2}$. I will talk about several decision problems regarding the Borel and continuous combinatorics of $F(2^{\mathbb{Z}^2})$. Specifically, we consider the Subshift Problem, the Tiling Problem, and the Graph Homomorphism Problem. As an example, the Graph Homomorphism Problem for $F(2^{\mathbb{Z}^2})$ asks for which finite graphs Γ there exist a continuous or Borel graph homomorphism from $F(2^{\mathbb{Z}^2})$ to Γ . We report some results on the continuous versions of these problems. We will demonstrate a theorem known as the Twelve Tiles Theorem that completely characterizes the positive answers to these problems. Using the Twelve Tiles Theorem, we will be able to show, for instance, that the set of (codes for) all finite graphs Γ such that there exist continuous graph homomorphisms from $F(2^{\mathbb{Z}^2})$ to Γ is a Σ_1^0 -complete set. This is joint work with Steve Jackson, Ed Krohne, and Brandon Seward.

Extending filters to ultrafilters

MOTI GITIK

Tel Aviv University, Israel

ABSTRACT

We will discuss possibilities of extending κ -complete filters over κ to κ -complete ultrafilters, for a regular $\kappa > \aleph_0$. A special attention will be given to filters related to forcing notions. Most of the results are joint with Tom Benhamou and Yair Hayut.

True stages and hyperarithmetical isomorphisms

NOAM GREENBERG

Victoria University of Wellington, New Zealand

ABSTRACT

The existence of a hyperarithmetical isomorphism is complete for Π_1^1 equivalence relations. To show this, we use Montalban's α -true stage machinery. We give a direct development of this machinery. Work with Dan Turetsky.

The implicitly constructible universe

MARCIA GROSZEK

Dartmouth College, USA

ABSTRACT

The implicitly constructible universe, \mathbf{Imp} , is a model of set theory intermediate between \mathbf{L} and \mathbf{HOD} . Hamkins and Leahy (2016) defined \mathbf{Imp} , showed that \mathbf{Imp} can be separated from \mathbf{L} (both $\mathbf{L} = \mathbf{Imp} \neq V$ and $\mathbf{L} \neq \mathbf{Imp}$ are possible), and made further key observations, in particular about $\mathbf{Imp}^{M[G]}$ where $M[G]$ is a generic extension of M . Groszek and Hamkins (to appear) showed that \mathbf{Imp} can be separated from \mathbf{HOD} , that the continuum hypothesis can fail in \mathbf{Imp} , and that \mathbf{Imp} is not absolute ($\mathbf{Imp}^{\mathbf{Imp}} \neq \mathbf{Imp}$ is possible). We discuss these and further questions about \mathbf{Imp} .

Definable unions of smooth equivalence relations

STEPHEN JACKSON

University of North Texas, USA

ABSTRACT

We consider sets in models of determinacy which can be represented as unions of smooth equivalence relations. We show in natural models of determinacy that when the classes are countable, such sets are isomorphic to the product of the reals with an ordinal, and so for example are Jonsson. We characterize when subsets of $(\omega_1)^{<\omega_1}$ have such representations.

BQO-Wadge degrees on nonseparable ultrametric spaces and computability on uncountable cardinals

TAKAYUKI KIHARA

Nagoya University, Japan

ABSTRACT

Recently, Kihara and Montalban have given a complete classification of the BQO-valued Borel maps on Baire space (indeed, any separable ultrametric space) in terms of the Wadge degrees. Roughly speaking, the authors have shown that any BQO-valued Borel map on Baire space is generated by (suitable iterations of) very simple construction principles, and is completely classified by detecting how it is constructed.

In this talk, we discuss an extension of the theory of Wadge degrees on BQO-valued maps to κ^ω for any uncountable cardinal κ . We apply suitable variants of computability theory over arbitrary uncountable cardinals to generalize Kihara-Montalban's result of the BQO-valued Borel maps to the uncountable setting.

Permutations of the integers, automorphisms of the Turing degrees, and arithmetically presentable groups

BJØRN KJOS-HANSEN

University of Hawai‘i at Mānoa, USA

ABSTRACT

Slaman and Woodin showed that the automorphism group of the Turing degrees $\text{Aut}(\mathcal{D})$ is countable, and in fact each automorphism is represented by an arithmetic function. We use statistics and algorithmic randomness to show that the SW result cannot be strengthened to representation by a permutation of the integers, unless $\text{Aut}(\mathcal{D})$ is trivial. We also use analyze the SW results to show that $\text{Aut}(\mathcal{D})$ is contained in a Δ_{19}^0 -presentable group. This yields an example of a countable group G which we can prove is not automorphic to $\text{Aut}(\mathcal{D})$, answering a question of Schweber.

Tutorial on AD+

PAUL LARSON

Miami University, USA

ABSTRACT

We will give an overview of Woodin's initial work on AD+ from the 1990's. We will start with the definitions and statements of the basic theorems and open problems, and then proceed to the details. Some prerequisites from the Cabal seminar will also be reviewed. Time permitting, we will spend approximately one lecture apiece on infinity-Borel sets, the Vopenka algebra and deriving real determinacy from AD plus the assumption that all sets of reals are Suslin.

Are all singular cardinals born equal? The case of \aleph_ω and \aleph_{ω^2}

MENACHEM MAGIDOR

The Hebrew University of Jerusalem, Israel

ABSTRACT

\aleph_ω and \aleph_{ω^2} looks rather similar: they are both singular cardinals of cofinality ω , but a series of old and new results indicate that there may be fundamental difference in their properties or properties of their successors $\aleph_{\omega+1}$ and \aleph_{ω^2+1} .

We shall give few examples and list some open problems. Typical problems: suppose that λ is a singular cardinal. Is the tree property at λ^+ consistent with every stationary subset of λ^+ reflecting? Is the tree property at λ^+ preserved under forcing of cardinality $< \lambda$?

Relativizing to an enumeration oracle

JOE MILLER

University of Wisconsin–Madison, USA

ABSTRACT

It is natural to relativize essentially c.e. notions to an enumeration degree. We will consider several examples. For example, a Σ_1^0 class is generated by a c.e. set of strings. To relativize such a class to an enumeration oracle A , we would simply require that the class be generated by a set of strings that is enumeration reducible to A . This immediately allows us to relativize other notions, such as 1-genericity, Π_1^0 classes, and PA degrees. While some of the expected properties are preserved by relativizing to an enumeration oracle, others are not. This leads to questions about which enumeration degrees behave well under relativization. For example, it turns out that the enumeration degree of A is continuous (i.e., the degree of a point in a computable metric space) if and only if every PA degree relative to A (as an enumeration oracle) is above A in the enumeration degrees. We will survey such results.

SRT_2^2 vs RT_2^2 in ω -models

BENOÎT MONIN

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ABSTRACT

We present a new forcing notion that we use to show the following core theorem: For any Δ_2^0 set A , A or $\omega - A$ contains an infinite subset whose jump is not $PA(\emptyset')$. This result can be used to separate SRT_2^2 from RT_2^2 in omega-models. The forcing notion consists in two successive modification of computable Mathias forcing : A first modification allows to controls the truth of Σ_2^0 statements. A second modification leaves the possibility to obtain sufficiently generic non-cohesive sets, via the use of multiple reservoirs. This is joint work with Ludovic Patey.

Hyperarithmetical sets and computable structure theory

ANTONIO MONTALBAN

University of California, Berkeley, USA

ABSTRACT

We will start with an introduction to hyperarithmetical sets. We will end with some applications on computable structure theory.

On Σ -preorderings over the reals

ANDREY MOROZOV

Sobolev Institute of Mathematics, Novosibirsk State University, Russia

ABSTRACT

We study preorderings Σ -presentable with parameters in the hereditarily finite superstructure over the reals, $\text{HF}(\mathbb{R})$. The main result is that any such preordering does not contain an isomorphic copy of ω_1 . From this we obtain characterizations of ordinals and Gödel constructive sets that admit such Σ -presentations. We also prove that the semilattices of some popular reducibilities (including Turing reducibility) fail to have such presentations.

Effective metric Scott analysis

ANDRÉ NIES

University of Auckland, New Zealand

ABSTRACT

The Scott rank of a countable structure is a countable ordinal that measures the complexity of deciding whether two tuples of the same length are automorphic. We study analogues of Scott rank for Polish metric structures. There are two possible settings:

1. view the metric structure as a classical structure with distance relations $R_q(x, y) < \text{---} > d(x, y) < q$, for each rational $q > 0$, and rely on the usual infinitary logic;
2. view the metric structure as a continuous structure (where the relations are real-valued), and use infinitary continuous logic.

In the setting of (1), with Turetsky we give a recursion theoretic proof of the result of Doucha that the rank of a pair of tuples of the same length is countable: we show in fact that this rank is an ordinal recursive in the pair of tuples, assuming the structure is computable. It remains open whether the overall Scott rank of a Polish metric structure in this setting is countable; this question was asked by Sy Friedman, Fokina, Koerwien and Nies in 2012.

In the setting (2) we show that the continuous Scott rank is countable (Metric Scott Analysis, with Ben Yaacov, Doucha, and Tsankov, *Adv. in Maths* 318, 2017). This follows from our continuous analogue of the Lopez Escobar theorem. In this talk we also provide an effective view of the argument.

Measurable Hall's theorem for actions of abelian groups

MARCIN SABOK

McGill University, Canada

ABSTRACT

I will discuss a measurable version of the Hall marriage theorem for actions of abelian groups. In particular, it implies that for free measure-preserving actions of such groups, if two equidistributed measurable sets are equidecomposable, then they are equidecomposable using measurable pieces. The latter generalizes a recent result of Grabowski, Máthé and Pikhurko on the measurable circle squaring and confirms a special case of a conjecture of Gardner. This is joint work with Tomasz Cieśla.

MM^{++} implies (*)

RALF SCHINDLER

Universität Münster, Germany

ABSTRACT

We show that Martin's Maximum⁺⁺ implies Woodin's P_{max} axiom (*). This is joint work with David Asperó.

Randomness notions for infinite time Turing machines

PHILIP SCHLICHT

University of Bristol, UK

ABSTRACT

I will give an overview of result about ITTM-randomness and related notions.

Uniform reductions in computable structure theory

NOAH SCHWEBER

University of Wisconsin–Madison, USA

ABSTRACT

There are two natural ways to compare the computability-theoretic complexity of countable structures by looking at explicit copies (i.e. copies with domain ω) of those structures: every copy of one computes a copy of the other, and copies of one **uniformly** compute copies of the other. I'll talk about the latter, which is much less understood - in particular, I'll focus on the behavior of the ordinals with respect to this reducibility (whereas their nonuniform behavior is totally understood). Most of the results I'll present use set-theoretic forcing and absoluteness, and some interesting questions of a set-theoretic flavor emerge.

Σ_1^1 in every real in a Σ_1^1 class or reals is Σ_1^1

RICHARD SHORE

Cornell University, USA

ABSTRACT

We first prove a theorem about reals (subsets of \mathbb{N}) and classes of reals: If a real X is Σ_1^1 in every member G of a nonempty Σ_1^1 class \mathcal{K} of reals then X is itself Σ_1^1 . We then explore the relationship between this theorem, various basis results in hyperarithmetic theory and omitting types theorems in ω - and other logics. The analog of our first theorem also holds for classes of reals: If a class \mathcal{A} of reals is Σ_1^1 in every member of a nonempty Σ_1^1 class \mathcal{B} of reals then \mathcal{A} is itself Σ_1^1 .

Joint work with Leo Harrington and Ted Slaman.

Turing degrees of hyperjumps

STEPHEN G. SIMPSON

Vanderbilt University, USA

ABSTRACT

This talk is about ongoing work with my Ph.D. student Hayden Jananthan. It is well known that the hyperjump of a hyperdegree is well defined up to recursive isomorphism, hence well defined up to Turing degree. In a 1977 paper, John MacIntyre proved that every Turing degree above the Turing degree of Kleene's \mathcal{O} is the Turing degree of the hyperjump of some hyperdegree. In some partly unpublished work, Slaman and Woodin have proved hyperdegree analogs of the 1981 theorem of Posner and Robinson. We obtain some improvements of these results. For example, the hyperdegree in MacIntyre's result can be chosen to be minimal. The following question seems to be open. Does there exist a real X such that the hyperjump of X is of the same hyperdegree, but not the same Turing degree, as the join of X with Kleene's \mathcal{O} ?

Finding randomness

THEODORE A. SLAMAN

University of California, Berkeley, USA

ABSTRACT

We will discuss a family of inverse problems of the form “For which reals x does there exist a probability measure m of type T such that x is random in the sense of m ?” In this lecture, we will focus on continuous measures. In joint work with Jan Reimann, we show that randomness relative to continuous measures is related to the fine structure of Goedel’s L , the universe of constructible sets.

Finding better randomness

THEODORE A. SLAMAN

University of California, Berkeley, USA

ABSTRACT

We will discuss randomness relative to measures with the mass distribution property, which by work of Reimann is related to effective Hausdorff dimension, and also randomness relative to measures with well-behaved Fourier transform, which is related to properties of uniform distributions and the theory of Diophantine Approximation.

Polishable equivalence relations

SLAWOMIR SOLECKI

Cornell University, USA

ABSTRACT

We will define a class of equivalence relations called Polishable equivalence relations, which lies between the class of orbit equivalence relations of Polish group actions and the class of idealistic equivalence relations of Kechris and Louveau. We will present a Scott analysis for such equivalence relations. We will compare this analysis with the Scott analysis for isomorphism equivalence relations from continuous model theory and with versions of the Scott analysis for (certain) orbit equivalence relations of Polish group actions.

Coding in graphs and linear orderings

ALEXANDRA SOSKOVA

Sofia University, Bulgaria

ABSTRACT

Friedman and Stanley [2] introduced Borel embeddings as a way of comparing classification problems for different classes of structures. A Borel embedding of a class K in a class K' represents a uniform procedure for coding structures from K in structures from K' . Many Borel embeddings are actually Turing computable [1].

When a structure \mathcal{A} is coded in a structure \mathcal{B} , effective decoding is represented by a Medvedev reduction of \mathcal{A} to \mathcal{B} . Harrison-Trainor, Melnikov, Miller, and Montalbán [3] defined a notion of effective interpretation of \mathcal{A} in \mathcal{B} and proved that this is equivalent with the existing of computable functor, i.e. a pair of Turing operators, one taking copies of \mathcal{B} to copies of \mathcal{A} , and the other taking isomorphisms between copies of \mathcal{B} to isomorphisms between the corresponding copies of \mathcal{A} . The first operator is a Medvedev reduction. For some Turing computable embeddings Φ , there are *uniform* formulas that effectively interpret the input structure in the output structure, so we get a uniform Medvedev reduction.

The class of undirected graphs and the class of linear orderings both lie on top under Turing computable embeddings. The standard Turing computable embeddings of directed graphs (or structures for an arbitrary computable relational language) in undirected graphs come with uniform effective interpretations. We give examples of graphs that are not Medvedev reducible to any linear ordering, or to the jump of any linear ordering. Any graph can be interpreted in a linear ordering using computable Σ_3 formulas. Friedman and Stanley [2] gave a Turing computable embedding L of directed graphs in linear orderings. We show that there do not exist $L_{\omega_1\omega}$ -formulas that uniformly interpret the input graph G in the output linear ordering $L(G)$.

This is joint work of J. Knight, A. Soskova and S. Vatev and will be presented by A. Soskova.

References

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- [2] H. Friedman and L. Stanley, “A Borel reducibility theory for classes of countable structures,” *JSL*, vol. 54(1989), pp. 894-914.
- [3] M. Harrison-Trainor, A. Melnikov, R. Miller, and A. Montalbán, “Computable functors and effective interpretability” *JSL*, vol. 82(2017), pp. 77-97

The enumeration degrees: an overview

MARIYA SOSKVA

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ABSTRACT

Enumeration reducibility captures a natural relationship between sets of natural numbers in which positive information about the first set is used to produce positive information about the second set. By identifying sets that are reducible to each other we obtain an algebraic representation of this reducibility as a degree structure. The structure of the enumeration degrees has been the focus of active research in recent years. Motivation for the interest in this area comes from its nontrivial connections to the study of the Turing degrees, specifically with respect to longstanding open problems, such as the rigidity problem, and to problems from effective mathematics, where Turing reducibility is sometimes not sufficient to measure the effective content of an object. I will give an overview of this research. I will focus on work done by Ted Slaman and Hugh Woodin in this area, as well consequences of this work.

Mouse pairs and Suslin cardinals

JOHN STEEL

University of California, Berkeley, USA

ABSTRACT

A *mouse pair* is a pair (P, Σ) such that P is a premouse and Σ is an iteration strategy for P having a certain condensation property. The basic theorems of inner model theory (e.g. the Comparison Lemma and the Dodd-Jensen Lemma) are best stated as theorems about mouse pairs.

One type of mouse pair can be used to analyze the HODs of models of $\text{AD}_{\mathbb{R}}$, leading to

Theorem 0.1 *Assume $\text{AD}_{\mathbb{R}} + \text{HPC}$; then $\text{HOD} \models \text{GCH}$.*

Here HPC stands for “hod pair capturing”, a natural assumption concerning the existence of mouse pairs.

An analysis of optimal Suslin representations for mouse pairs leads to

Theorem 0.2 *Assume $\text{AD}_{\mathbb{R}} + \text{HPC}$; then the following are equivalent:*

- (a) δ is Woodin in HOD , and a cutpoint of the extender sequence of HOD ,
- (b) $\delta = \theta_0$, or $\delta = \theta_{\alpha+1}$ for some α .

Here the θ_α 's are the *Solovay sequence*, that is, θ_0 is the sup of the lengths of the ordinal definable prewellorderings of \mathbb{R} , $\theta_{\alpha+1}$ is the sup of the lengths of prewellorderings of \mathbb{R} ordinal definable from some set of Wadge rank θ_α , and $\theta_\lambda = \sup_{\alpha < \lambda} \theta_\alpha$ for λ a limit.

Grigor Sargsyan introduced a refinement of the Solovay sequence in which ordinal definability from sets of reals is replaced by ordinal definability from countable sequences of ordinals. Calling these ordinals η_α , we have

Theorem 0.3 *Assume $\text{AD}_{\mathbb{R}} + \text{HPC}$; then the following are equivalent*

- (a) δ is a successor Woodin in HOD ,
- (b) $\delta = \eta_0$, or $\delta = \eta_{\alpha+1}$ for some α .

This theorem was conjectured by Sargsyan.

Finally, we have the following conjecture

Conjecture. Assume $\text{AD}_{\mathbb{R}} + \text{HPC}$; then the following are equivalent

- (a) κ is a Suslin cardinal,
- (b) κ is a cardinal of V , and a cutpoint of the extender sequence of HOD.

We can prove (b) \Rightarrow (a). With Jackson and Sargsyan, we have shown that (a) \Rightarrow (b) holds whenever κ is a limit of Suslin cardinals, or the next Suslin after a limit of Suslin cardinals.

Caristi's theorem and approximating Π_1^1 comprehension

HENRY TOWNSNER

University of Pennsylvania, USA

ABSTRACT

Caristi's fixed point theorem is a wide-reaching generalization of Banach's fixed point theorem whose usual proof requires a transfinite induction up to ω_1 . To figure out in what sense this induction is necessary to prove the theorem we investigate the reverse mathematical strength of this theorem. Coding issues suggest that we should consider the theorem for Borel functions, which invites us to consider a certain kind of approximation to Π_1^1 comprehension.

Choiceless set-theoretic geology

TOSHIMICHI USUBA

Waseda University, Japan

ABSTRACT

The set-theoretic geology, which was initiated by Fuchs-Hamkins-Reitz, is a study of the structure of all ground models of the universe. The standard geology is assuming the axiom of choice, but in the modern set theory, the forcing method over choiceless model become a common tool. In this talk, we try to develop the set-theoretic geology without the axiom of choice. We show that if the universe satisfies some downward Loewenheim-Skolem type property holds, then every ground model is uniformly definable, and such a property follows from large cardinal axioms.

Quasi-inductive definitions and generalised recursion in higher types

PHILIP WELCH

University of Bristol, UK

ABSTRACT

We find an analogue to Kleene-Recursion in Type-2 functionals as an exemplification of *elementary quasi-inductive*, as opposed to regular positive elementary inductive, *definitions*. Much of the early theory of elementary induction over Moschovakis's *acceptable structures* can be reworked to deliver a theory of elementary quasi-inductions over an abstract structure. This is in particular smoother if we assume a quasi-hyper elementary wellorder of the structure. We find characterisations corresponding to Kleene's results that the sets of integers Kleene-recursive in 2E are the hyperarithmetical ones, and that the semi-recursive sets are those that are Π_1^1 . This enables us to give some characterisations and some conjectures about the determinacy of pointclasses provable within analysis.

Determined Borel sets and measurability

LINDA BROWN WESTRICK

Pennsylvania State University, USA

ABSTRACT

We describe the effective and Reverse Math content of the statement “Every determined Borel set is measurable”, where a determined Borel set is a Borel set such that every element of the space is either in the set or in its complement.

A new basis theorem for Σ_3^1 sets

W. HUGH WOODIN

Harvard University and University of California, Berkeley, USA

ABSTRACT

This is the first lecture in a two part series on recent applications of the fine-structure of inner models to problems in descriptive set theory. The focus of this first lecture will be on the projective sets and simple generalizations. The context will be determinacy hypotheses.

Counting Woodin cardinals in HOD

W. HUGH WOODIN

Harvard University and University of California, Berkeley, USA

ABSTRACT

The final synthesis of fine-structure and determinacy will yield a number of theorems about HOD in the context of the Axiom of Determinacy. However, there are some of these expected theorems which can be proved now, before that final synthesis is achieved. We focus on one such recent theorem which concerns the relationship between the number of Woodin cardinals in HOD and the descriptive set theory of the universe within which HOD is defined.

One application shows that the axiom $V = \text{Ultimate L}$ implies the Ω Conjecture.

On the minimal size of a basis for uncountable linear order

LIUZHEN WU

Chinese Academy of Sciences, China

ABSTRACT

Moore proves that the minimal size of basis for uncountable linear order is 5 under PFA. Under suitable large cardinal assumption, for any $n < \omega_2$, we construct a ZFC model where the minimal size of a basis is $2^n + 3$. This is a joint work with Yinhe Peng.

Some applications of recursion theoretical methods to set theory

LIANG YU

Nanjing University, China

ABSTRACT

We prove some results in analysis or descriptive set theory via recursion theoretical argument.

Indiscernibles for $L[T_3]$

YIZHENG ZHU

University of Chinese Academy of Sciences, China

ABSTRACT

We define an EM-blueprint from level-3 indiscernibles for $L[T_3]$. This generalizes the EM-blueprint of $0^\#$.